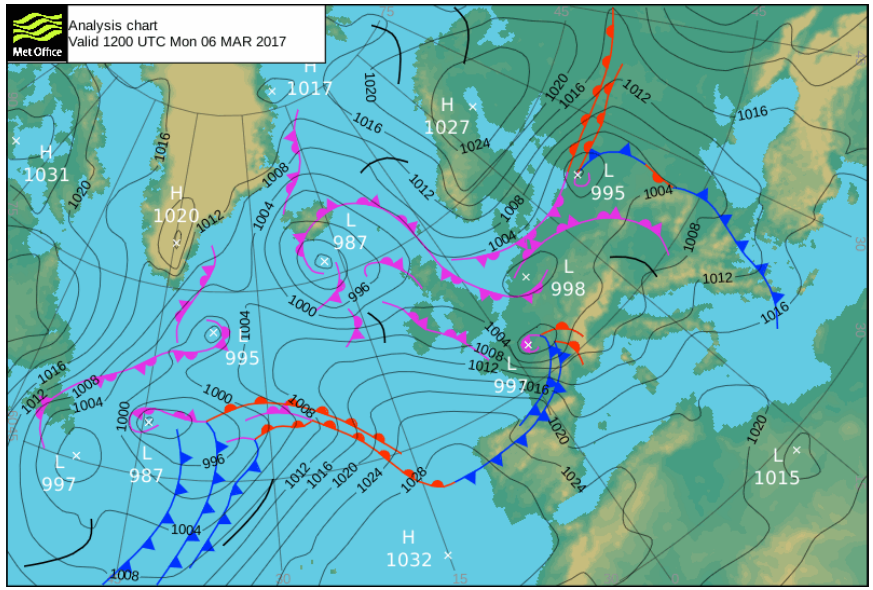
New Applications and Challenges In Data Assimilation



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Part 1.
Applications

Coupled Ocean-Atmosphere

Ensemble covariances for coupled atmosphere-ocean data assimilation

Challenge: determine covariances between atmosphere and ocean variables at interface for hybrid assimilation

Smith, Lawless & Nichols

Ocean-atmosphere interaction

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Carbon Cycle Balance

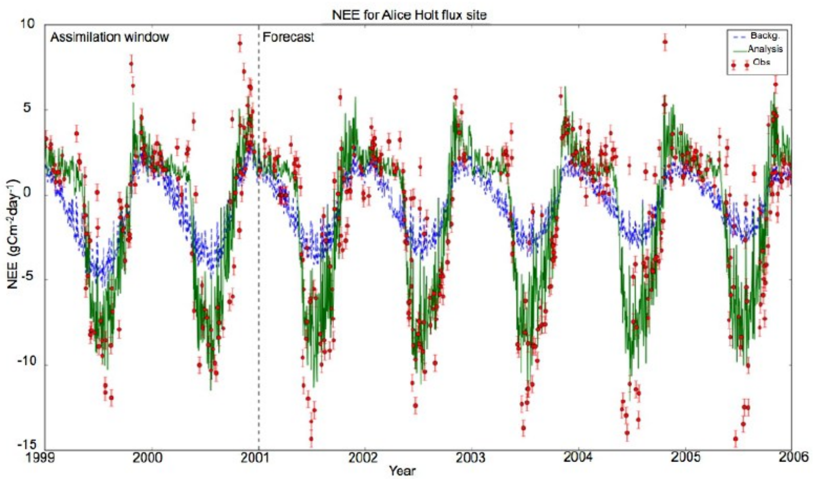

Figure 1: Two year assimilation and five year forecast of Alice Holt NEE with DALEC2, blue dotted line: background guess, green line: analysis after assimilation, red dots: observations from Alice Holt flux site with error bars.

Understanding the information content in observations of forest carbon balance

Challenge: state-parameter estimation - determining constrained prior and time correlated observation errors to improve estimates of forest CO_2 balance

Pinnington, Quaife, Dance, Lawless & Nichols, with Forest Research Team

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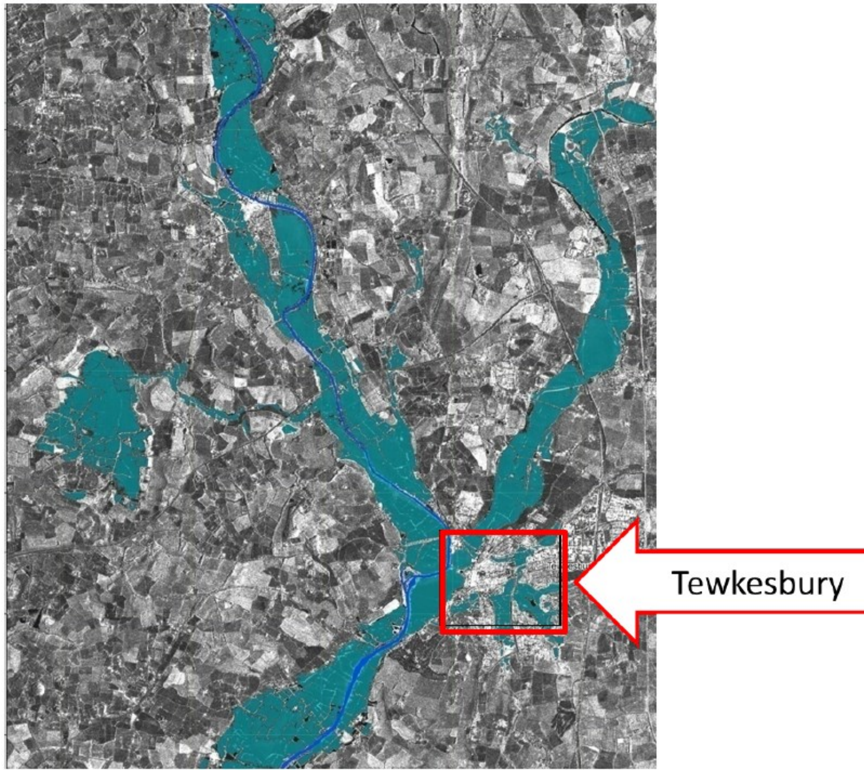
Hydrology - River Flooding

