Simplified ensemble-based forecast sensitivity analysis

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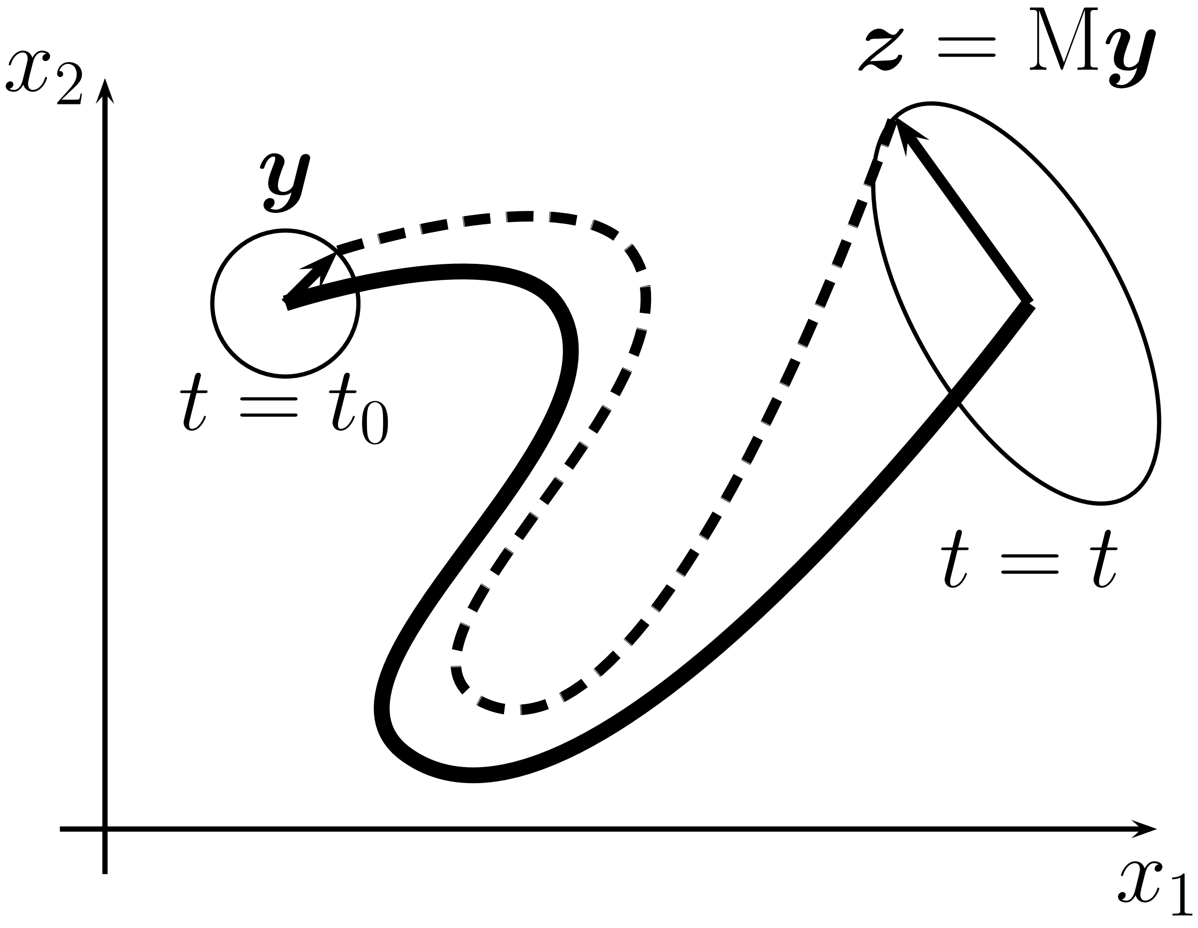
Sensitivity analysis

- conducted to find the target region for additional observation.
- finds the initial perturbation that optimally grows in the verification region at the verification time.
- has been conducted by the adjoint (Rabier et al 1996), SV (Palmer et al 1998), ensemble-based (Bishop and Toth 1999; Bishop et al 2000; Ancell and Hakim 2007) methods.



Sensitivity analysis using the adjoint method

The gradient of a cost function J is

$$\delta J = \left.\frac{\partial J}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}=\boldsymbol{x}(t)} \boldsymbol{z} = \left.\frac{\partial J}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}=\boldsymbol{x}(t)} \mathsf{M}\boldsymbol{y} \qquad (1)$$

Find the extremum δJ with $\boldsymbol{y}^{\mathsf{T}}\mathsf{G}\boldsymbol{y} = 1$ by the Lagrange multiplier method:

$$F(\boldsymbol{y}, \lambda) \equiv \delta J + \lambda(1 - \boldsymbol{y}^{\mathsf{T}}\mathsf{G}\boldsymbol{y}) \qquad (2)$$

to obtain

$$2\lambda\boldsymbol{y} = \mathsf{G}^{-1}\mathsf{M}^{\mathsf{T}} \left[\left.\frac{\partial J}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}=\boldsymbol{x}(t)} \right]^{\mathsf{T}}. \qquad (3)$$

Sensitivity analysis using the singular vectors

Maximize $||\boldsymbol{z}|| \equiv \sqrt{\boldsymbol{z}^\mathsf{T}\mathsf{H}\boldsymbol{z}}$ with $||\boldsymbol{y}|| \equiv \sqrt{\boldsymbol{y}^\mathsf{T}\mathsf{G}\boldsymbol{y}} = 1.$

The Lagrangian function

$$\begin{aligned} F(\boldsymbol{y},\lambda) &= \boldsymbol{z}^\mathsf{T}\mathsf{H}\boldsymbol{z} + \lambda(1-\boldsymbol{y}^\mathsf{T}\mathsf{G}\boldsymbol{y}) \\ &= \boldsymbol{y}^\mathsf{T}\mathsf{M}^\mathsf{T}\mathsf{H}\mathsf{M}\boldsymbol{y} + \lambda(1-\boldsymbol{y}^\mathsf{T}\mathsf{G}\boldsymbol{y}), \qquad (4) \\ \frac{\partial F(\boldsymbol{y},\lambda)}{\partial \boldsymbol{y}} &= \boldsymbol{y}^\mathsf{T}\mathsf{M}^\mathsf{T}\mathsf{H}\mathsf{M} - \lambda\boldsymbol{y}^\mathsf{T}\mathsf{G} = \mathbf{0}. \qquad (5) \end{aligned}$$

The solutions are the eigenvectors of $\mathsf{G}^{-1}\mathsf{M}^\mathsf{T}\mathsf{H}\mathsf{M}$.

Ensemble perturbations

Nonlinear time evolution of ensemble perturbations:

$$\boldsymbol{z}_i = M(\boldsymbol{y}_i), \quad i = 1, 2, \cdots, n. \tag{6}$$

Find the coefficients for a linear combination of initial perturbation

$$\boldsymbol{y} = a_1\boldsymbol{y}_1 + a_2\boldsymbol{y}_2 + \cdots + a_n\boldsymbol{y}_n. \tag{7}$$

Let

$$\mathsf{Y} = (\boldsymbol{y}_1, \boldsymbol{y}_2, \cdots, \boldsymbol{y}_n),\ \mathsf{Z} = (\boldsymbol{z}_1, \boldsymbol{z}_2, \cdots, \boldsymbol{z}_n),\ \boldsymbol{a} = (a_1, a_2, \cdots, a_n)^{\mathsf{T}}, \tag{8}$$

then

$$\boldsymbol{y} = \mathsf{Y}\boldsymbol{a},\ \boldsymbol{z} = M(\mathsf{Y})\boldsymbol{a} = \mathsf{Z}\boldsymbol{a}. \tag{9}$$

Ensemble adjoint sensitivity analysis

With ensemble forecast,

$$\delta J = \boldsymbol{\delta J}^{\mathsf{T}} \boldsymbol{a} \tag{10}$$

where $\boldsymbol{\delta J}^{\mathsf{T}} = (\delta J_1, \delta J_2, \cdots, \delta_m)$. The Lagrangian function becomes

$$F(\boldsymbol{a}, \lambda) = \boldsymbol{\delta J}^{\mathsf{T}} \boldsymbol{a} + \lambda(1 - \boldsymbol{a}^{\mathsf{T}} \mathsf{Y}^{\mathsf{T}} \mathsf{G} \mathsf{Y} \boldsymbol{a}), \tag{11}$$

$$\frac{\partial F(\boldsymbol{a}, \lambda)}{\partial \boldsymbol{a}} = \boldsymbol{\delta J} - 2\lambda \boldsymbol{a}^{\mathsf{T}} \mathsf{Y}^{\mathsf{T}} \mathsf{G} \mathsf{Y} = \mathbf{0}. \tag{12}$$

If $\mathsf{Y}^{\mathsf{T}}\mathsf{G}\mathsf{Y} \propto \mathsf{I}$, the optimal initial perturbations are

$$\boldsymbol{y} = \mathsf{Y}\boldsymbol{\delta J} = \delta J_1 \boldsymbol{y}_1 + \delta J_2 \boldsymbol{y}_2 + \cdots + \delta J_m \boldsymbol{y}_m. \tag{13}$$

Ensemble singular-vector sensitivity analysis

With ensemble forecast, the Lagrangian function becomes

$$F(\boldsymbol{a}, \lambda) = \boldsymbol{a}^{\mathsf{T}}\mathsf{Z}^{\mathsf{T}}\mathsf{H}\mathsf{Z}\boldsymbol{a} + \lambda(1 - \boldsymbol{a}^{\mathsf{T}}\mathsf{Y}^{\mathsf{T}}\mathsf{G}\mathsf{Y}\boldsymbol{a}) \tag{14}$$

and its derivative

$$\frac{\partial F(\boldsymbol{a}, \lambda)}{\partial \boldsymbol{a}} = \boldsymbol{a}^{\mathsf{T}}\mathsf{Z}^{\mathsf{T}}\mathsf{H}\mathsf{Z} - \lambda\boldsymbol{a}^{\mathsf{T}}\mathsf{Y}^{\mathsf{T}}\mathsf{G}\mathsf{Y} = \mathbf{0}. \tag{15}$$

Fields an eigenvalue problem

$$(\mathsf{Y}^{\mathsf{T}}\mathsf{G}\mathsf{Y})^{-1}\mathsf{Z}^{\mathsf{T}}\mathsf{H}\mathsf{Z}\boldsymbol{p} = \lambda\boldsymbol{p} \tag{16}$$

If $\mathsf{Y}^{\mathsf{T}}\mathsf{G}\mathsf{Y} \propto \mathsf{I}$, the fastest growing modes can be found as the eigenvectors of $\mathsf{Z}^{\mathsf{T}}\mathsf{H}\mathsf{Z}$.

