Ensemble Data Assimilation and Particle Filters for NWP

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NWP @ DWD &
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With the help of many people, in particular:

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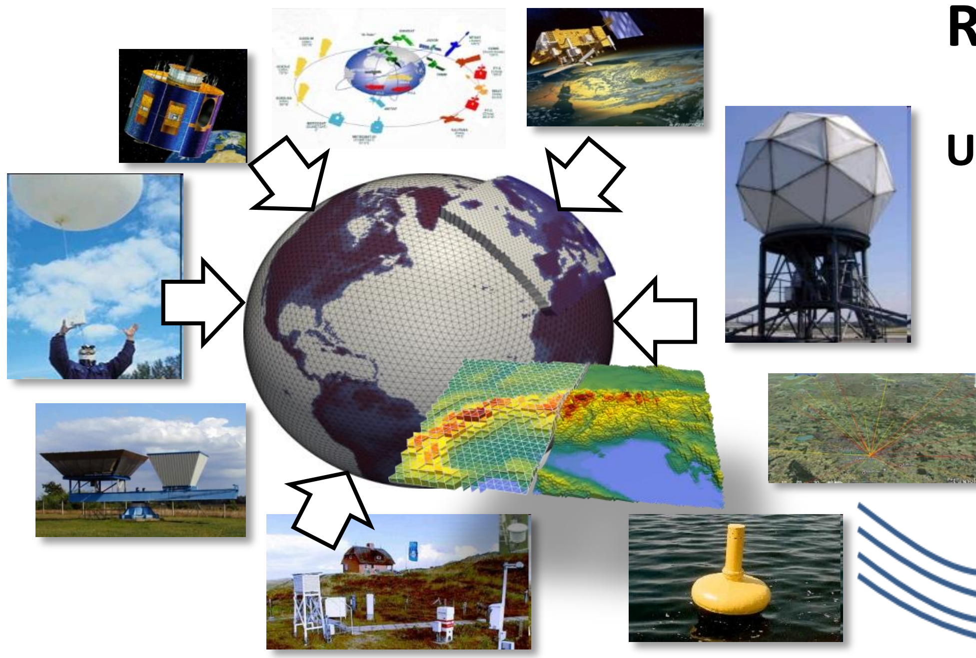

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Kobe November 2018

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Deutscher Wetterdienst
Wetter und Klima aus einer Hand

1. Why and Where **Distributions, Risk** and **Uncertainty?**
2. Discussion of **Ensemble (+Particle) Methods**
3. Framework Global+LAM+LES Model: **ICON** and **ICON-EPS** and the **LEKTF+EnVAR/KENDA** System
4. **LAPF & LMCPF Particle Filters** for Non-Gaussian Distributions – Details and Results

Why Distributions , Risk, Uncertainty?

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Wetter und Klima aus einer Hand

Extreme Weather Even… triggered by … threaten … ture and economy!

Lightning, C… precipitatio… impact the life of inc…viduals ans the society.

…es:s	Level
	Moderate
…-10)	Strong
…nfall (10-25mm/h)	Strong
Storm Force Gusts, Heavy Rainfall	Strong
Storm Force Gusts, Heavy Rainfall, Hail	Strong

Protect Lifes!

Protect Property!

Protect Society and Business

We need Uncertainty Estimates for Risk Prediction!

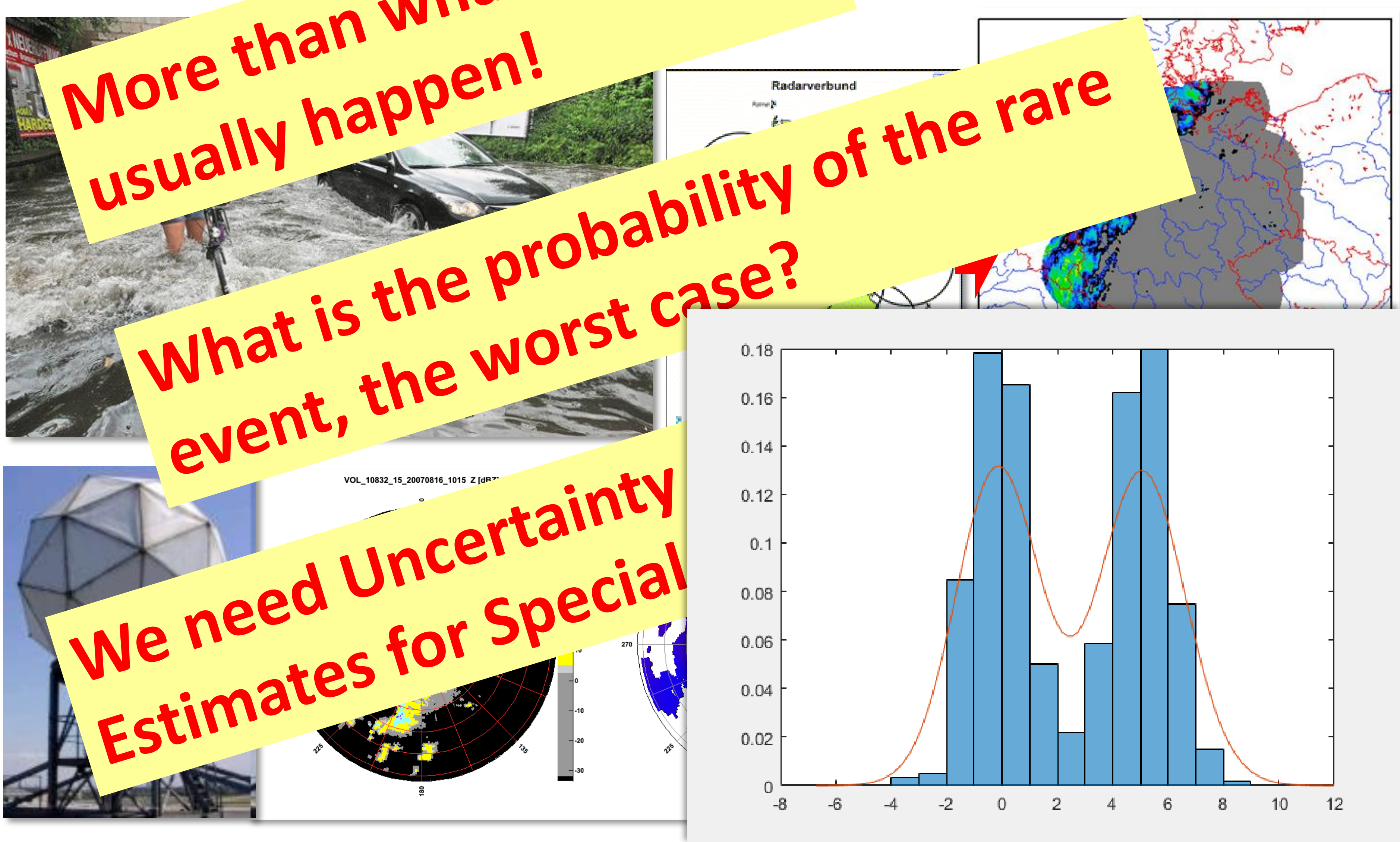
NOW, 5min, 30min, … 1h, 2h, 6h, 24h, 72h, …

Why Distributions , Risk, Un… nty?