Improving flood predictions using data assimilation

Challenge: inflow estimation together with state-parameter estimation with real topography and SAR observations giving waterline information

Cooper, Dance, Smith & Garcia-Pintado



Flooding in the midlands

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DA for Moving Boundary Problems

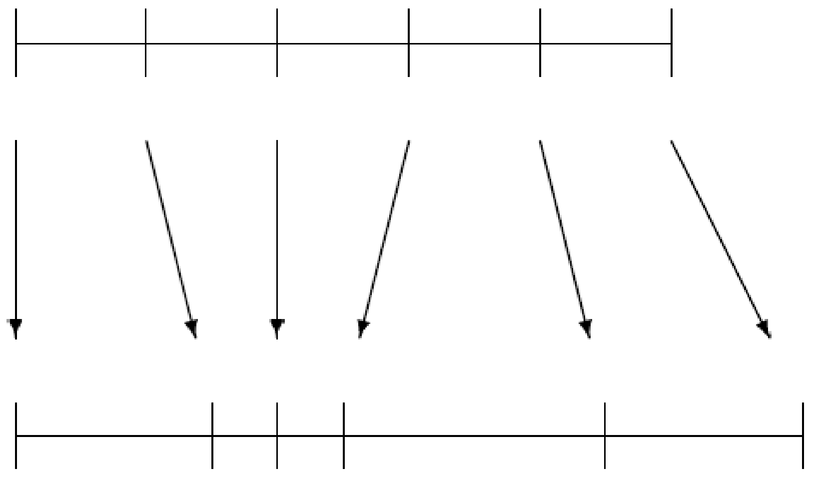
Challenge: Follow moving boundaries accurately and efficiently using data assimilation

Bonan, Nichols,
Baines & Partridge

Ice sheet model

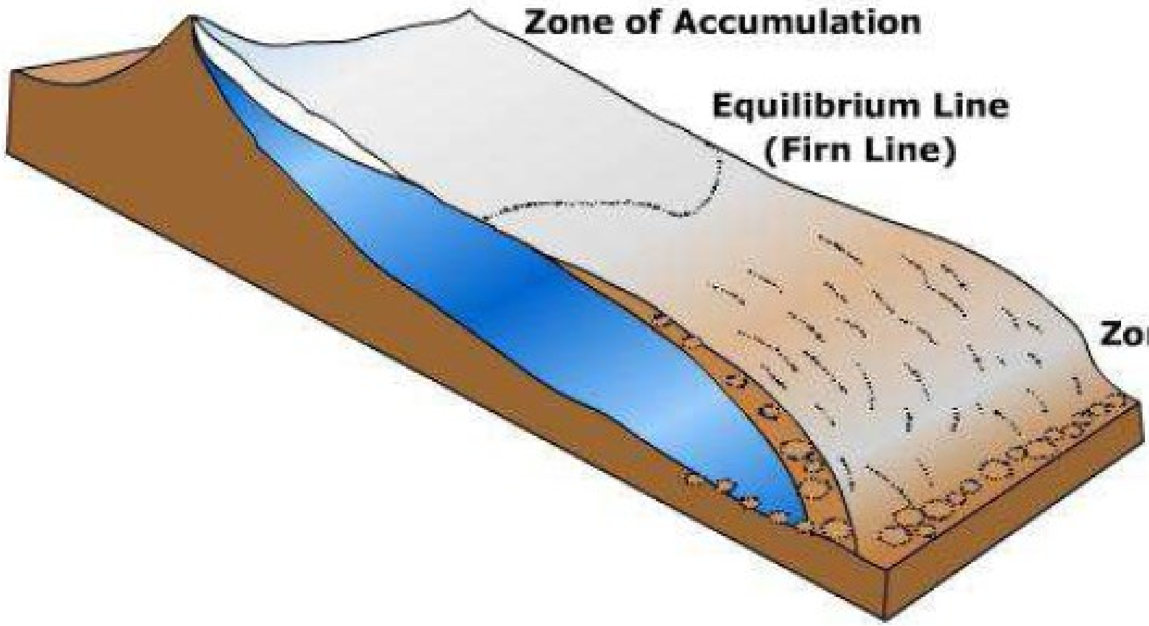
Moving Framework – 1D

Moving mesh methods use varying techniques to
to move the nodes of the mesh



Here we will be using ***physical properties***
to generate the movement

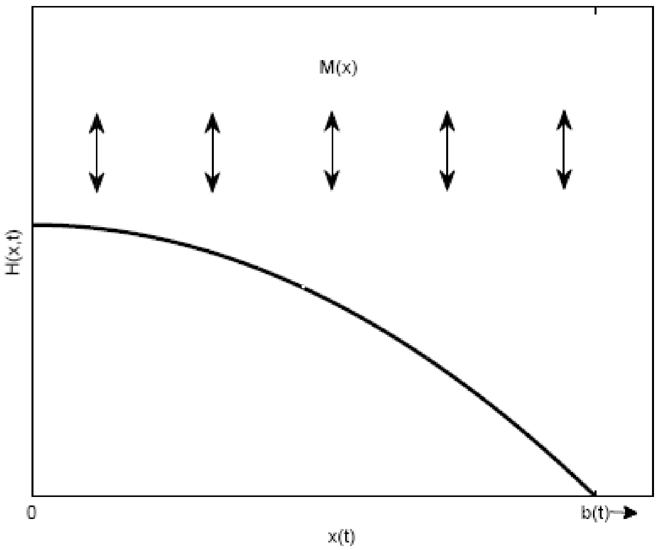
Ice Sheet Model - Schematic Form



Glacier Zones Courtesy of Michael Ritter

Model Domain

Consider a flat bed domain $x \in [0, b(t)]$,



where $\mathsf{h}(x, t)$ is the ice thickness, with boundary conditions:

$$\mathsf{h}_x(0, t) = 0 \quad \text{and} \quad \mathsf{h}(b(t), t) = 0$$

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Model Assumptions

- Flat bedrock topography
- No basal sliding
- Isothermal ice sheet
- Grounded ice
- Radially-symmetric ice sheet, so mass balance equation is

$$\frac{\partial h}{\partial t} = m - \frac{1}{r}\frac{\partial (r\,hU_r)}{\partial r}$$

