Simplified ensemble-based forecast sensitivity analysis

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Sensitivity analysis

- conducted to find the target region for additional observation.
- finds the initial perturbation that optimally grows in the verification region at the verification time.
- has been conducted by the adjoint (Rabier et al 1996), SV (Palmer et al 1998), ensemble-based (Bishop and Toth 1999; Bishop et al 2000; Ancell and Hakim 2007) methods.
Sensitivity analysis using the adjoint method

The gradient of a cost function $J$ is

$$
\delta J = \frac{\partial J}{\partial x} \bigg|_{x=x(t)} \quad z = \frac{\partial J}{\partial x} \bigg|_{x=x(t)} \quad M y
$$

Find the extremum $\delta J$ with $y^T G y = 1$ by the Lagrange multiplier method:

$$
F(y, \lambda) \equiv \delta J + \lambda (1 - y^T G y)
$$

(2)

to obtain

$$
2\lambda y = G^{-1} M^T \left[ \frac{\partial J}{\partial x} \bigg|_{x=x(t)} \right]^T
$$

(3)
Sensitivity analysis using the singular vectors

Maximize $||z|| \equiv \sqrt{z^T H z}$ with $||y|| \equiv \sqrt{y^T G y} = 1$.

The Lagrangian function

$$F(y, \lambda) = z^T H z + \lambda (1 - y^T G y)$$

$$= y^T M^T H M y + \lambda (1 - y^T G y),$$

$$\frac{\partial F(y, \lambda)}{\partial y} = y^T M^T H M - \lambda y^T G = 0. \tag{5}$$

The solutions are the eigenvectors of $G^{-1} M^T H M$.
Ensemble perturbations

Nonlinear time evolution of ensemble perturbations:

$$z_i = M(y_i), \quad i = 1, 2, \cdots, n.$$  \hfill (6)

Find the coefficients for a linear combination of initial perturbation

$$y = a_1 y_1 + a_2 y_2 + \cdots + a_n y_n.$$  \hfill (7)

Let

$$Y = (y_1, y_2, \cdots, y_n), \quad Z = (z_1, z_2, \cdots, z_n), \quad a = (a_1, a_2, \cdots, a_n)^T,$$ \hfill (8)

then

$$y = Ya, \quad z = M(Y)a = Za.$$  \hfill (9)
Ensemble adjoint sensitivity analysis

With ensemble forecast,

$$\delta J = \delta J^T a$$

(10)

where $\delta J^T = (\delta J_1, \delta J_2, \cdots, \delta_m)$. The Lagrangian function becomes

$$F(a, \lambda) = \delta J^T a + \lambda (1 - a^T Y^T G Y a),$$

(11)

$$\frac{\partial F(a, \lambda)}{\partial a} = \delta J - 2 \lambda a^T Y^T G Y = 0.$$ 

(12)

If $Y^T G Y \propto I$, the optimal initial perturbations are

$$y = Y \delta J = \delta J_1 y_1 + \delta J_2 y_2 + \cdots + \delta J_m y_m.$$ 

(13)
Simplified Sensitivity Analysis

Enomoto, Yamane and Ohfuchi

**Ensemble singular-vector sensitivity analysis**

With ensemble forecast, the Lagrangian function becomes

\[
F(a, \lambda) = a^T Z^T H Z a + \lambda (1 - a^T Y^T G Y a) \quad (14)
\]

and its derivative

\[
\frac{\partial F(a, \lambda)}{\partial a} = a^T Z^T H Z - \lambda a^T Y^T G Y = 0. \quad (15)
\]

Fields an eigenvalue problem

\[
(Y^T G Y)^{-1} Z^T H Z p = \lambda p \quad (16)
\]

If \( Y^T G Y \propto I \), the fastest growing modes can be found as the eigenvectors of \( Z^T H Z \).
Simplified ensemble-based sensitivity analysis