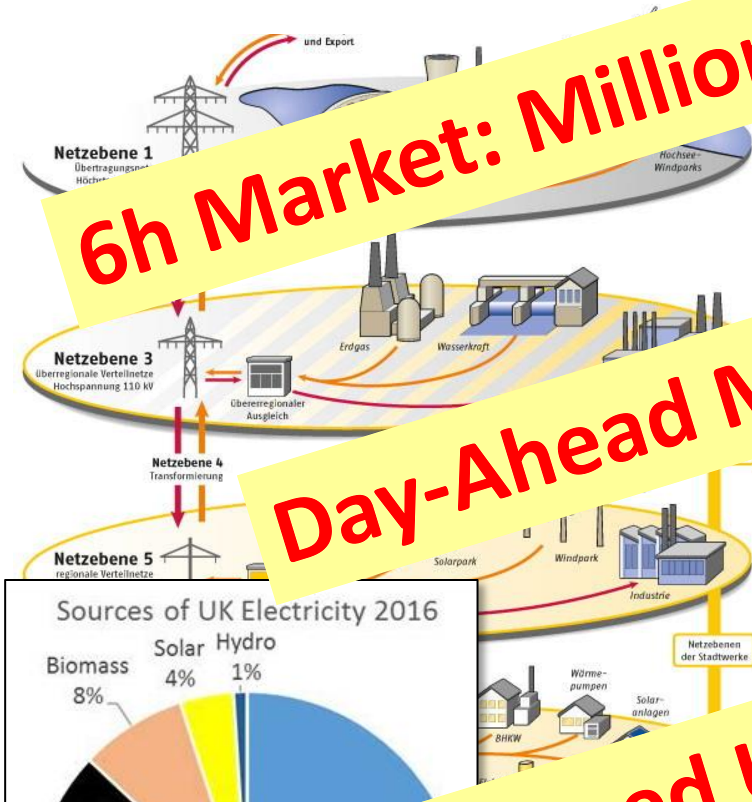
More than what would usually happen!

What is the probability of the rare event, the worst case?

We need Uncertainty Estimates for Special



Why Distributions , Risk, Uncertainty?

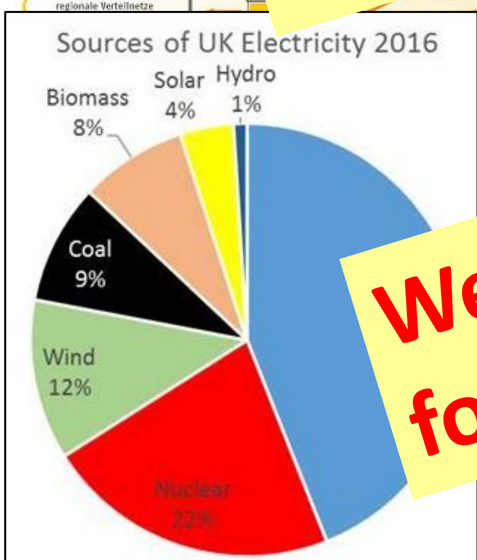


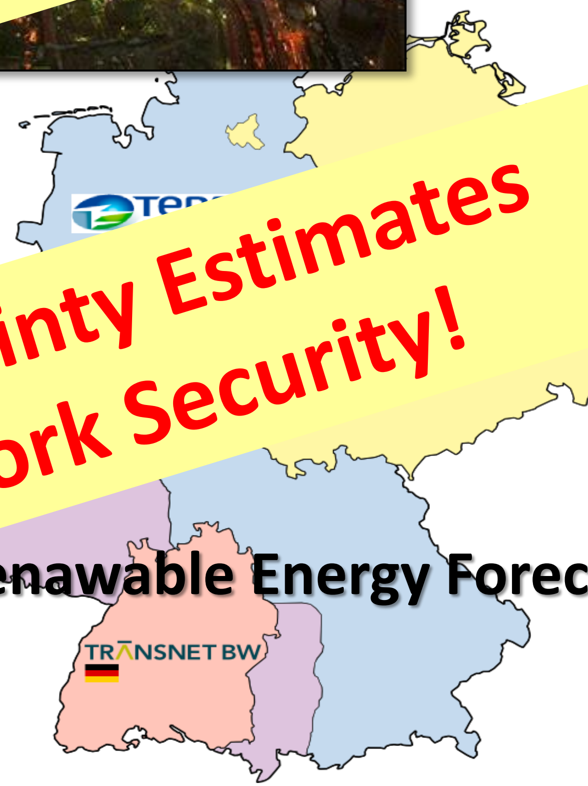
.net/pressemitteilung-76

6h Market: Millions

Day-Ahead Market: Billions!

We need Uncertainty Estimates for Energy Network Security!