- SIA, so vertically averaged velocity is

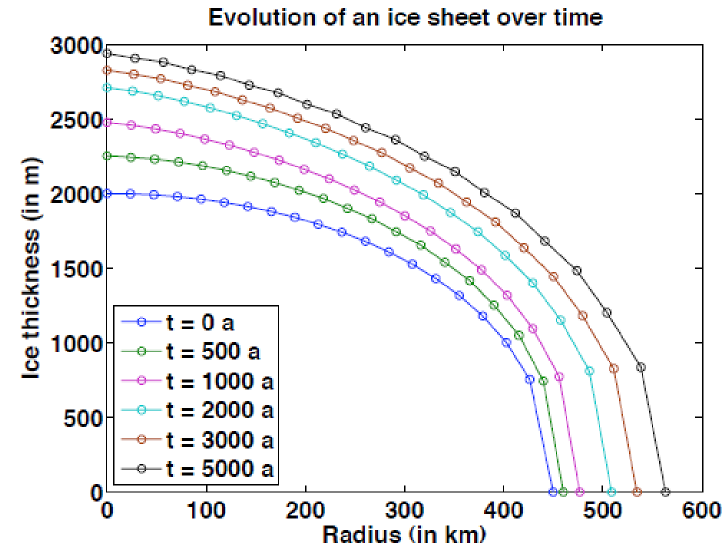
$$U_r = -\frac{2A\left(\rho_i\, g\right)^n}{n+2}\, h^{n+1} \left|\frac{\partial h}{\partial r}\right|^{n-1} \frac{\partial h}{\partial r}$$

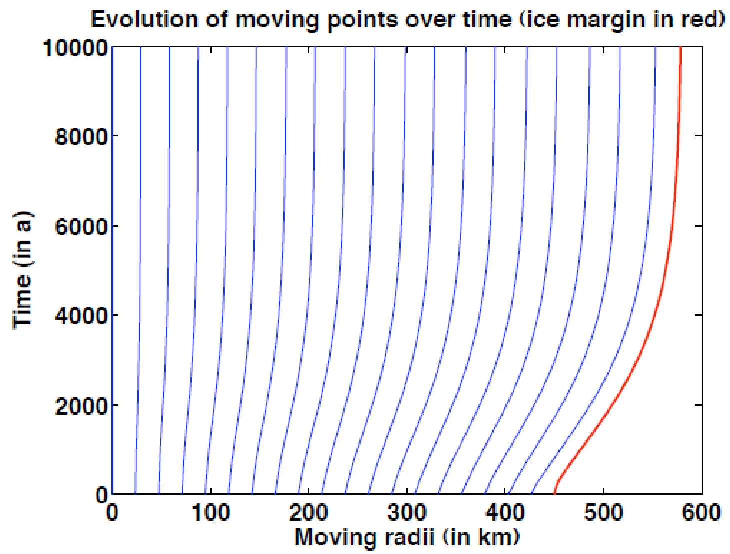
- Physical quantities (h, U_r, ...) calculated on a moving grid with a fixed number of evolving radii $\widehat{r}_i(t)$.

- One moving radius for
  - ice divide $\widehat{r}_1(t) = 0$
  - ice margin $\widehat{r}_{n_r}(t) = r_l(t)$

- **Strategy**:
  At given time, geometry of ice sheet known
  - calculate velocity of moving radii.

  $\longrightarrow$ next time step
  - update radii
  - update ice sheet geometry





Data Assimilation - Twin Experiments

- What we estimate: moving points and ice thickness.

State vector is: $\begin{pmatrix} \mathbf{r} \\ \mathbf{h} \end{pmatrix}$

- What we observe:

Observations are obtained from a reference run at different times ($t =$ 500, 600, 700, 800, 900, 1000 a) and perturbed with a Gaussian noise.

  - ice thickness at different locations, $\sigma_h = 100$ m.
  - position of ice margin, $\sigma_r = 1$ km.

- DA system: Ensemble Kalman Filter.