Simplified ensemble-based sensitivity analysis

- does not need the tangent-linear and adjoint models or a DA system.
- expected to approach the adjoint or SV sensitivity asymptotically as $n \to \infty$.
- might be affected by ensemble size and the perturbation method.
- does not consider observations.
- may conducted with arbitrary verification regions.

Total energy norm

Here we use the total energy norm (Talagrand 1981, Ehrendorfer *et al.* 1999),

$$\mathrm{TE} = \frac{1}{2}\iint_A \quad u'^2 + v'^2 + \frac{c_p}{T_\mathrm{r}}T'^2 + RT_\mathrm{r}\left(\frac{p'_s}{p_\mathrm{r}}\right)^2 + \varepsilon\frac{L^2}{c_pT_\mathrm{r}}q'^2\,\mathrm{d}A\,\mathrm{d}p, \tag{17}$$

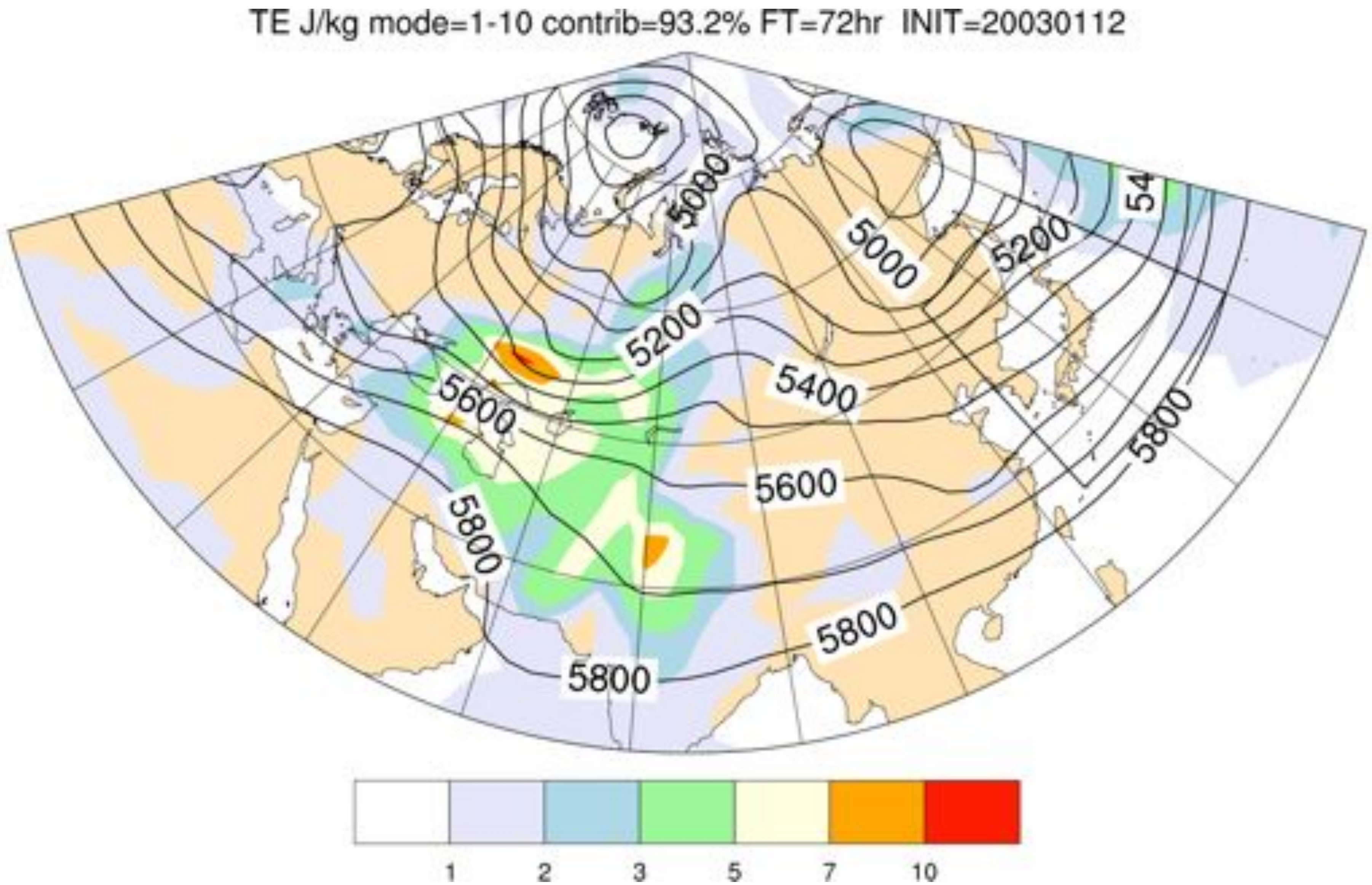
where $'$ denotes anomaly from the control run, $T_\mathrm{r} = 270$ K the reference temperature $p_\mathrm{r} = 1000$ hPa, $\varepsilon = 0$ and $\varepsilon = 1$ for the dry and moist total energy norm, respectively.

Here the dry TE $\varepsilon = 0$ is used but note that the forward model includes all physical parametrizations.

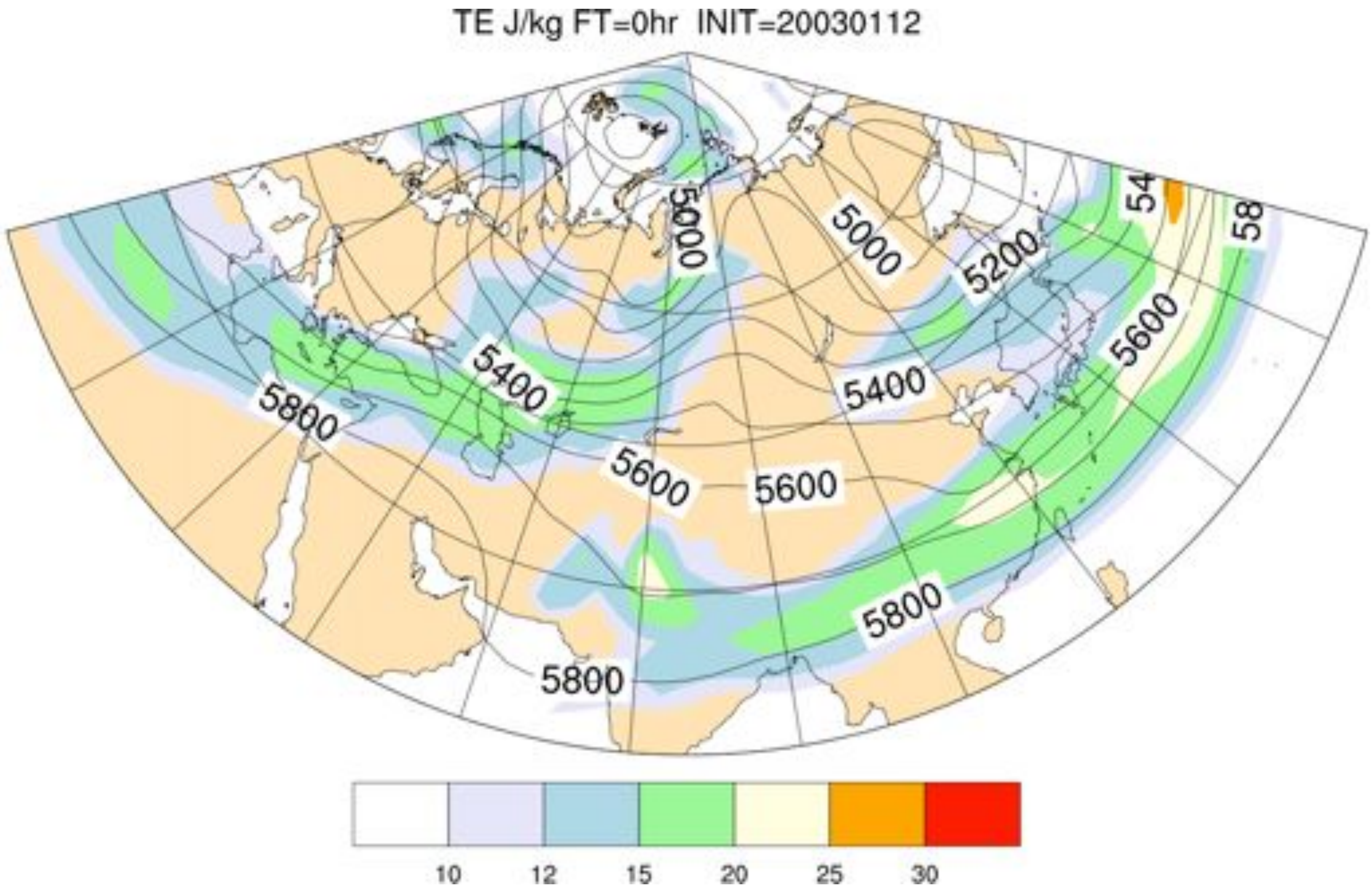
JMA weekly ensemble forecast

- Perturbations generated with the breeding method.
- T106, but interpolated to $2.5^\circ \times 2.5^\circ$grids for distribution.
- Full 25 members provided in the GRIB format.
- u, v, z, T at 300, 500, 850 hPa and RH only at 850 hPa.
- Additionally 100, 200, 700, and 1000 hPa-level data is provided for this study by courtesy of NPD/JMA.
- Also (u, v) at the surface, precip and slp.