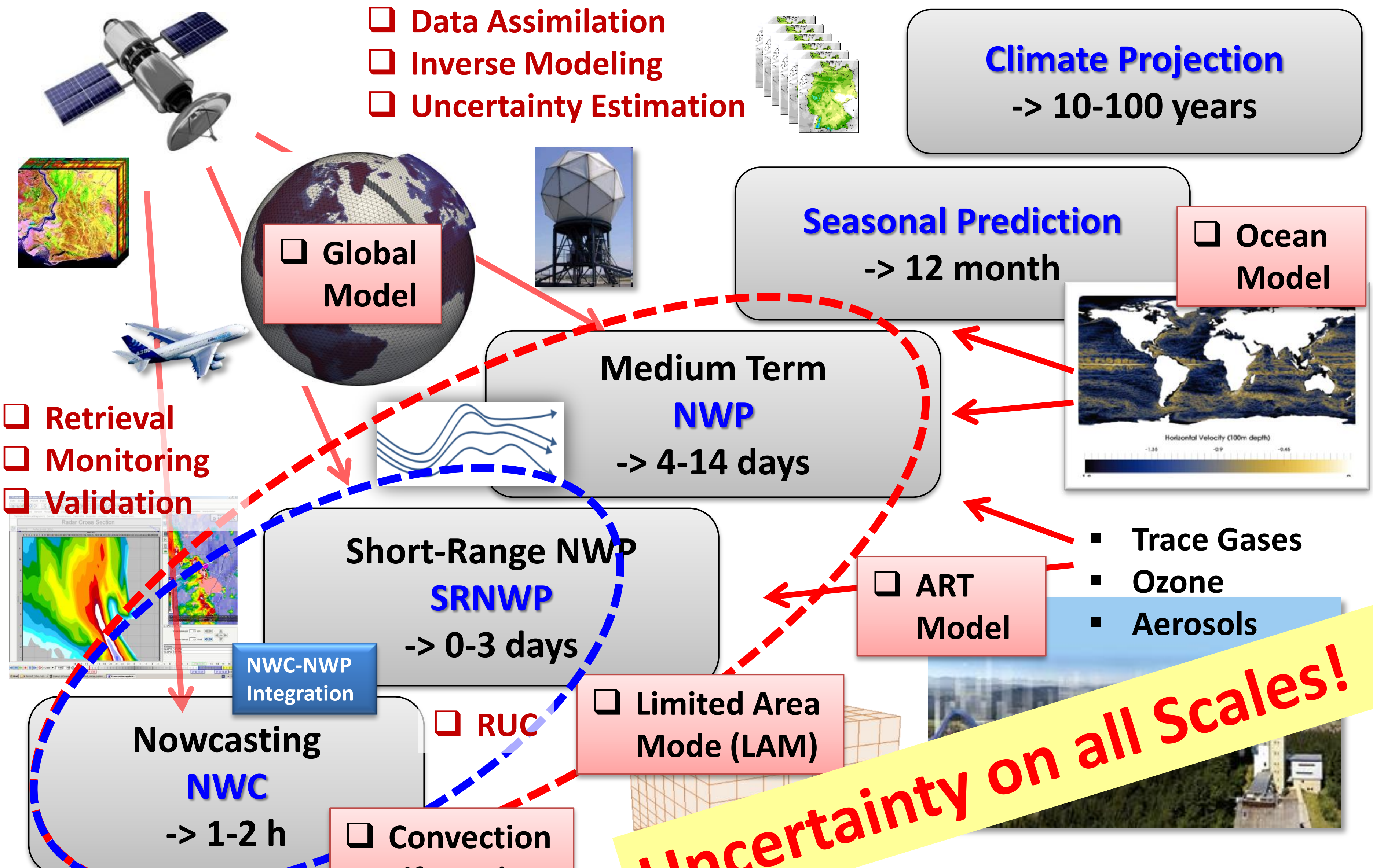




Renawable Energy Forecasting

Bildquelle: https://de.wikipedia.org/wiki/%C3%9Cbertragungsnetzbetreiber

Framework Numerical Weather Prediction



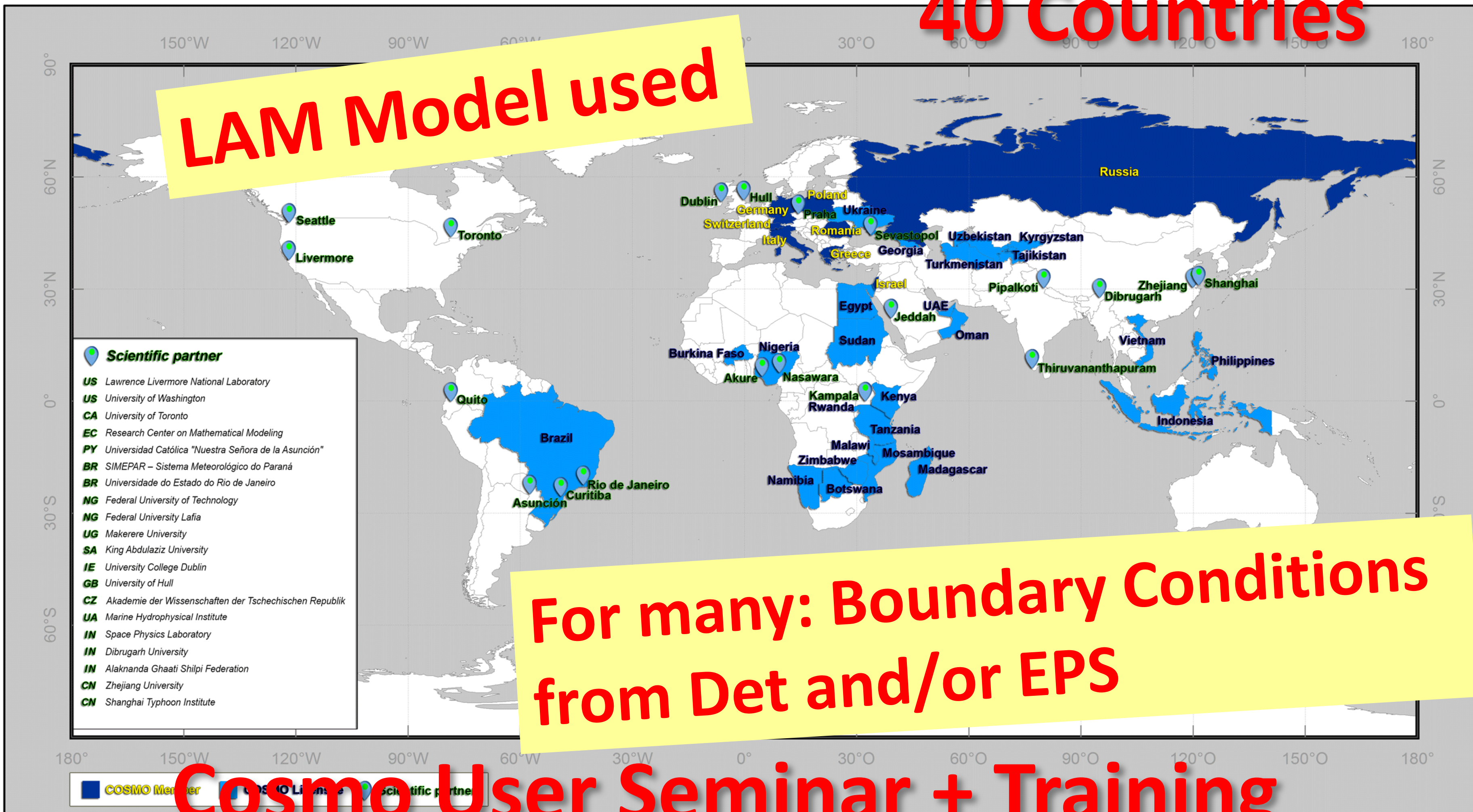
Roland Potthast, 2018

CONSORTIUM FOR SMALL SCALE MODELING

COSMO

40 Countries

LAM Model used



For many: Boundary Conditions from Det and/or EPS

Cosmo User Seminar + Training Course + Symposium

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Data Assimilation Methods

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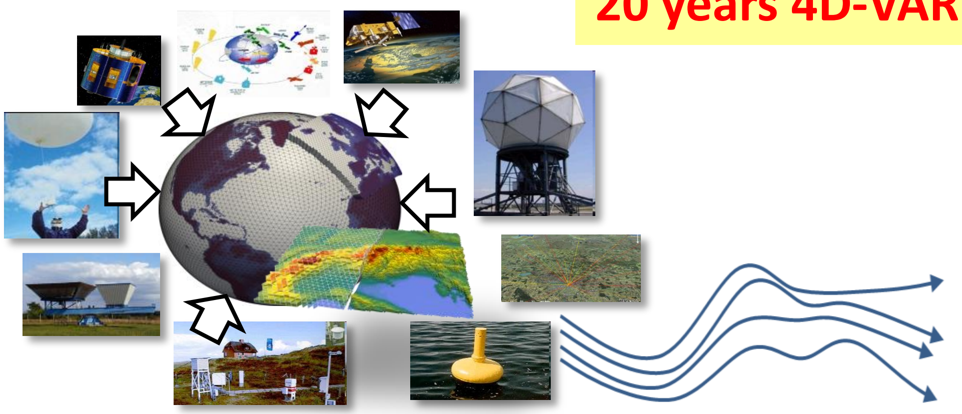
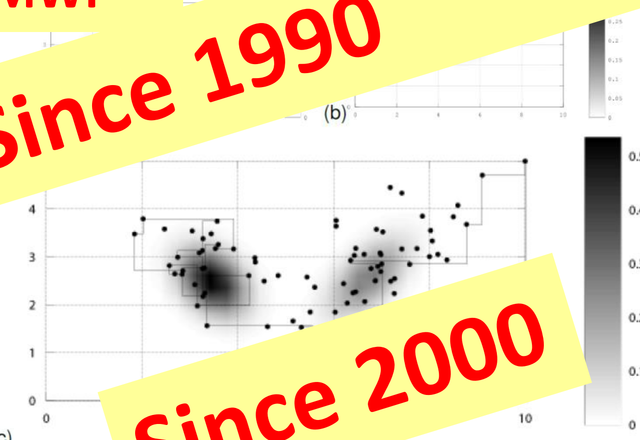
- Why **variational** Data Assimilation (3D/4D-VAR)?

20 years 4D-VAR at ECMWF





Since 1990

- Why **Ensemble** Data Assimilation (EDA)?