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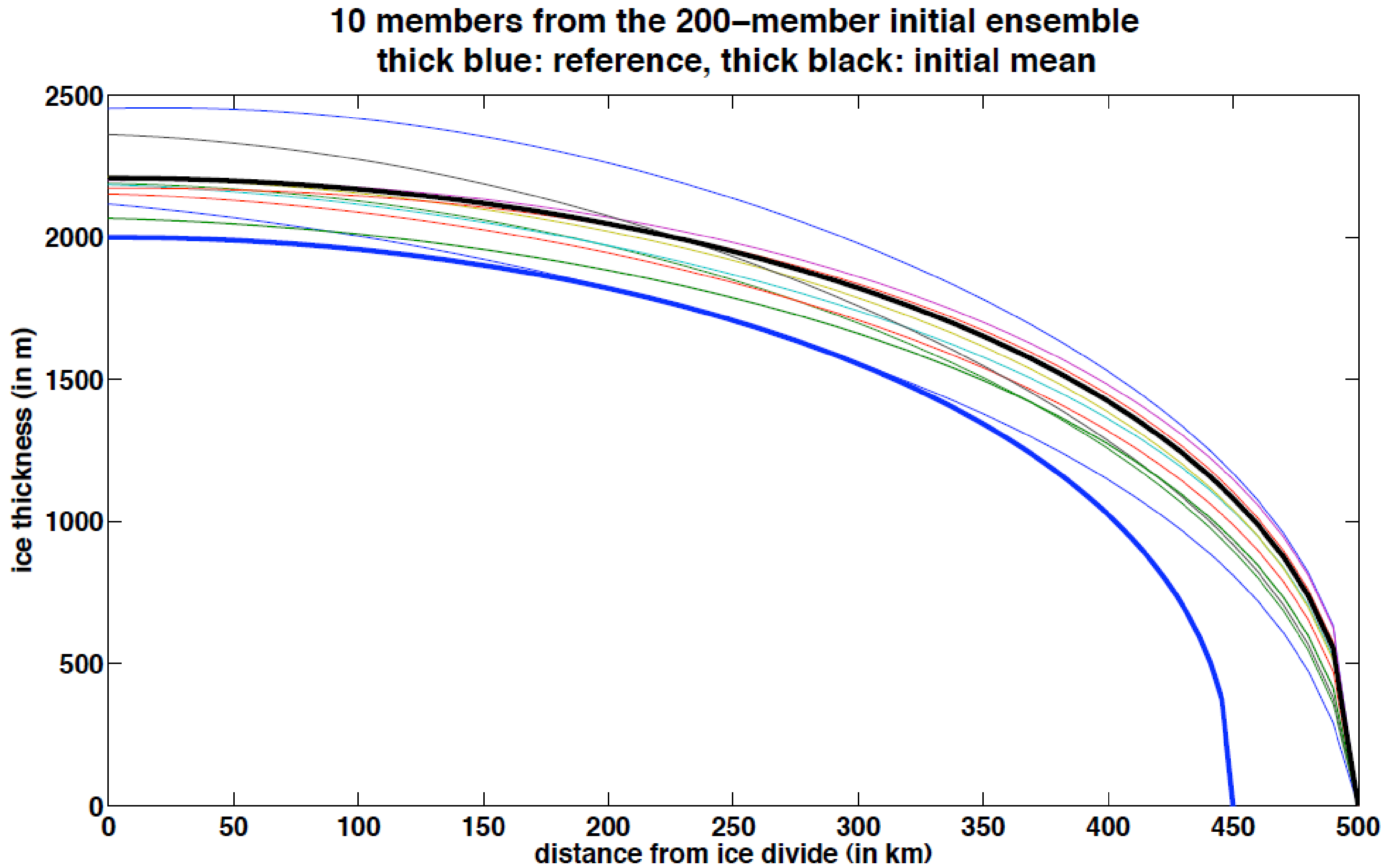
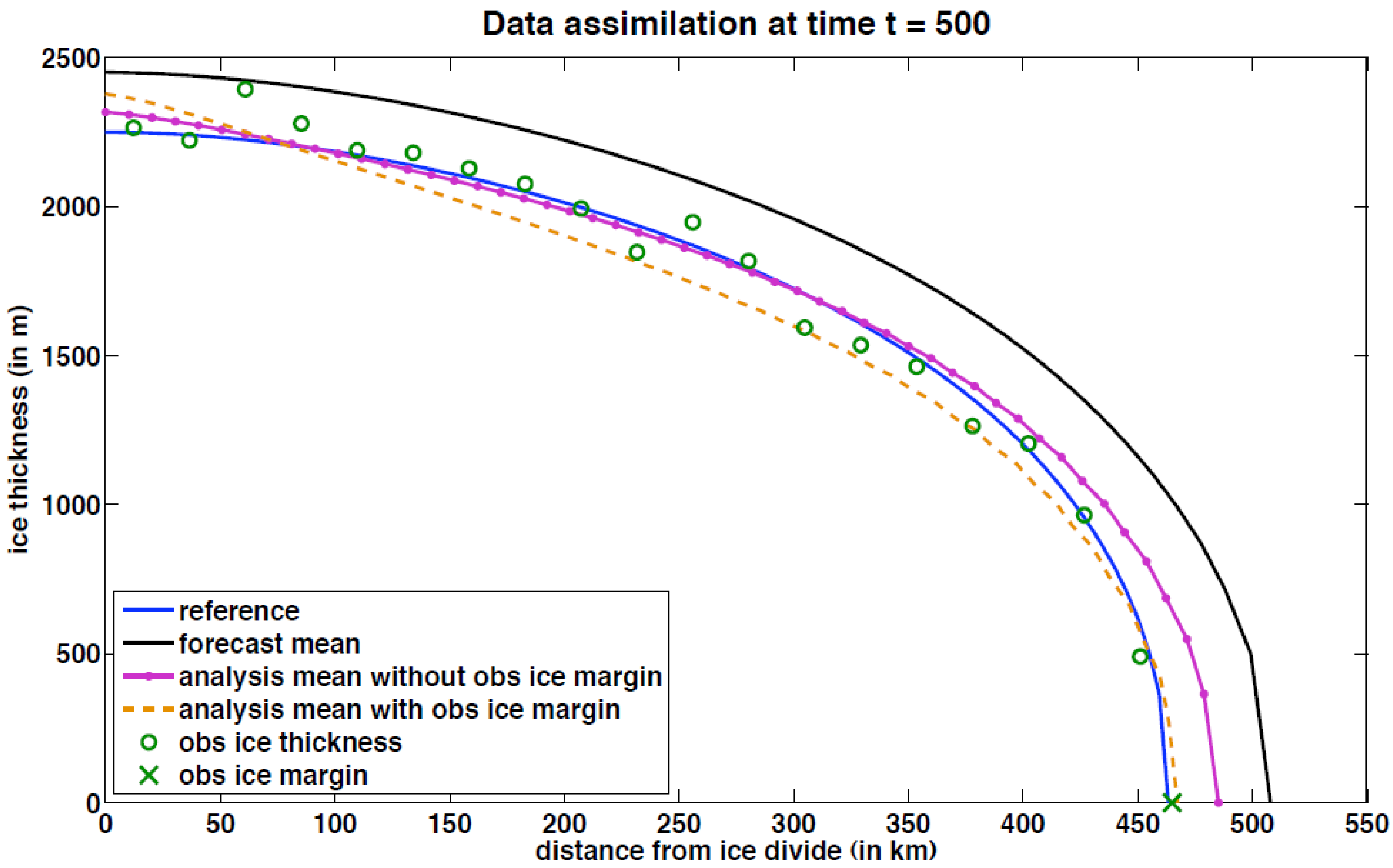
Initial Ensemble – 200 members