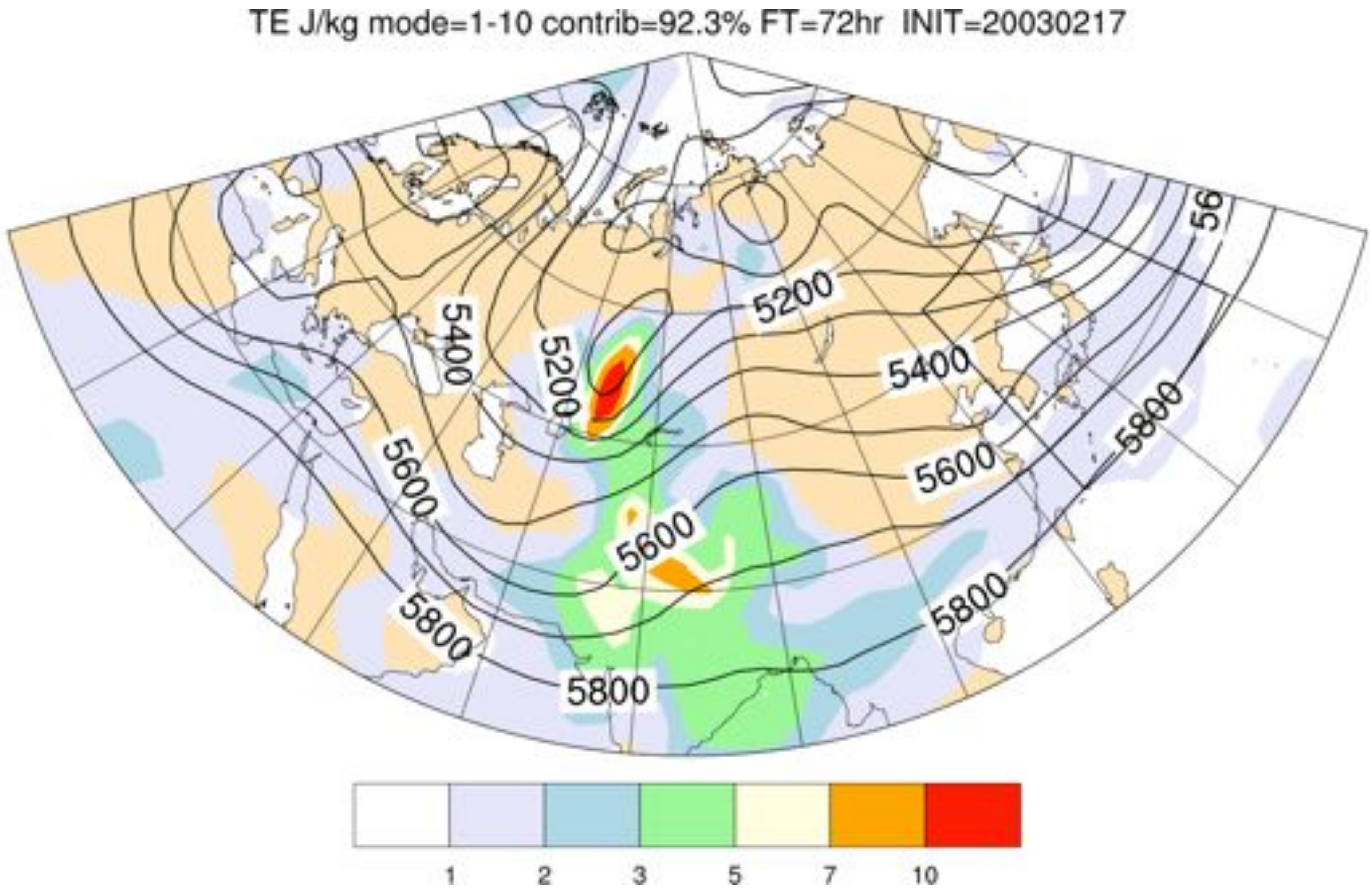
72-h sensitivity for a cold surge



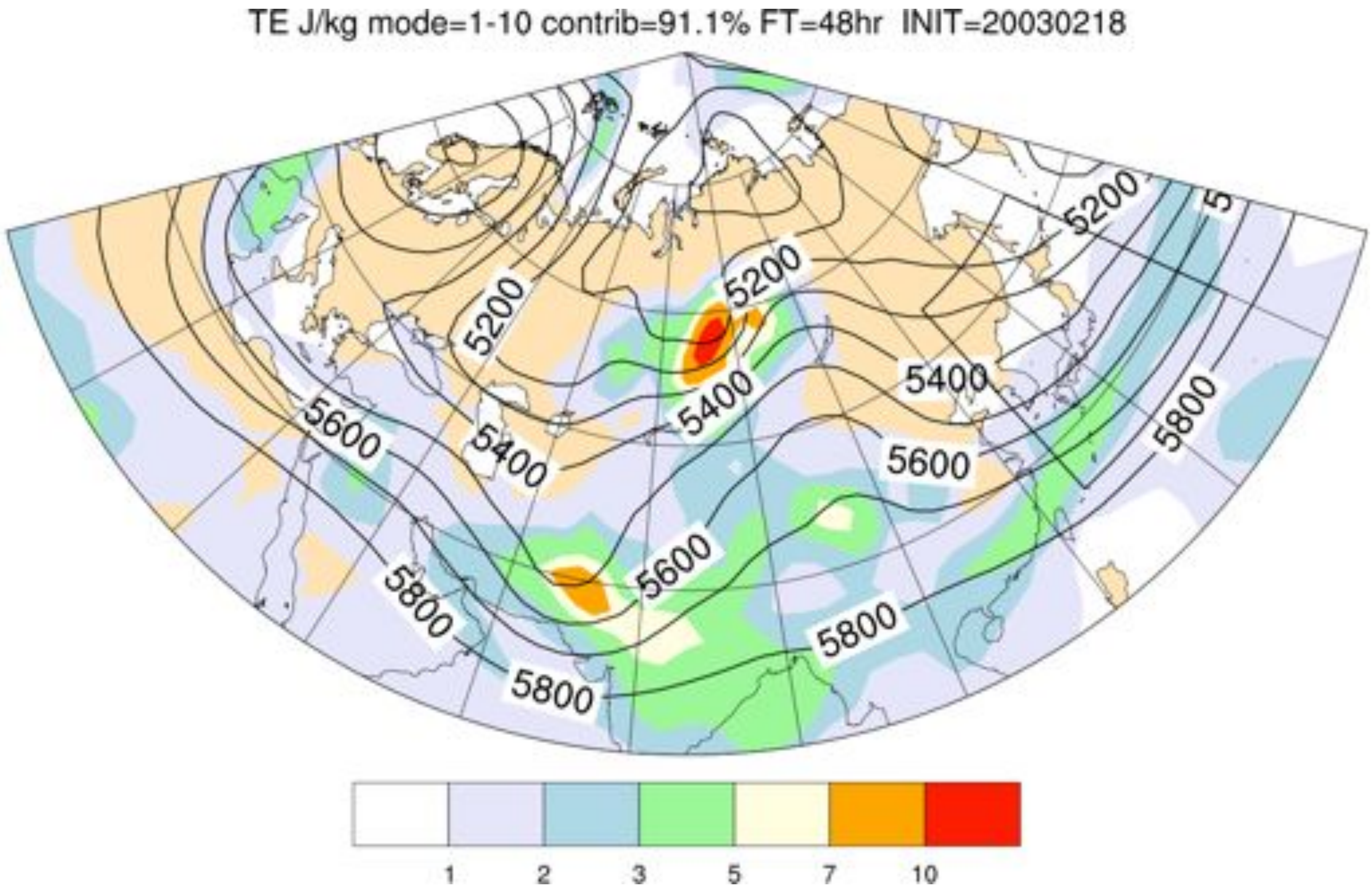
The spread of the initial total energy norm