Since 2000

- Why **Hybrid** Methods? (3D/4D-EnVAR)

Since 2010

- Why Particle **Filters**? (PF,GPF,ETPF,LAPF,LMCPF)

Since 2020

☺

Variational Analysis (3D/4D-VAR)

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Recall where we came from …

The minimization of

Obs Operator

$$J(x) = \|x - x^b\|^2_{B^{-1}} + \|y - Hx\|^2_{R^{-1}}$$

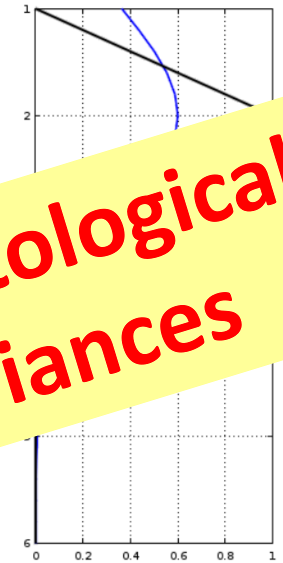
leads to the **analysis**

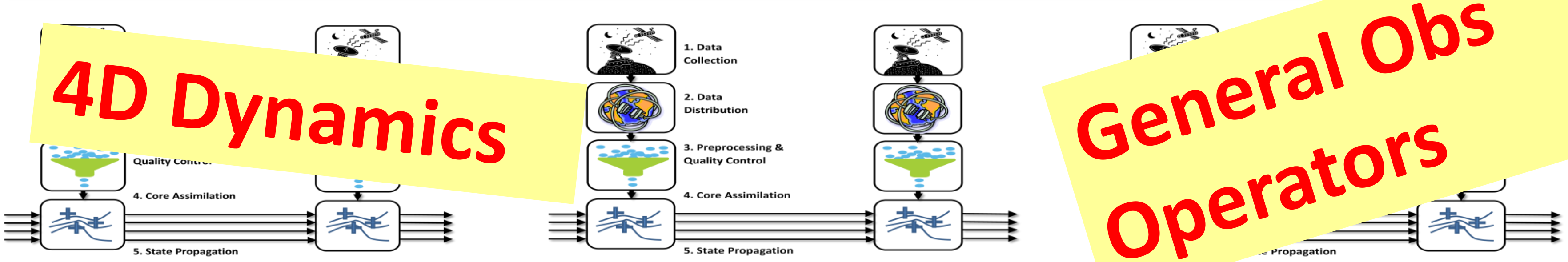
$$x^a = x^b + B H^T \left(R + H B H^T \right)^{-1} (y - Hx^b)$$

Climatological covariances

Minimization

This is the **mother** of all data assimilation formulas

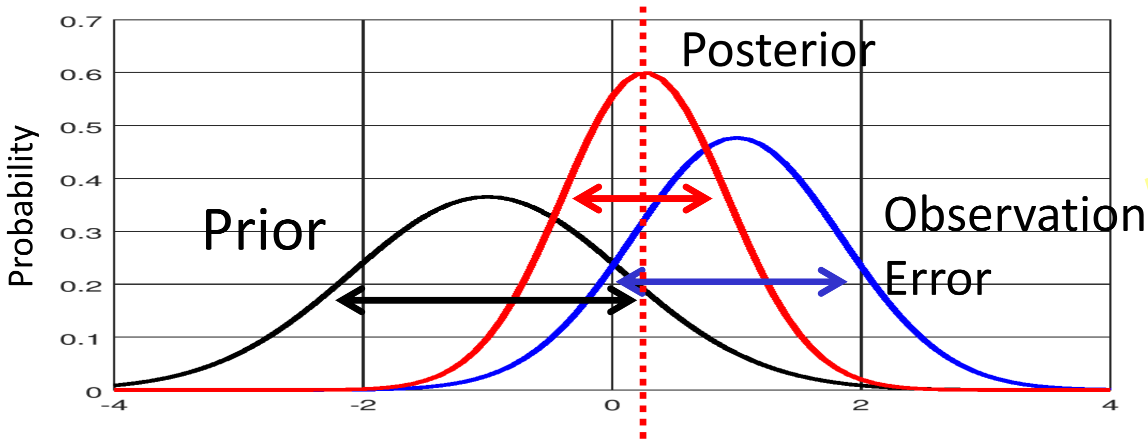




4D Dynamics

General Obs Operators

Stochastic View ⇔ Minimization



Var == Mean/ ML estimator!