Results



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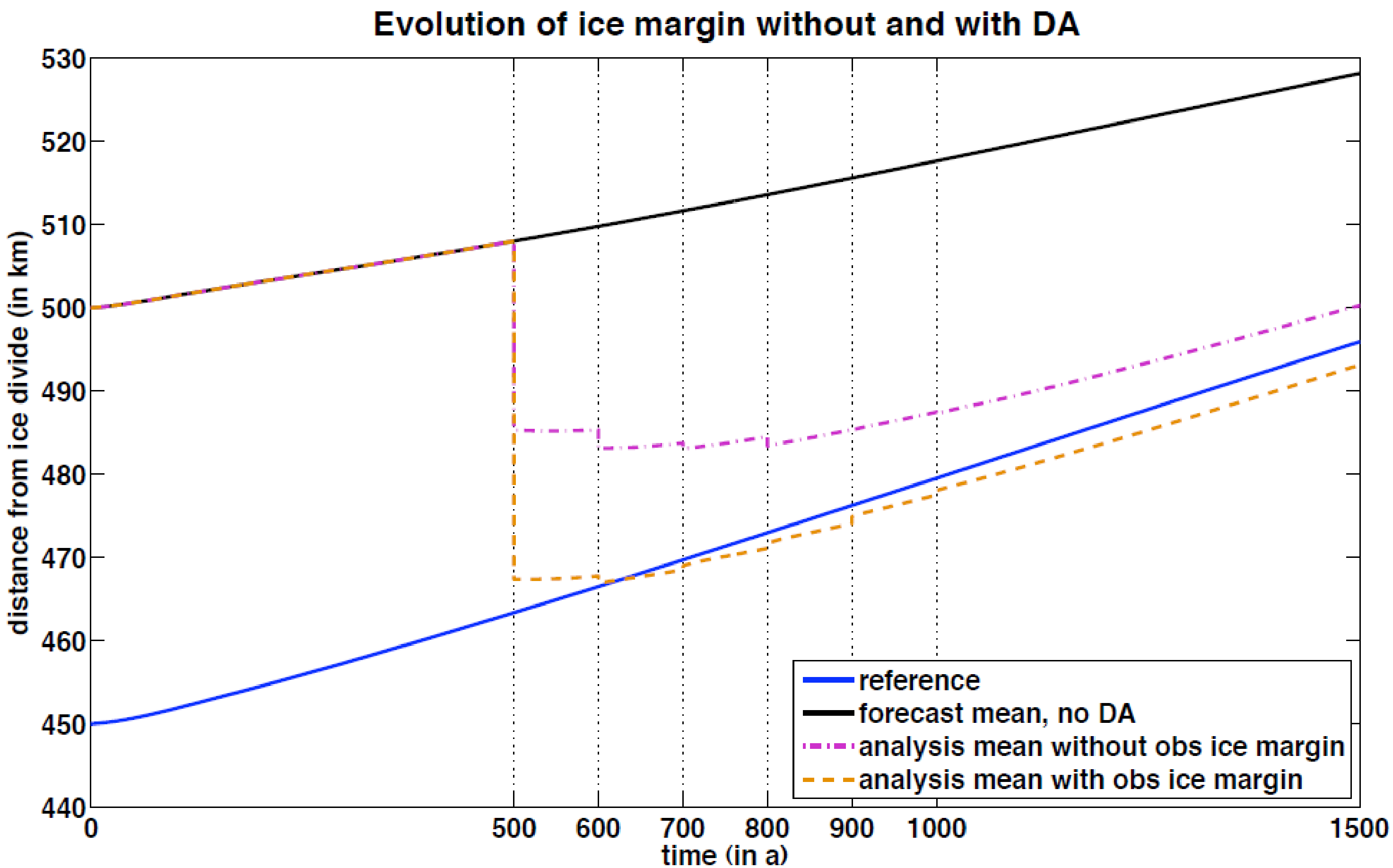
Results



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Future

Challenge: Treat the problem in 2D - moving mesh model is developed, but data assimilation presents new issues!!

Part 2.
Observation Errors and Conditioning of the Assimilation Problem

Conditioning of the Problem

Rate of convergence and accuracy of the solution are bounded in terms of the condition number of the Hessian:

$$\mathbf{S} = \mathbf{B}^{-1} + (\mathbf{H})^T \mathbf{R}^{-1} \mathbf{H}$$

where $\mathbf{B}$ and $\mathbf{R}$ are covariance matrices with special structures that depend on the variances and correlation length scales of the errors.

Summary: Conditioning of the Problem

We find that the condition number of S increases as:

- the observations become more accurate
- the observation spacing decreases
- the prior (background) becomes less accurate
- the prior error correlation length scales increase
- the observation error covariance becomes ill-conditioned.

Haben et al, 2011; Haben 2011, Tabeart, 2016

Condition of Hessian

$$\mathbf{S} = \mathbf{B}^{-1} + (\mathbf{H})^T \mathbf{R}^{-1} \mathbf{H}$$

- $\mathbf{B} = \sigma_b{}^2 \, \mathbf{C}$, where **C** is a correlation matrix
- Assume observations are at grid points
- Assume observation errors uncorrelated
- σ_0^2 variance observation errors
- $\mathbf{R} = \sigma_0^2 \mathbf{I}$
- **H** linear.
- $\mathbf{H}^T\mathbf{H}$ diagonal.

Condition Number of Hessian

Assume **C** has a circulant covariance structure.