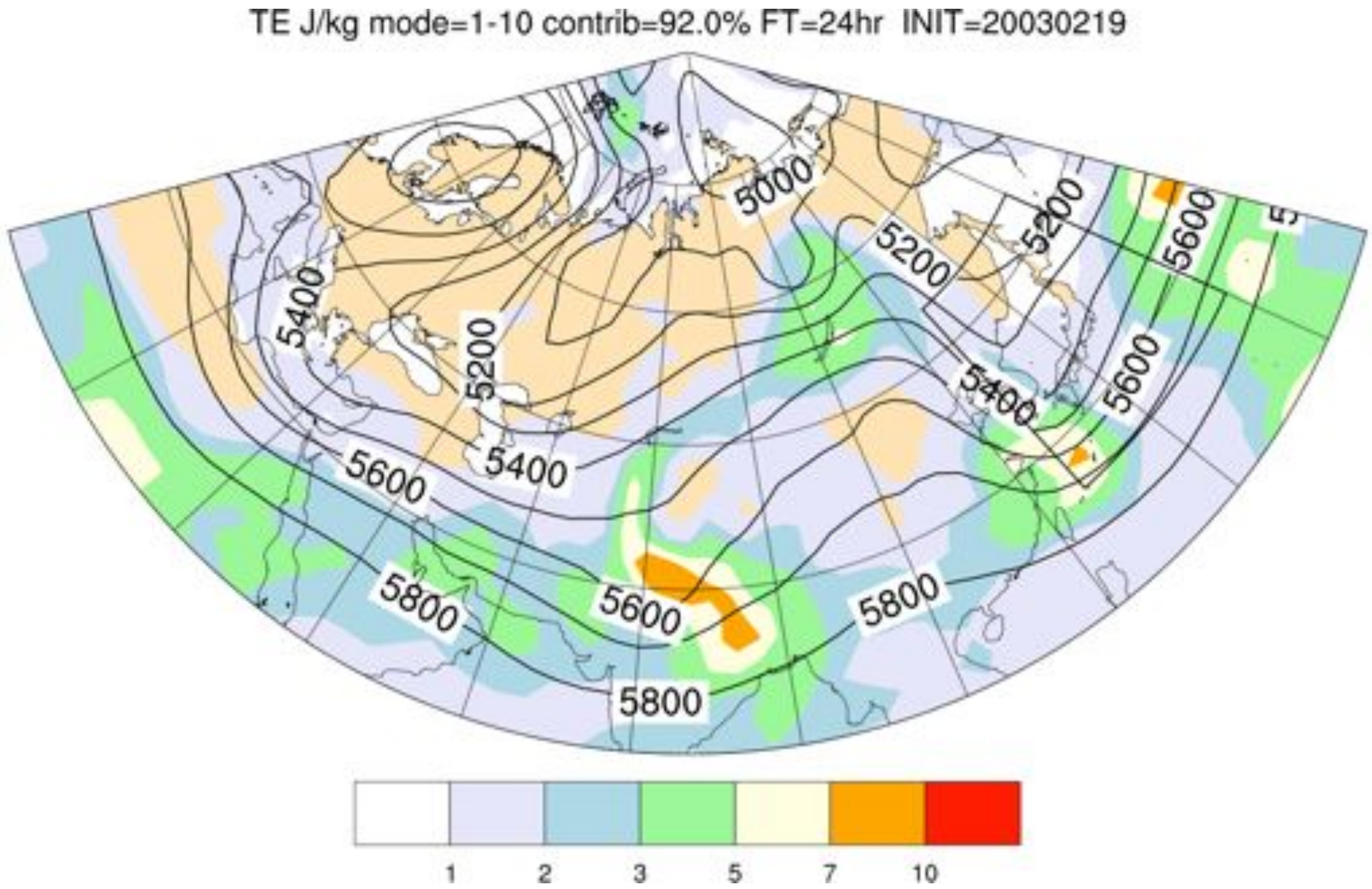
72-h sensitivity for a mid-latitude cyclone



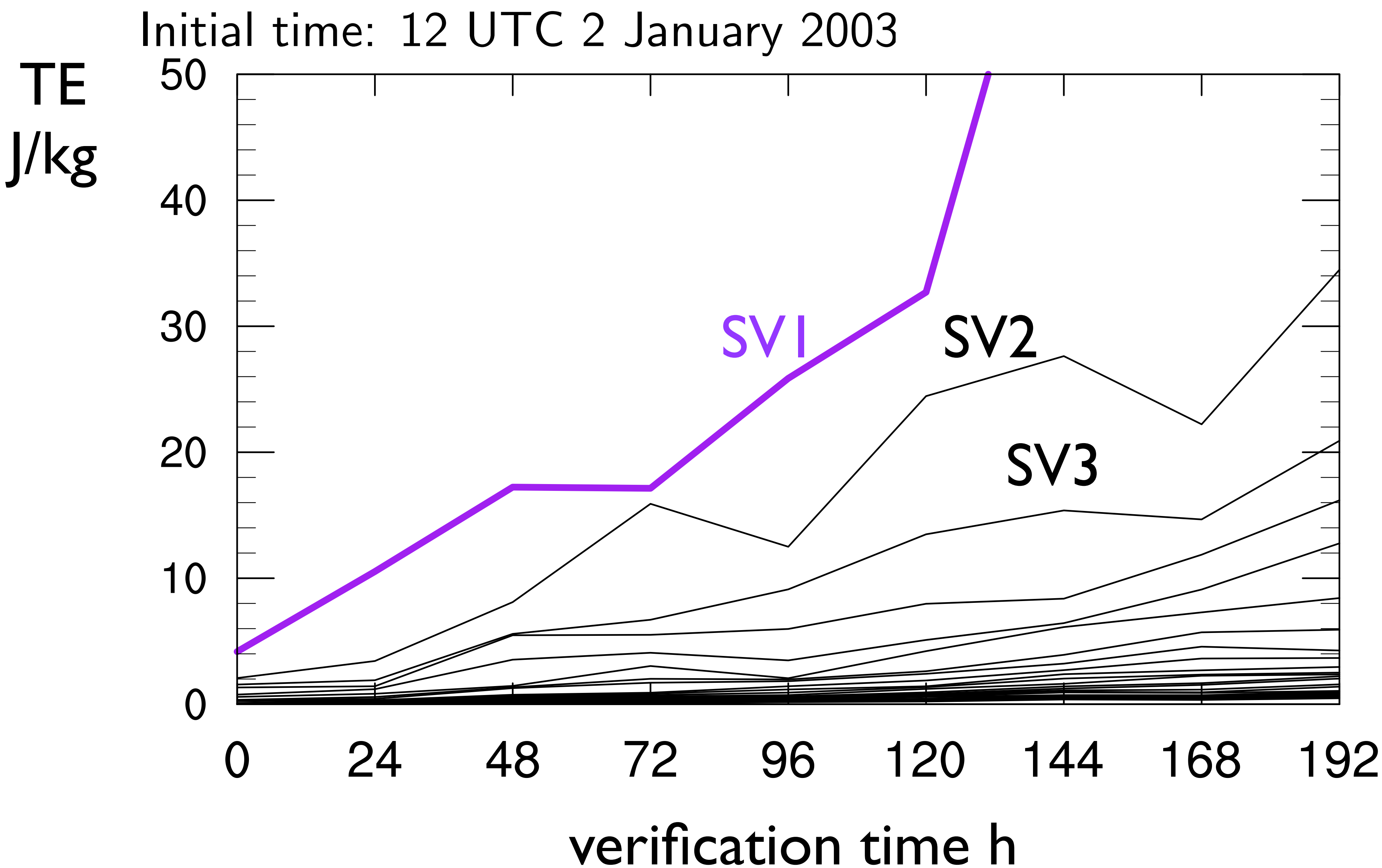
48-h sensitivity to a mid-latitude cyclone



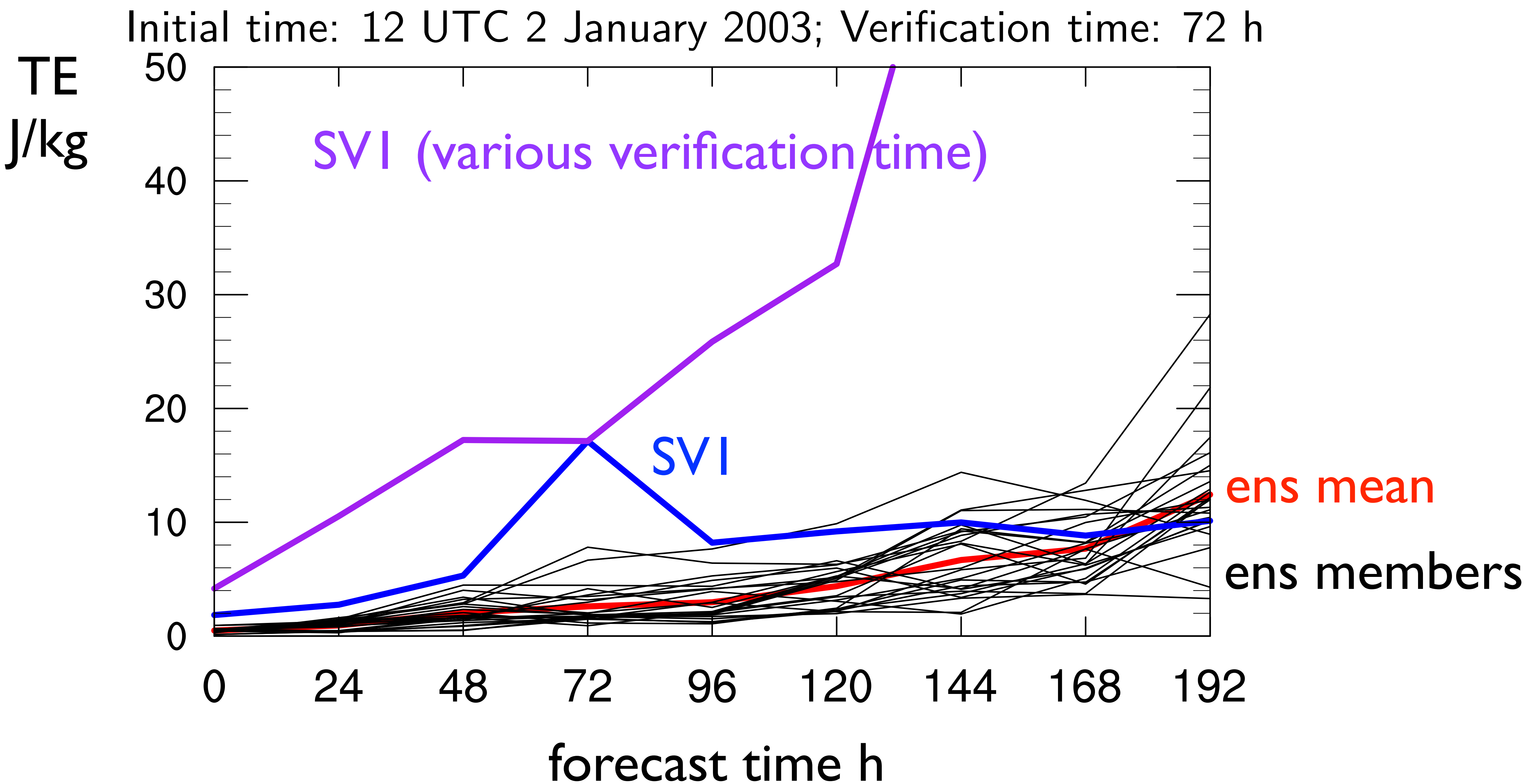
24-h sensitivity for a mid-latitude cyclone