- **Gaussian Prior** $$p(x) = e^{-\frac{1}{2}(x-x^b)^T B^{-1}(x-x^b)}$$
- **Gaussian Data Error** $$p(y|x) = e^{-\frac{1}{2}(y-Hx)^T R^{-1}(y-Hx)}$$
- **Gaussian Posterior**

Maximum Likelyhood Estimator = Minimization of Functional = 3/4DVar

$$p(x|y) = c\, e^{-\frac{1}{2}\left\{(x-x^b)^T B^{-1}(x-x^b)+(y-Hx)^T R^{-1}(y-Hx)\right\}}$$

Kalman Filter & Uncertainty Estimation

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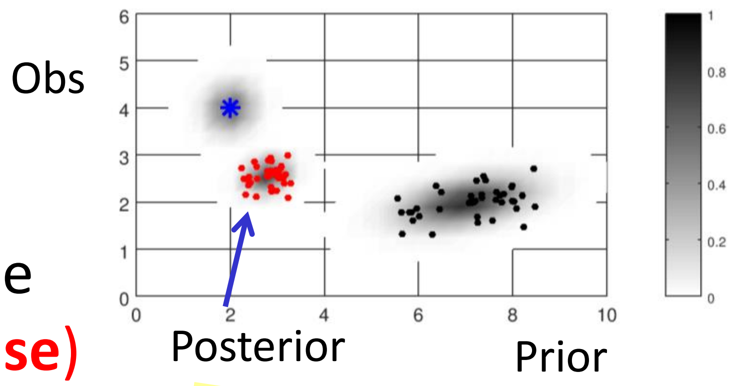
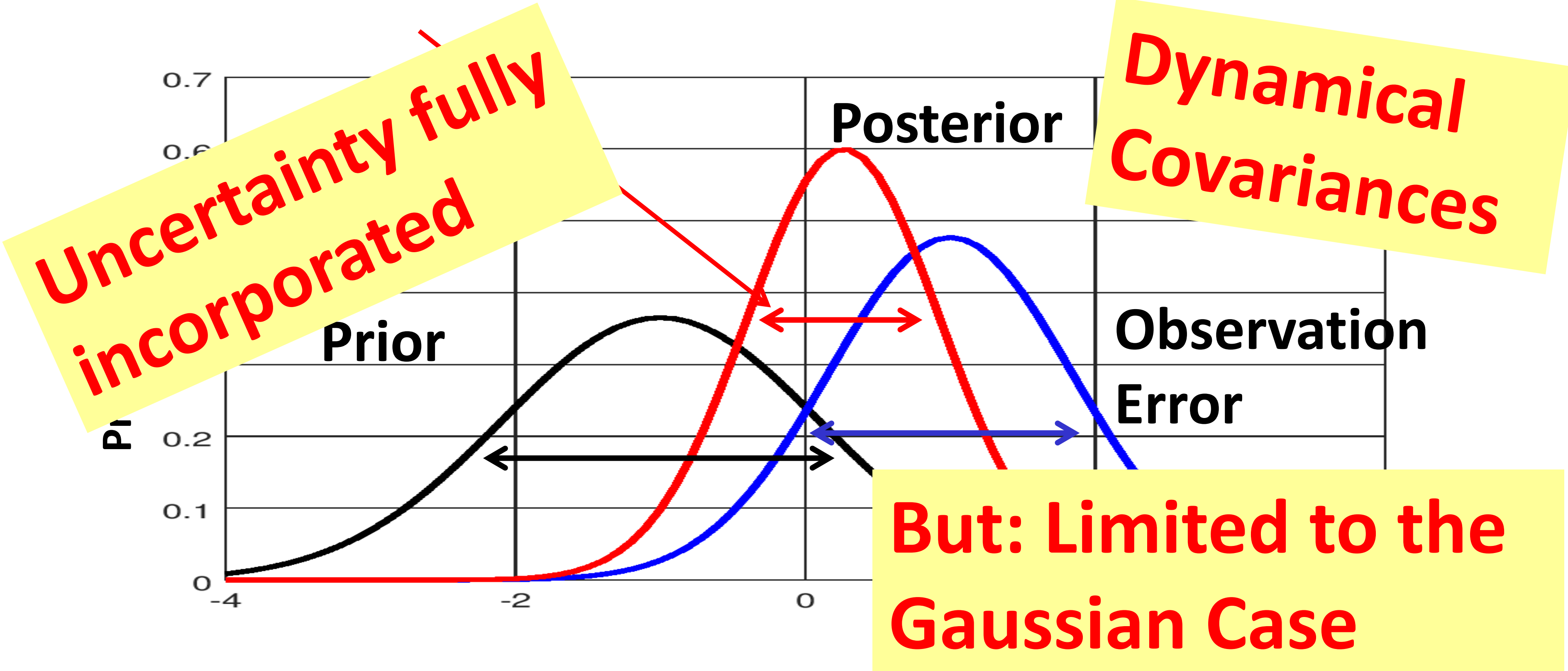
Basic Idea of the Kalman Filter

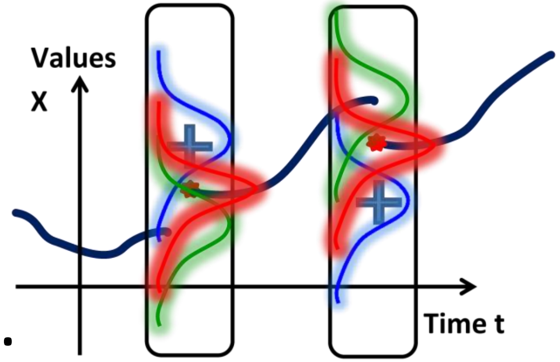


- **Sequentiel** Assimilation of Data
- Do not only adapt the mean, but also the **Covariance B (Uncertainty, Gaussian Case)**

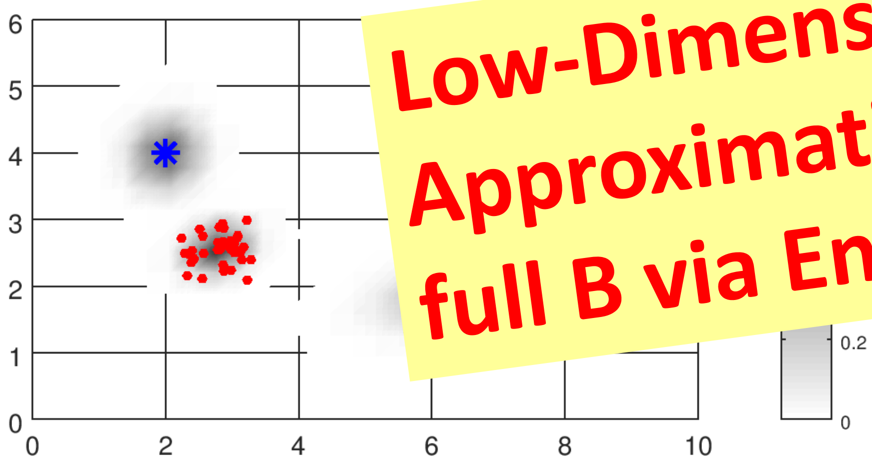


EDA: Ensemble Kalman Filter (EnKF)

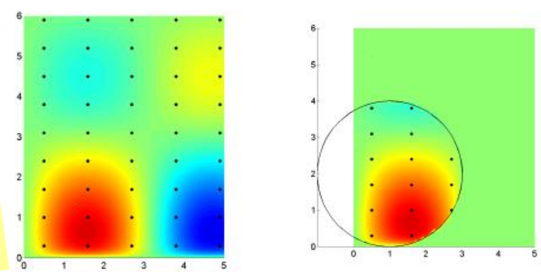
- Kalman Filter needs B update => **expensive**!
- **Estimate** B based on an ensemble of forecasted states (**stochastic estimator**).