Bounds on the conditioning of the Hessian are:

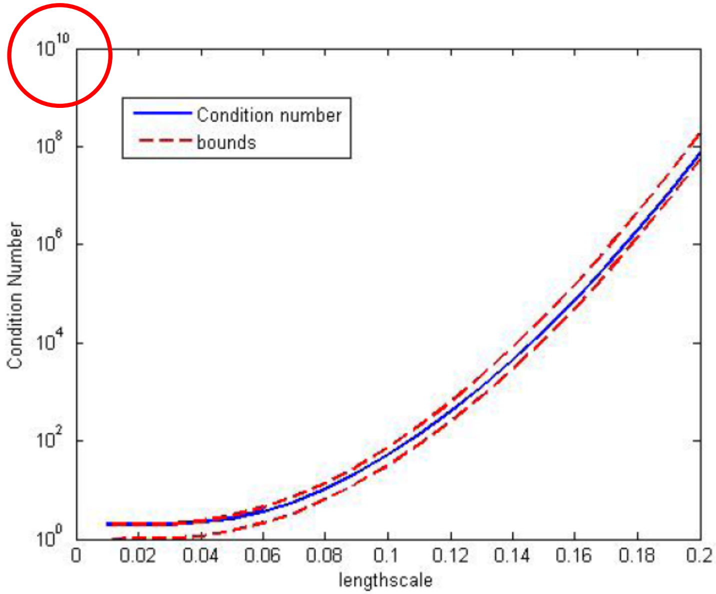
$$\alpha\,\kappa(\mathbf{C}) \leq \kappa(\mathbf{B}^{-1} + \mathbf{H}^T\mathbf{R}^{-1}\mathbf{H}) \leq \left(1 + \left(\frac{\sigma_b^2}{\sigma_o^2}\right)\lambda_{\min}(\mathbf{C})\right)\kappa(\mathbf{C})$$

where $\alpha = \left(\dfrac{1 + \frac{p}{N}\frac{\sigma_b^2}{\sigma_o^2}\lambda_{\min}(\mathbf{C})}{1 + \frac{p}{N}\frac{\sigma_b^2}{\sigma_o^2}\lambda_{\max}(\mathbf{C})}\right)$

and p = number of observations

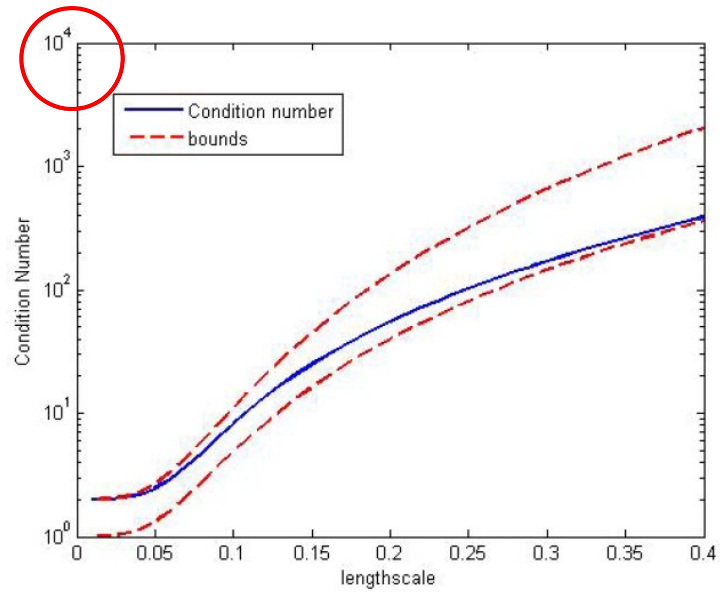
Conditioning of Hessian

Condition Number of ($B^{-1} + HR^{-1}H^T$) vs Length Scale



Periodic Gaussian Exponential

$$\mathbf{B}_{ij} = \sigma_b^2 \exp\left(\frac{-r_{i,j}^2}{2L^2}\right)$$



Laplacian 2nd Derivative

$$\mathbf{B}^{-1} = \gamma^{-1}\left(\mathbf{I} + \frac{l^4}{2\Delta x^4}(\mathbf{L})^2\right)$$

Blue = condition number   Red = bounds

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Preconditioning - Control Variable Transform

To improve conditioning transform to new variable :

- $\mathbf{z} = \mathrm{B}^{1/2}(\mathbf{x}_0 - \mathbf{x}_0^{\mathrm{b}})$
- Uncorrelated variable
- Equivalent to preconditioning by $\mathrm{B}^{1/2}$
- Hessian of transformed problem is

$$\boxed{\mathrm{I} + \mathrm{B}^{1/2}\hat{\mathrm{H}}^T\hat{\mathrm{R}}^{-1}\hat{\mathrm{H}}\mathrm{B}^{1/2}}$$

Preconditioned Hessian

Bounds on the conditioning of the preconditioned Hessian are:

$$1 + \frac{\sigma_b^2}{\sigma_o^2}\gamma \leq \kappa(\mathbf{I} + \sigma_o^{-2}(\mathbf{B}^{1/2}\mathbf{H}^T\mathbf{H}\mathbf{B}^{1/2})) \leq 1 + \frac{\sigma_b^2}{\sigma_o^2}\nu_0$$

where

$$\nu_0 = \|\mathbf{HCH^T}\|_\infty \,, \quad \gamma = \frac{1}{p}\sum_{i,j\in J} c_{i,j} \;.$$

ν_0 changes slowly as a function of length scale.