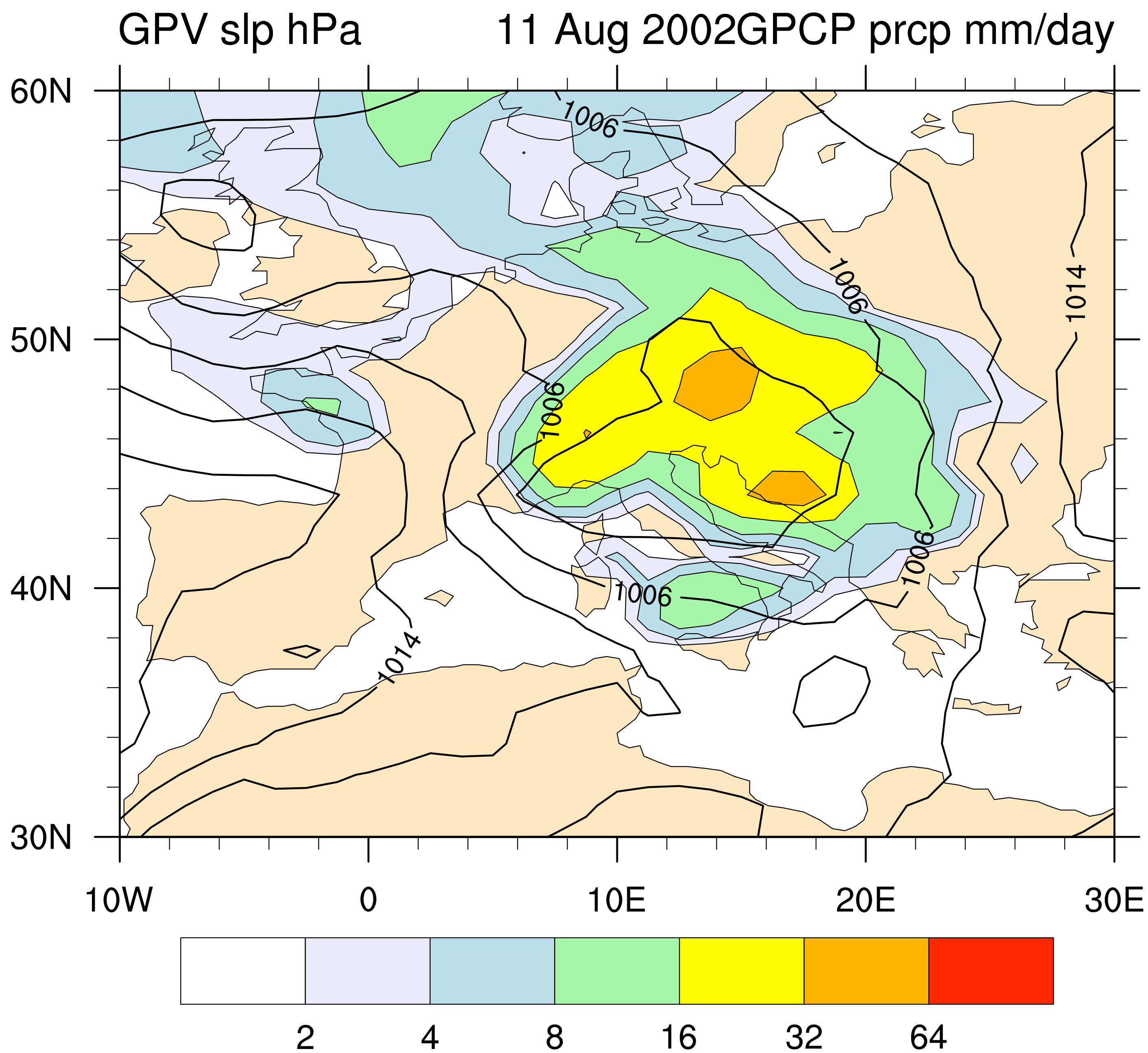
Growth of SV modes



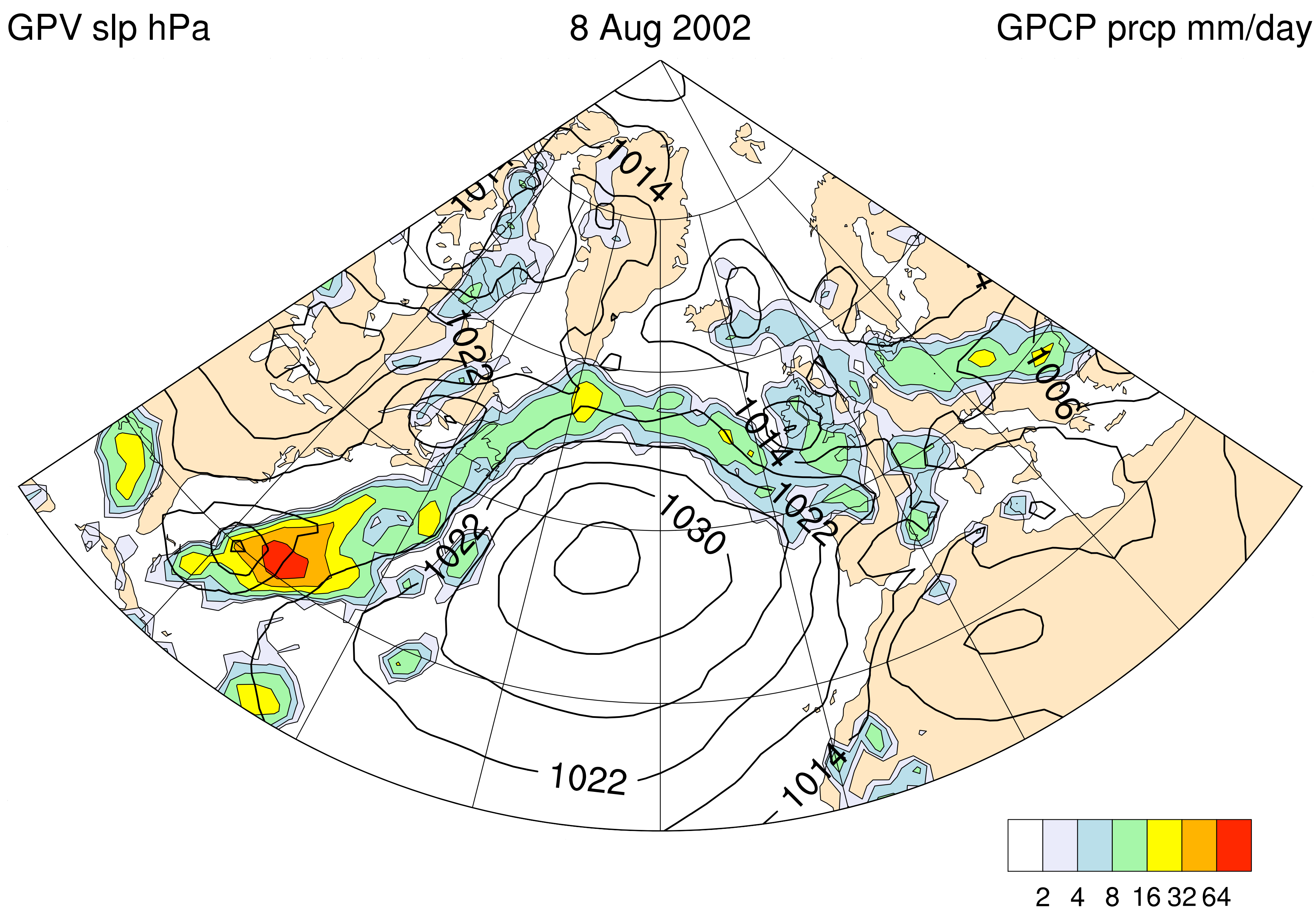
SV1 and perturbation growth

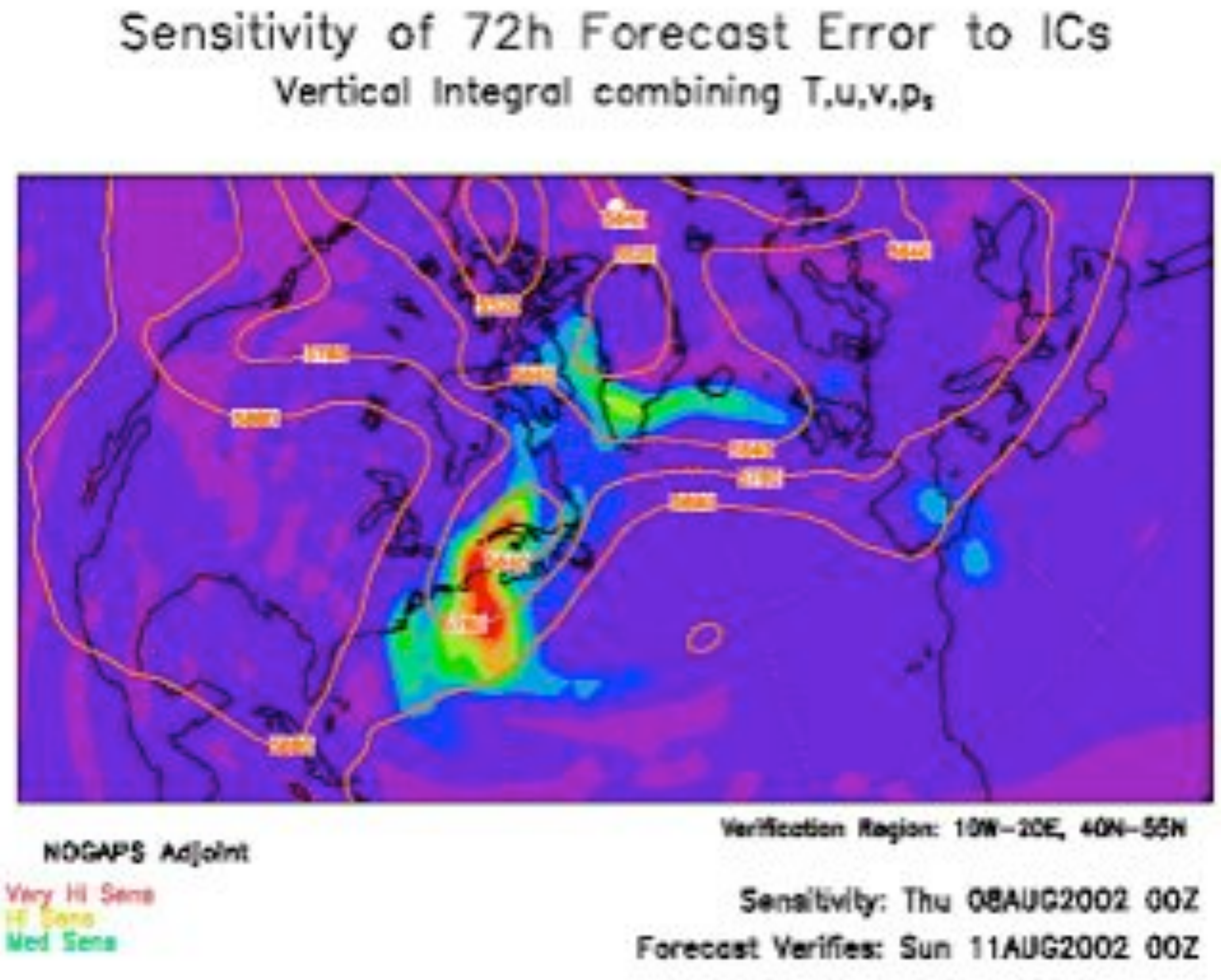


The August 2002 storm in Europe



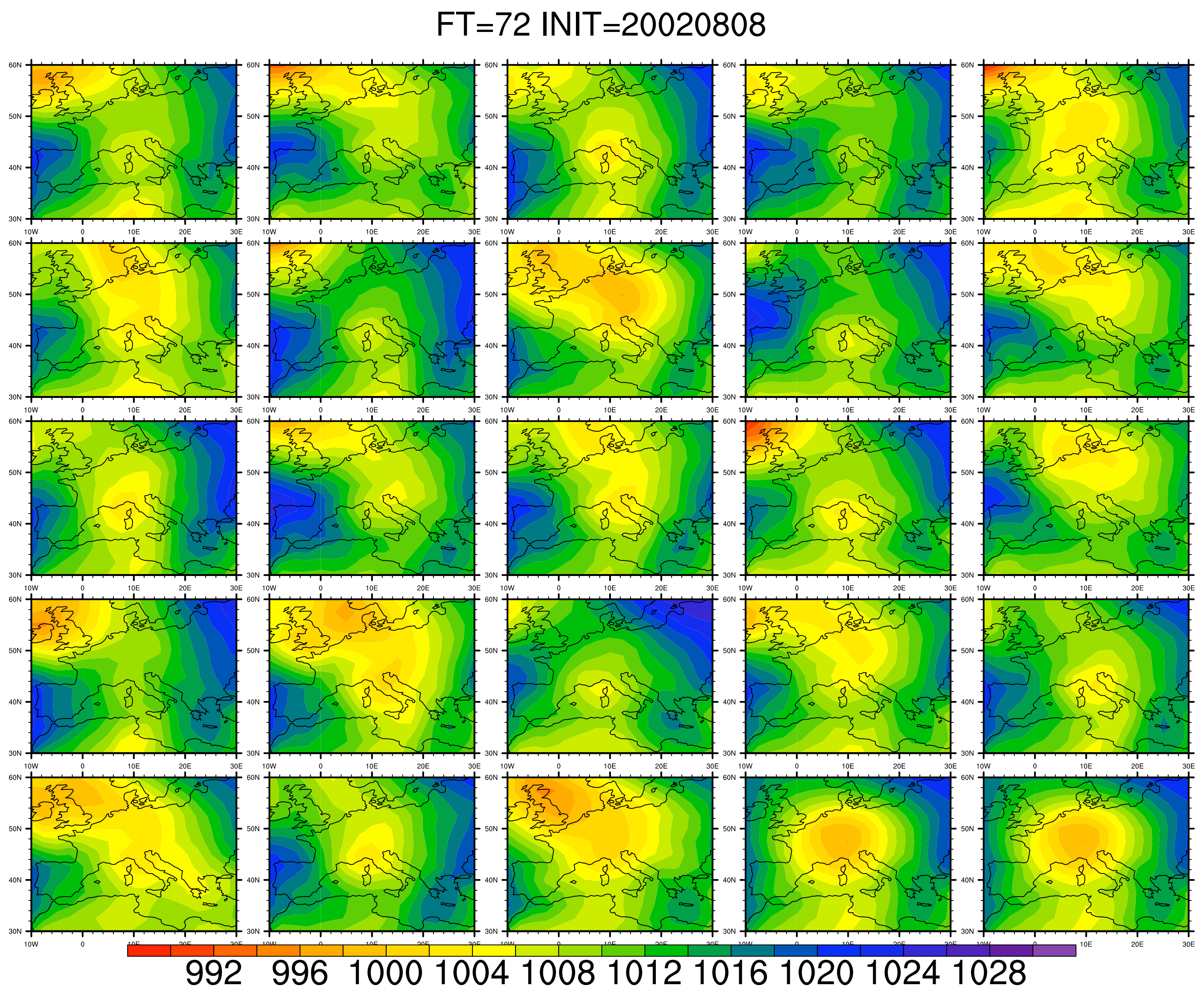