B will be **flow-dependent** and variable, depending on the **model dynamics** and on the **observations**



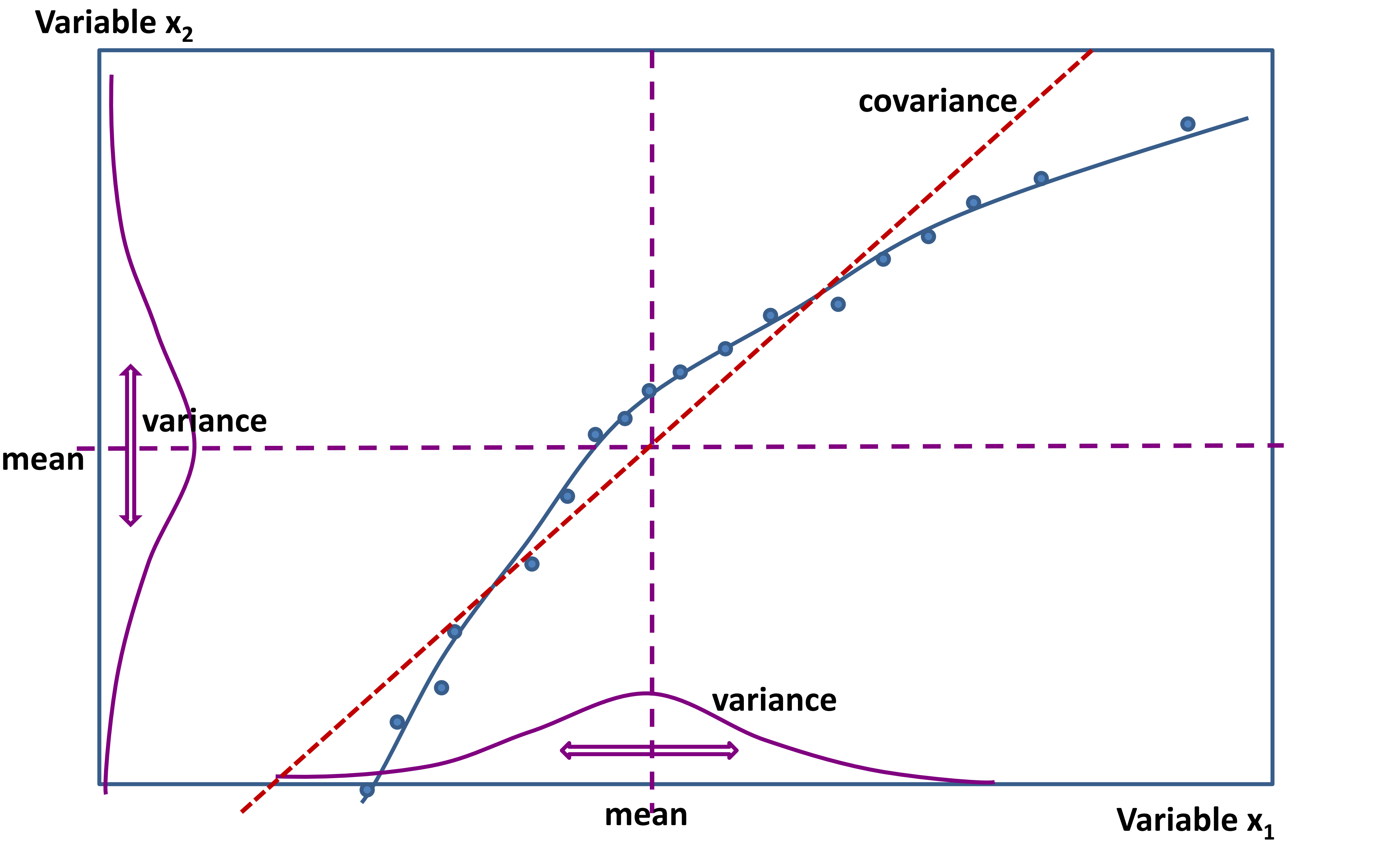
Low-Dimensional Approximation to full B via Ensemble!