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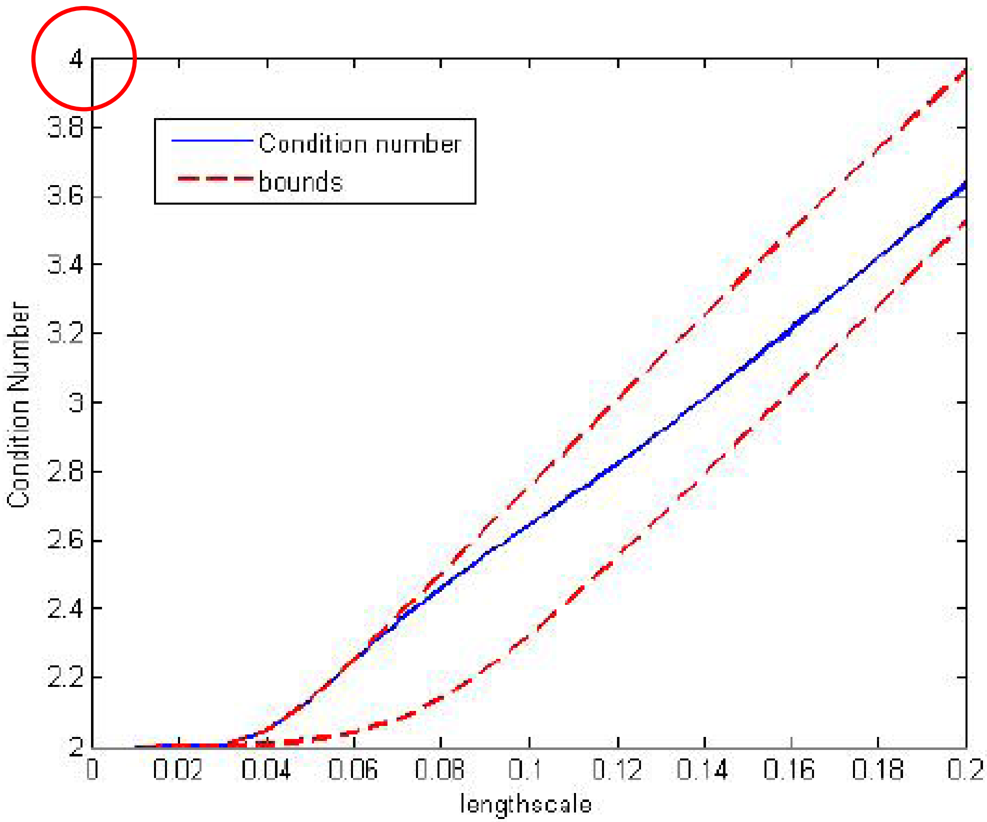
Preconditioned Hessian - Gaussian



Preconditioned (blue)
Bounds (red)

Condition number as a function of length scale

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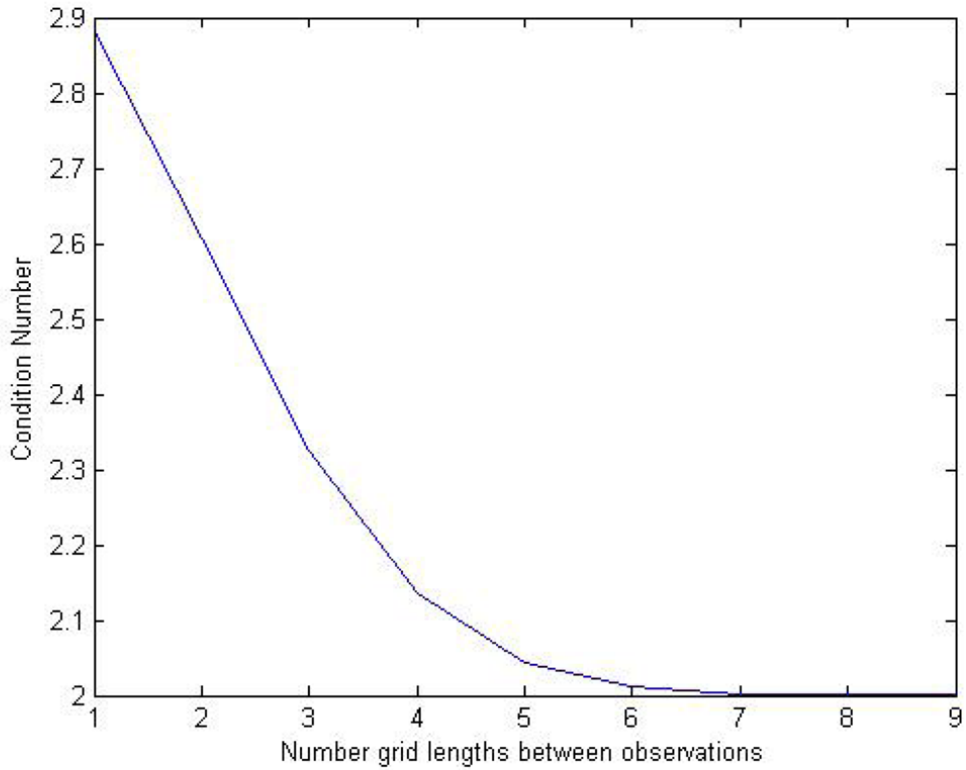
Preconditioned Hessian - Gaussian

Assume two observations at k^{th} and m^{th} grid points

$$\kappa(\mathbf{I} + \sigma_o^{-2}(\mathbf{B}^{1/2}\mathbf{H}^T\mathbf{H}\mathbf{B}^{1/2})) = 1 + \frac{\sigma_b^2}{\sigma_o^2} + \frac{\sigma_b^2}{\sigma_o^2}\,|c_{k,m}|$$