Remote influence





Langland (*pers. comm.*)

JMA EPS FT=72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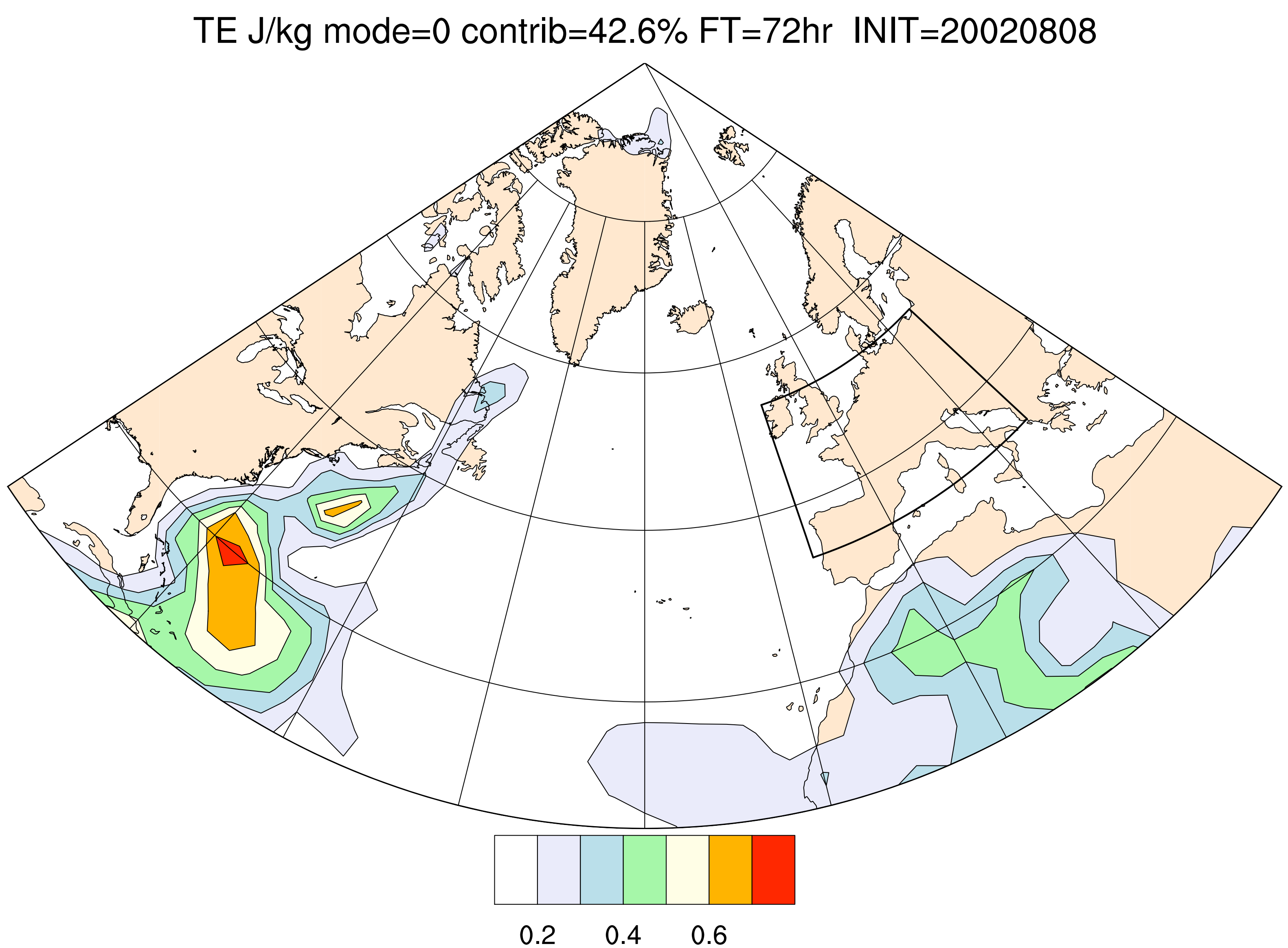
60N
50N
40N
30N
10W
0
10E
20E
30E
992 996 1000 1004 1008 1012 1016 1020 1024 1028