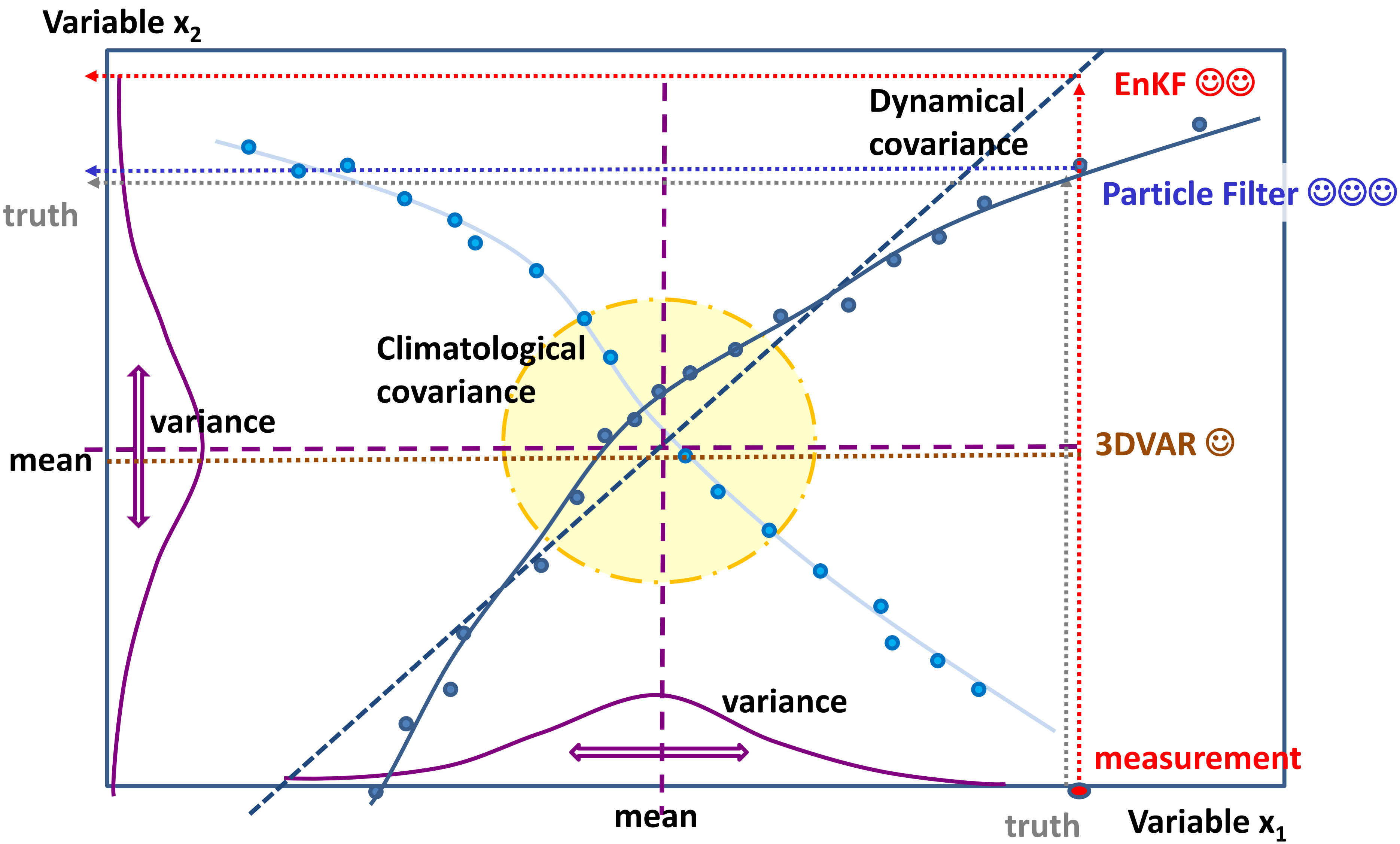
Needs Localization

Localized: LETKF

Using Perturbations, Climatological or Dynamical Covariances, and the Particle Filter



Using Perturbations, Climatological or Dynamical Covariances, and the Particle Filter



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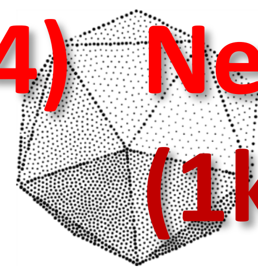
Wetter und Klima aus einer Hand

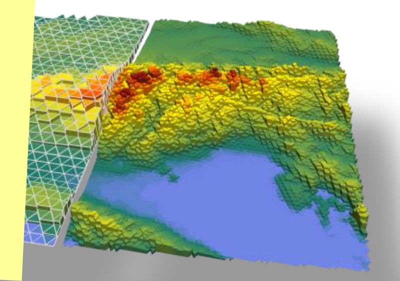
Global NWP Modelling: Det + EPS –
Reality + Goals



1) ICON Model 13km
2) Nest over Europe (6.5km; 2-way)
3) ICON-LAM D2
4) Nest over Germany (1km; 2-way) D1
5) NWC Ensemble

- 3h cycles global+Europe: 180h fc
- 1h ana cycle LAM, 3h fc cycle: 27h fc
- RUC cycle 1h: 6-12h fc

$$\frac{\partial \rho\theta_v}{\partial t} + \nabla\cdot(\vec{v}\rho\theta_v) = 0$$



Full Observation System

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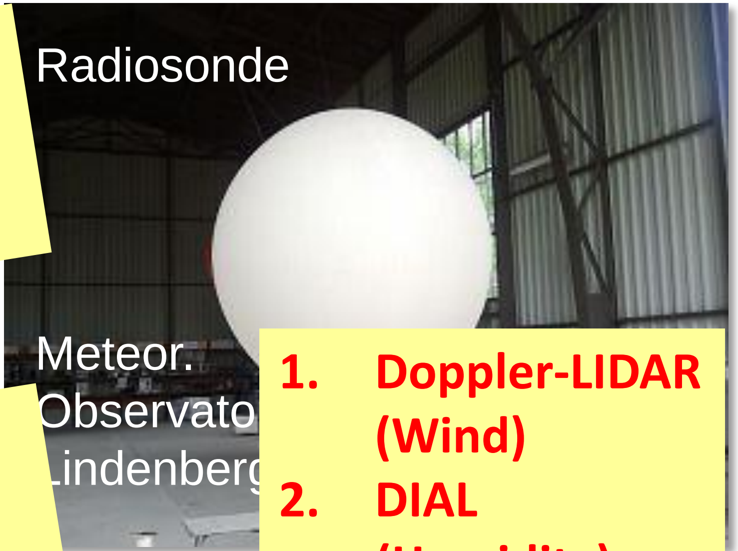
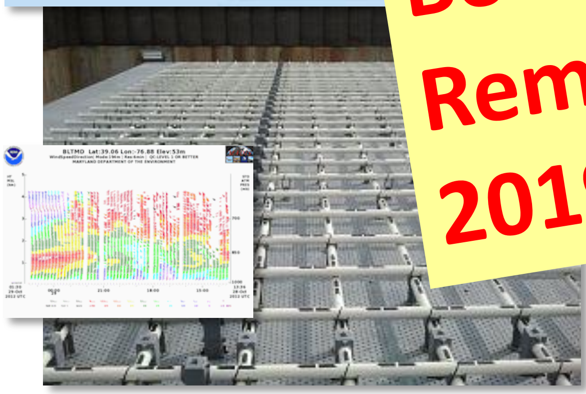
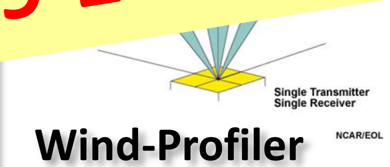