- does not need the tangent-linear and adjoint models or a DA system.
- expected to approach the adjoint or SV sensitivity asymptotically as $n \to \infty$.
- might be affected by ensemble size and the perturbation method.
- does not consider observations.
- may conducted with arbitrary verification regions.
Total energy norm

Here we use the total energy norm (Talagrand 1981, Ehrendorfer et al. 1999),

$$TE = \frac{1}{2} \int \int_A u'^2 + v'^2 + \frac{c_p}{T_r} T'^2$$

$$+ RT_r \left( \frac{p'_s}{p_r} \right)^2 + \varepsilon \frac{L^2}{c_p T_r} q'^2 \, dA \, dp,$$

where $'$ denotes anomaly from the control run, $T_r = 270$ K the reference temperature $p_r = 1000$ hPa, $\varepsilon = 0$ and $\varepsilon = 1$ for the dry and moist total energy norm, respectively.

Here the dry $TE \varepsilon = 0$ is used but note that the forward model includes all physical parametrizations.
JMA weekly ensemble forecast

- Perturbations generated with the breeding method.
- T106, but interpolated to $2.5^\circ \times 2.5^\circ$ grids for distribution.
- Full 25 members provided in the GRIB format.
- $u$, $v$, $z$, $T$ at 300, 500, 850 hPa and RH only at 850 hPa.
- Additionally 100, 200, 700, and 1000 hPa-level data is provided for this study by courtesy of NPD/JMA.
- Also ($u$, $v$) at the surface, precip and slp.
72-h sensitivity for a cold surge
The spread of the initial total energy norm
72-h sensitivity for a mid-latitude cyclone
48-h sensitivity to a mid-latitude cyclone
24-h sensitivity for a mid-latitude cyclone
Growth of SV modes

Initial time: 12 UTC 2 January 2003
SV1 and perturbation growth

Initial time: 12 UTC 2 January 2003; Verification time: 72 h

TE
J/kg

SV1 (various verification time)

ens mean

ens members

forecast time h
The August 2002 storm in Europe
Remote influence

GPV slp hPa  8 Aug 2002  GPCP prcp mm/day
Sensitivity of 72h Forecast Error to ICs
Vertical Integral combining T,u,v,p,z

Verification Region: 10W-20E, 40N-55N
Sensitivity: Thu 08AUG2002 00Z
Forecast Verifies: Sun 11AUG2002 00Z

Langland (pers. comm.)
Simplified Sensitivity Analysis

Enomoto, Yamane and Ohfuchi

JMA EPS FT=72

FT=72 INIT=20020808
Sensitivity analysis: first mode

TE J/kg mode=0 contrib=42.6% FT=72hr INIT=20020808

Vertically integrated energy norm
Sensitivity analysis: first mode

FT=0

FT=72

$p'_{S}$
Shanshan (13th) in 2006
Damages caused by Shanshan

- caused tornadoes in Kyushu and Shikoku Islands
- 9 dead, 1 missing and 448 inured
- 159 houses destroyed, 514 partially and 11211 damaged
- 189 houses flooded above floorboards and 1177 flooded up to floor boards
ALERA (Miyoshi et al. 2007a)

- AFES-LETKF experimental ensemble reanalysis
- T159L48M40
- Observations used in JMA NWP except for satellite radiances
- from May 2005 to January 2007
- produced under the collaboration among JMA, JAMSTEC and CIS
48-hr SV sensitivity to Shanshan 2006
48-hr adjoint sensitivity to Shanshan 2006
Summary

• Simplified sensitivity methods have been formulated using ensemble forecasts.

• Simplified methods only require simple matrix operations.

• Largest growing mode at the verification is correctly obtained.

• Sensitive regions are more focused than regions with large ensemble spread.

• Reasonable sensitive regions are found for cold surge and mid-latitude and tropical cyclones.

• Sensitive regions by ensemble adjoint and SV methods are consistent with some differences.