Condition number decreases as the separation of the observations increases

Preconditioned Hessian - Gaussian



Condition number as a function of observation spacing

Extension to 4DVar

Convergence depends on condition number of

$$\boxed{\mathbf{I} + \mathbf{B}^{1/2}\hat{\mathbf{H}}^T\hat{\mathbf{R}}^{-1}\hat{\mathbf{H}}\mathbf{B}^{1/2}}$$

where

$$\hat{\mathbf{H}} = \begin{pmatrix} \mathbf{H}_0 \\ \mathbf{H}_1\mathbf{M}_{0,1} \\ \vdots \\ \mathbf{H}_n\mathbf{M}_{0,n} \end{pmatrix} \qquad \hat{\mathbf{R}} = \begin{pmatrix} \mathbf{R}_0 & 0 & \cdots & 0 \\ 0 & \mathbf{R}_1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & \mathbf{R}_n \end{pmatrix}$$

$$\mathbf{M}_{0,k} = \frac{\partial \mathcal{M}_{0,k}}{\partial \mathbf{x}}|_{\mathbf{x}_0} \qquad \mathbf{H}_k = \frac{\partial \mathcal{H}_k}{\partial \mathbf{x}}|_{\mathcal{M}_{0,k}(\mathbf{x}_0)}$$

Preconditioned Hessian

Bounds on the conditioning of the preconditioned Hessian are:

$$1 + \frac{1}{p(n+1)} \frac{\sigma_b^2}{\sigma_o^2} \sum_{i,j=1}^{p(n+1)} (\hat{\mathbf{H}}\mathbf{C}\hat{\mathbf{H}}^T)_{i,j} \leq \kappa(\mathbf{A}_p) \leq 1 + \frac{\sigma_b^2}{\sigma_o^2} ||\hat{\mathbf{H}}\mathbf{C}\hat{\mathbf{H}}^T||_\infty$$

where

- $\mathbf{B} = \sigma_b{}^2\ \mathbf{C}$, $\mathbf{C}$ is correlation matrix
- $\mathbf{R}_k = \sigma_0^2 \mathbf{I}$ for $k = 0, \dots, n$
- Advection model discretized using upwind scheme

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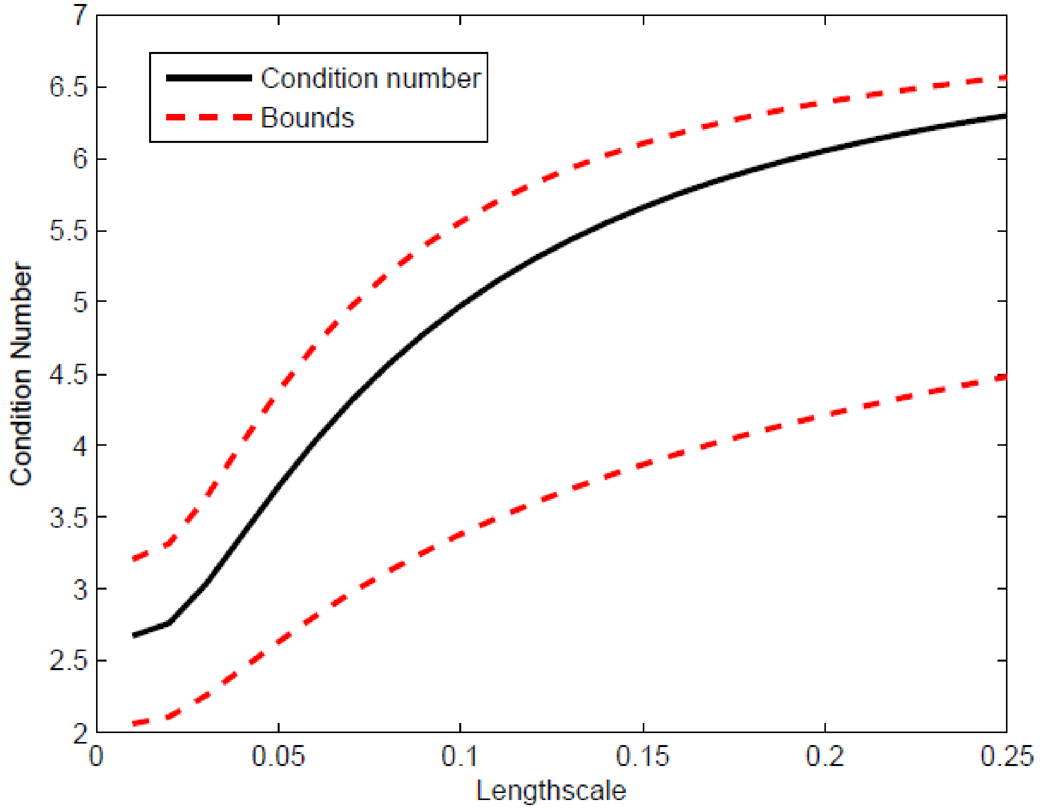
Condition of Preconditioned 4DVar – using SOAR Correlation Matrix



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Convergence Rates of CG in 4DVar – using SOAR Correlation Matrix

	Iterations	
Lengthscale	Unprecond	Precond
0.01	8	8
0.1	54	11
0.2	187	12
0.3	361	12

Haben et al, 2011

Conditioning – with Correlated Observation Errors

We can establish the following theorem:

Let $\mathbf{B} \in \mathbb{R}^{N \times N}$ *and* $\mathbf{R} \in \mathbb{R}^{p \times p}$*, with* $p < N$*, be the background and observation error covariance matrices respectively. Additionally, let* $\mathbf{H} \in \mathbb{R}^{p \times N}$ *be the observation operator. Then the following bounds are satisfied by the condition number of the Hessian,* $\mathbf{S} = \mathbf{B}^{-1} + \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H}$*,*

$$\frac{\kappa(\mathbf{B})}{(1 + \frac{\lambda_{min}(\mathbf{B})}{\lambda_{min}(\mathbf{R})} \lambda_{max}(\mathbf{H}\mathbf{H}^T))} \leq \kappa(\mathbf{S}) \leq (1 + \frac{\lambda_{min}(\mathbf{B})}{\lambda_{min}(\mathbf{R})} \lambda_{max}(\mathbf{H}\mathbf{H}^T))\kappa(\mathbf{B}).$$

Similar analysis – leads to the same conclusions, but reveals important relations between prior and observation covariances

Haben et al, 2011; Haben 2011, Tabeart, 2016

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Thank you for your attention!

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