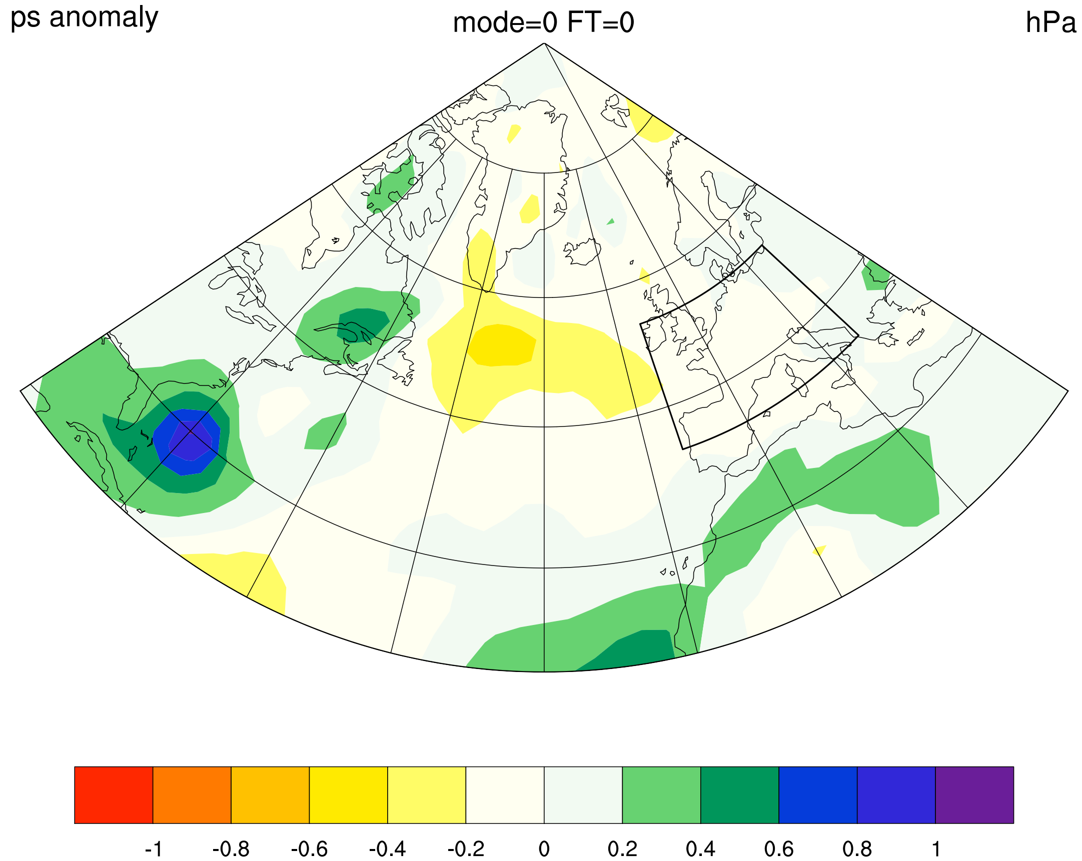
Sensitivity analysis: first mode



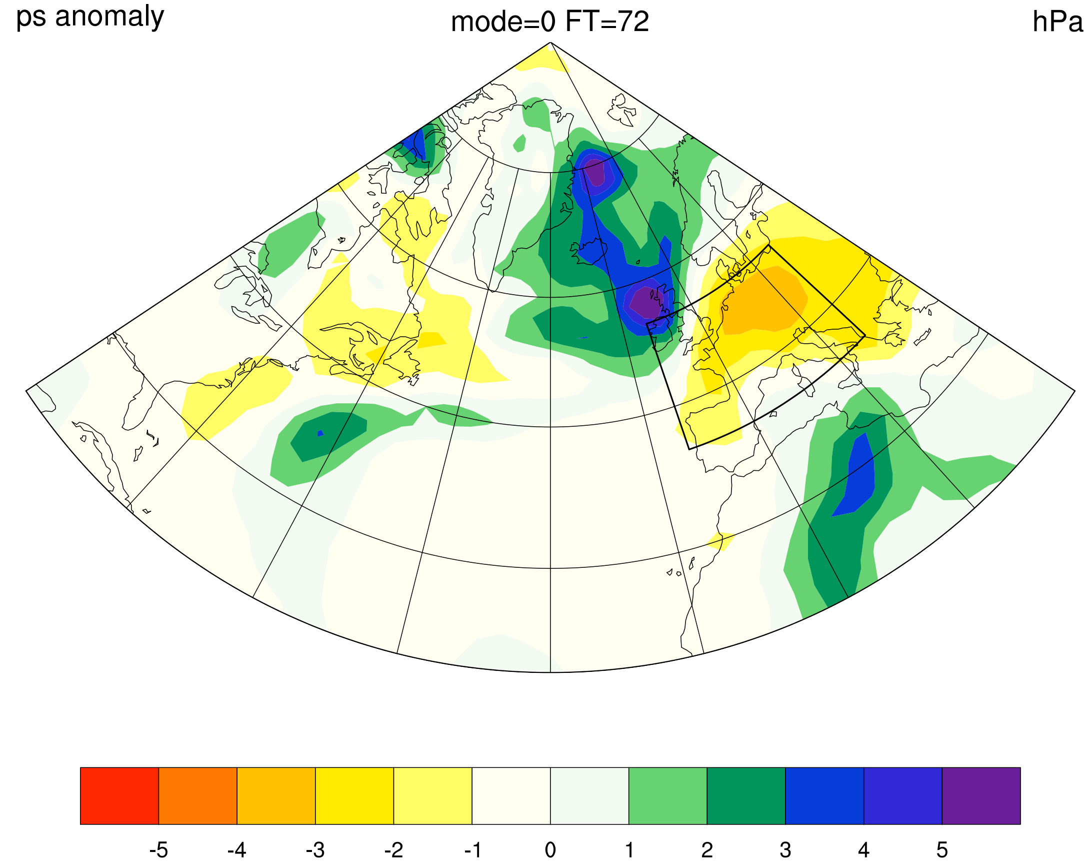
Vertically integrated energy norm

Sensitivity analysis: first mode

FT=0

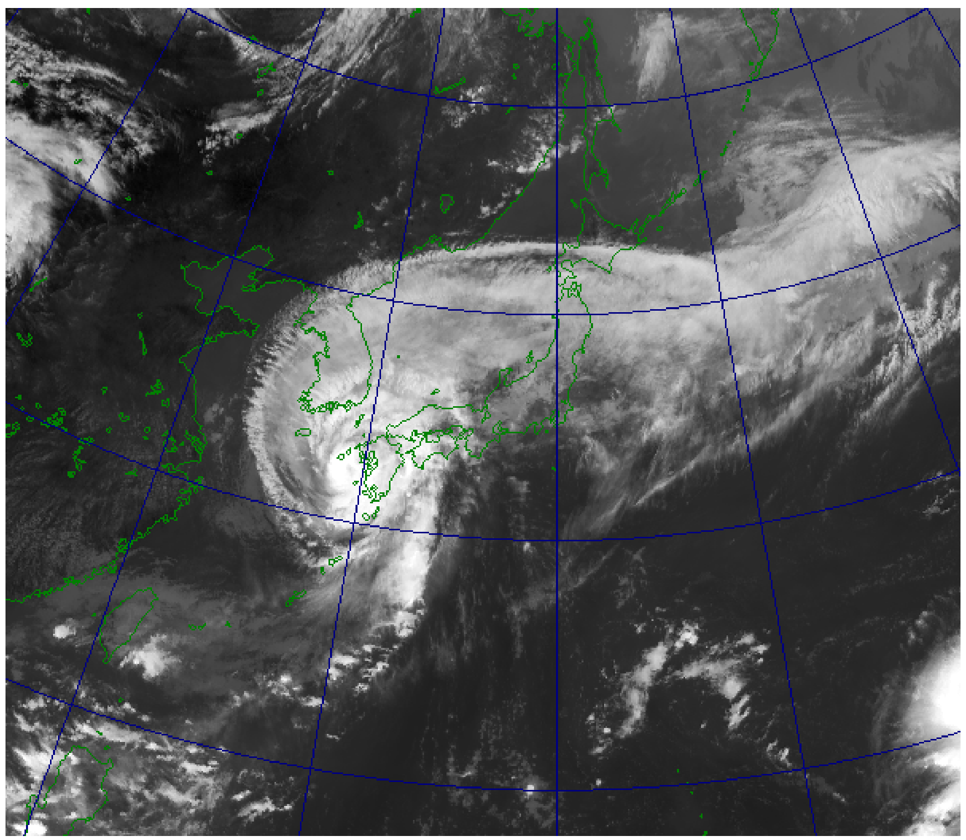


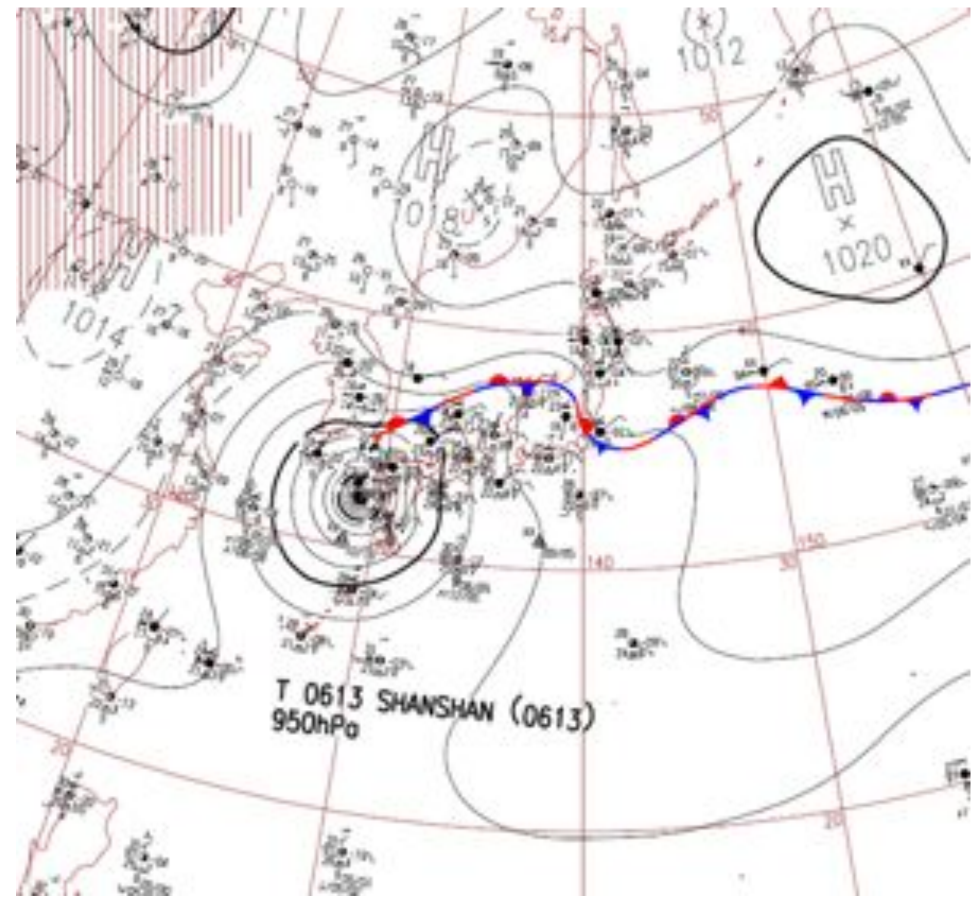
FT=72



p'_{S}

Shanshan (13th) in 2006





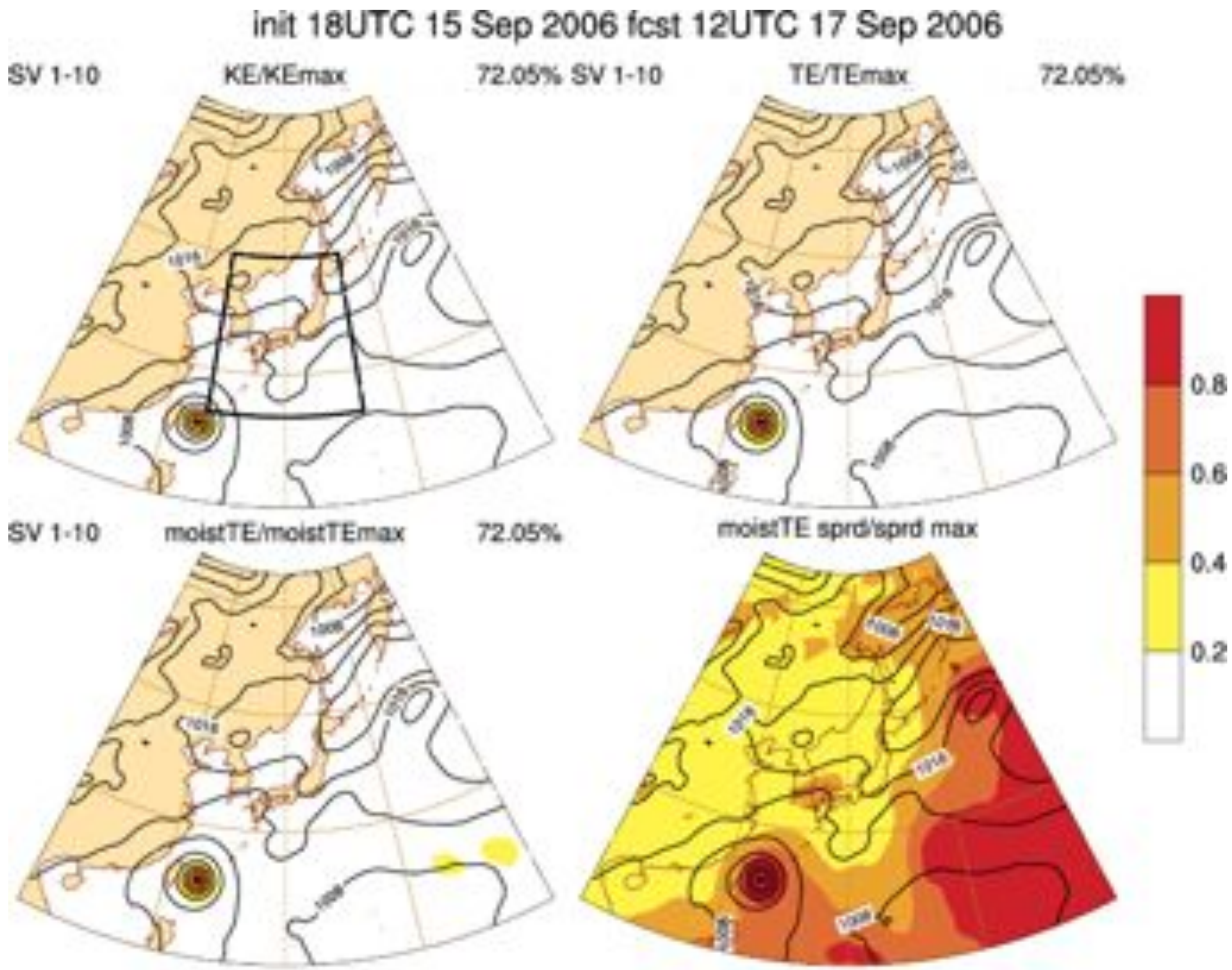
Damages caused by Shanshan

- caused tornadoes in Kyushu and Shikoku Islands
- 9 dead, 1 missing and 448 inured
- 159 houses destroyed, 514 partially and 11211 damaged
- 189 houses flooded above floorboards and 1177 flooded up to floor boards

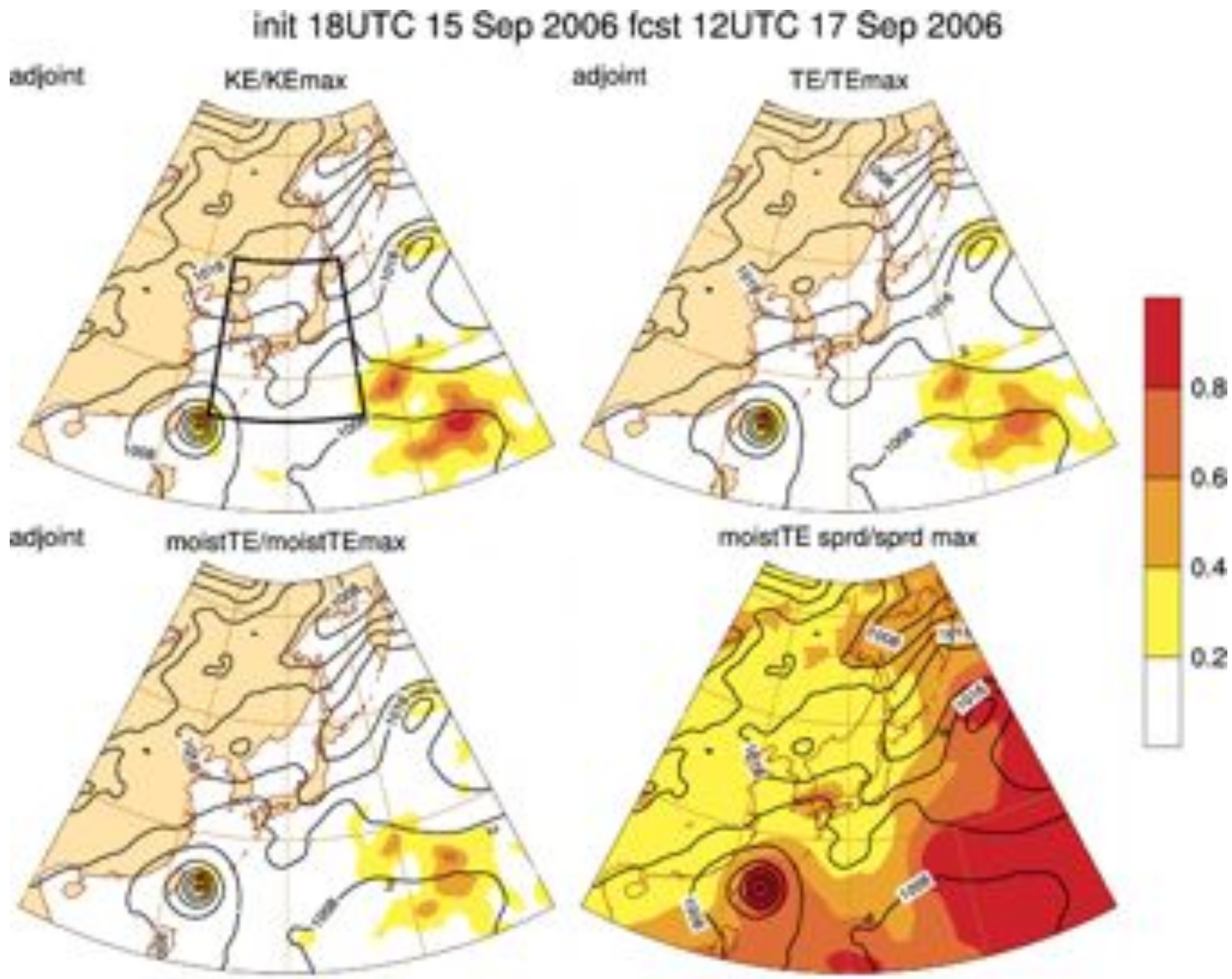
ALERA (Miyoshi et al. 2007a)

- AFES-LETKF experimental ensemble reanalysis
- T159L48M40
- Observations used in JMA NWP except for satellite radiances
- from May 2005 to January 2007
- produced under the collaboration among JMA, JAMSTEC and CIS

48-hr SV sensitivity to Shanshan 2006



48-hr adjoint sensitivity to Shanshan 2006



Summary

- Simplified sensitivity methods have been formulated using ensemble forecasts.
- Simplified methods only require simple matrix operations.
- Largest growing mode at the verification is correctly obtained.
- Sensitive regions are more focused than regions with large ensemble spread.
- Reasonable sensitive regions are found for cold surge and mid-latitude and tropical cyclones.
- Sensitive regions by ensemble adjoint and SV methods are consistent with some differences.