Multi-scale Localization in Ensemble-based Data Assimilation

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Motivation

- Due to the limited ensemble size, sampling error may be problematic.
- Localization plays an essential role.
  - Distance-dependent localization is applied to error covariance and reduces the sampling errors.

Analysis increments from a single profile observation (20 members)

- Higher resolution models require narrower localization which limits the influence of observations.
Motivated by Buehner (2012), we construct analysis increments as a sum of high- \((h)\) and low- \((l)\) wavenumber components.

\[ \delta x = \delta x_h + \delta x_l \]
We apply spatial smoothing to the ensemble perturbations to reduce noise in longer-range covariance.
Larger-scale localization

○ Applying a 1000-km (larger scale) localization.

Full-range (T30) analysis increment

Analysis increment from reduced-resolution (T21) ensemble perturbations

Noisier in distance
Applying a 500-km (smaller scale) localization.

Full-range (T30) analysis increment

Analysis increment from reduced-resolution (T21) ensemble perturbations

More structure in short range
Merging the two scales

Original covariance with 500-km (smaller scale) localization

\[ \delta x_h \]

Preserve the smaller-scale structure in short range

Large-scale covariance with 1000-km (larger scale) localization

\[ \delta x_l \]

Removing the short-range structure

\[ \delta x = \delta x_h + \delta x_l \]
We merge the high ($h$) and low ($l$) wavenumber components.

\[ \delta x = \delta x_h + \delta x_l \]
Summary of the algorithm

1. Compute the analysis increment regularly
   (with smaller-scale localization)

2. Compute the analysis increment with smoothed ensemble perturbations
   (with larger-scale localization)

3. Compute the analysis increment with smoothed ensemble perturbations
   (with smaller-scale localization)

4. Take the difference between 2 and 3

5. Add 1 and 4
## Settings of perfect model experiments

<table>
<thead>
<tr>
<th></th>
<th>CTL(L=500)</th>
<th>CTL(L=1000)</th>
<th>Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>SPEEDY, T30L7 (Molteni 2003)</td>
<td></td>
<td></td>
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<tr>
<td>Observation</td>
<td>Radiosonde-like</td>
<td></td>
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<tr>
<td>Ensemble size</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Localization scale</td>
<td><strong>500 km</strong> <em>(small)</em></td>
<td><strong>1000 km</strong> <em>(large)</em></td>
<td><strong>500 km</strong> <strong>1000 km</strong></td>
</tr>
</tbody>
</table>

- Test experiment: Dual Localization LETKF
23-month average global analysis error power spectrum.

○ Successfully reducing the errors at all scales.
General improvements for mid-level U

23-month average RMS errors

- 500-km regular
- 1000-km regular
- 500-1000-km dual

○ Successfully improving analysis RMS errors in the Northern Hemisphere.
Impressive improvements for low-Q

23-month average RMS errors

- **500-km regular**
- **1000-km regular**
- **500-1000-km dual**

○ Greatly improving analysis RMS errors almost everywhere.
Summary

○ Dual-localization LETKF analysis showed promising results.
  • Improvements at almost all scales
  • Improvements almost everywhere for all variables
  • Impressive improvements for humidity

○ Drawback: LETKF computations are tripled.

○ Future plans
  • Improving the algorithm for saving computations.
  • Applying to higher-resolution models
    – Multi-scale considerations are more important with higher resolutions.
Thank you for your attention!
23-month average global analysis error power spectrum.

- 500-km localization standard LETKF
- 1000-km localization standard LETKF
- 500-1000-km dual-localization LETKF

- Successfully reducing the errors slightly.
Regular analysis increments at the full resolution (T30) with two different localization scales:

- **Small scale localization**
- **Large scale localization**

Noisier in distance
Reducing sampling noise in a longer range

- We apply spatial smoothing to the ensemble perturbations to reduce noise in longer-range covariance.

Full-range (T30) analysis increment

Analysis increment from reduced-resolution (T21) ensemble perturbations
Longer-range component

Reduced-resolution (T21) perturbations with **large** scale localization

Reduced-resolution (T21) perturbations with **small** scale localization

Removing the small scale structure