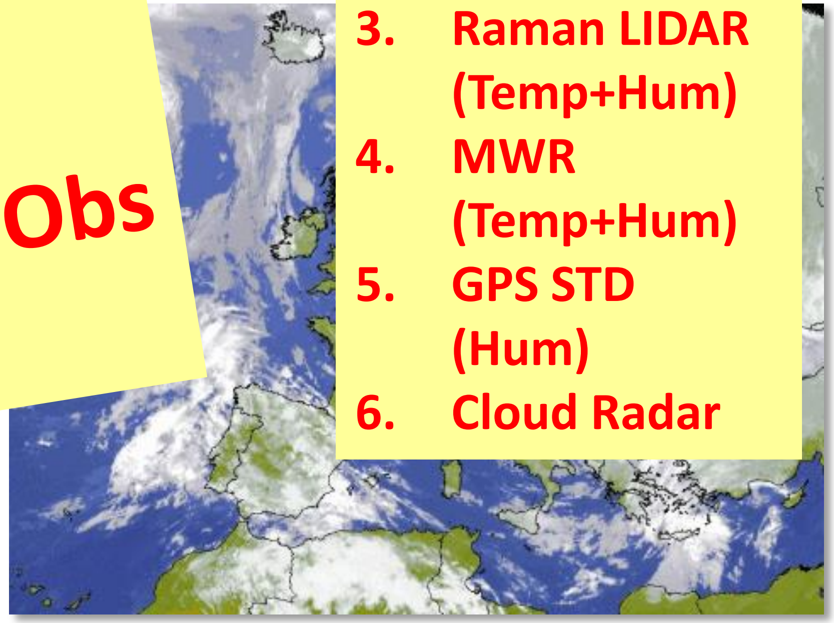
Conventional and Remote Sensing

Exploring Pilot Stations for Boundary Layer Remote Sensing Obs 2019-2025

1. Doppler-LIDAR (Wind)
2. DIAL (Humidity)
3. Raman LIDAR (Temp+Hum)
4. MWR (Temp+Hum)
5. GPS STD (Hum)
6. Cloud Radar







Conventional Synop + Airplanes

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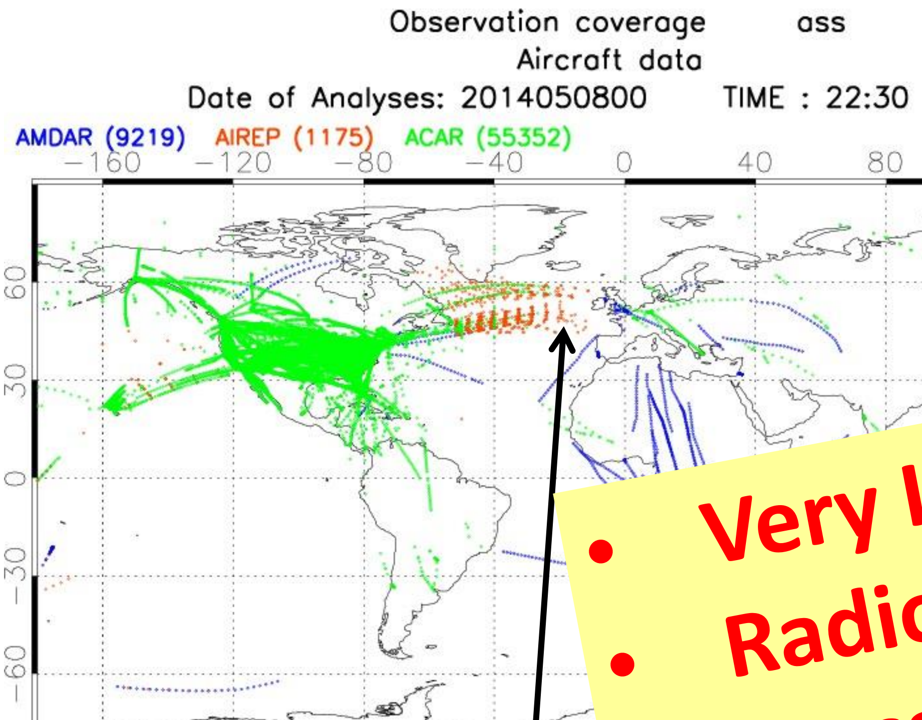
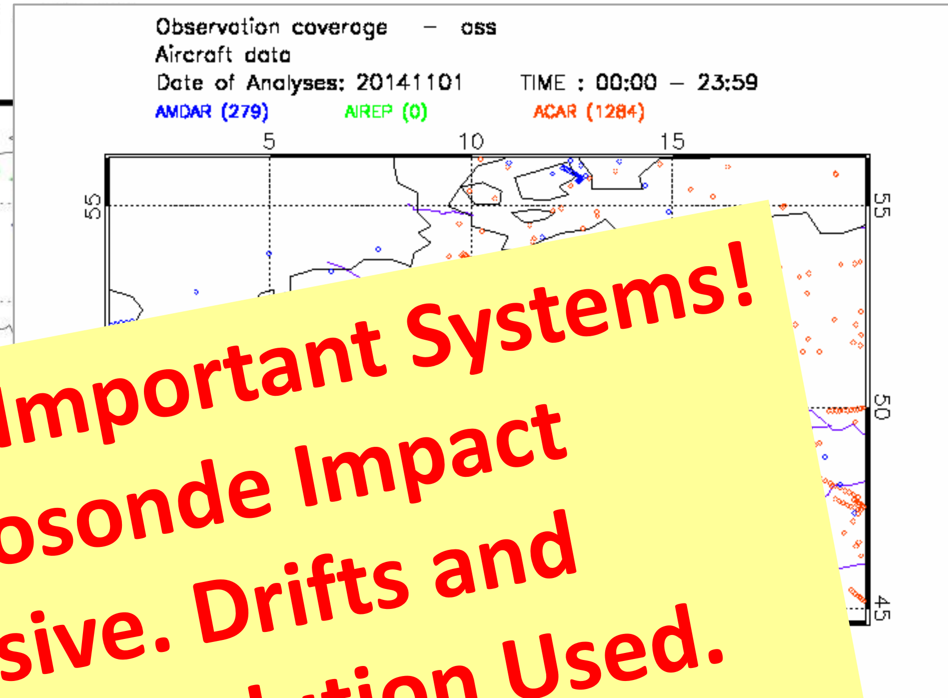
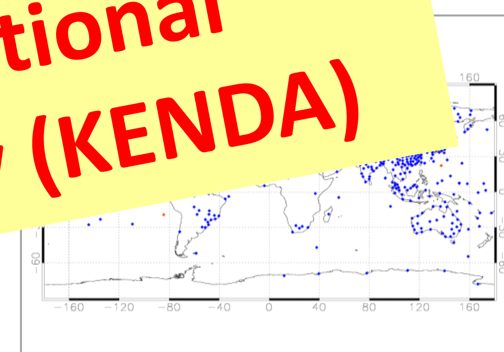
Wetter und Klima aus einer Hand







- Very Important Systems!
- Radiosonde Impact massive. Drifts and High-Resolution Used.
- Mode-S Operational over Germany (KENDA)

Observations: Geostationary Satellites

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Wetter und Klima aus einer Hand

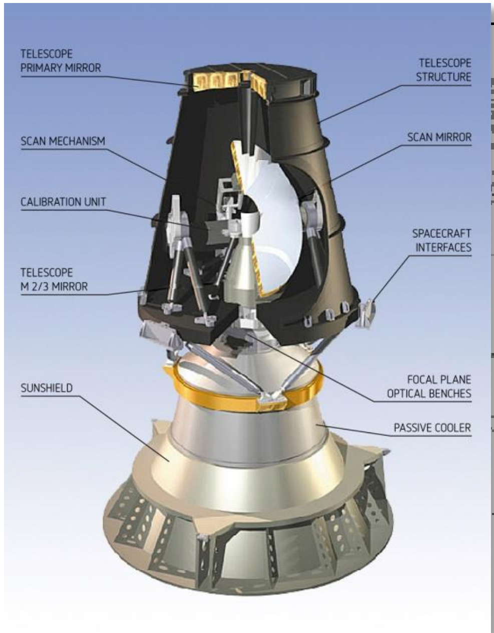
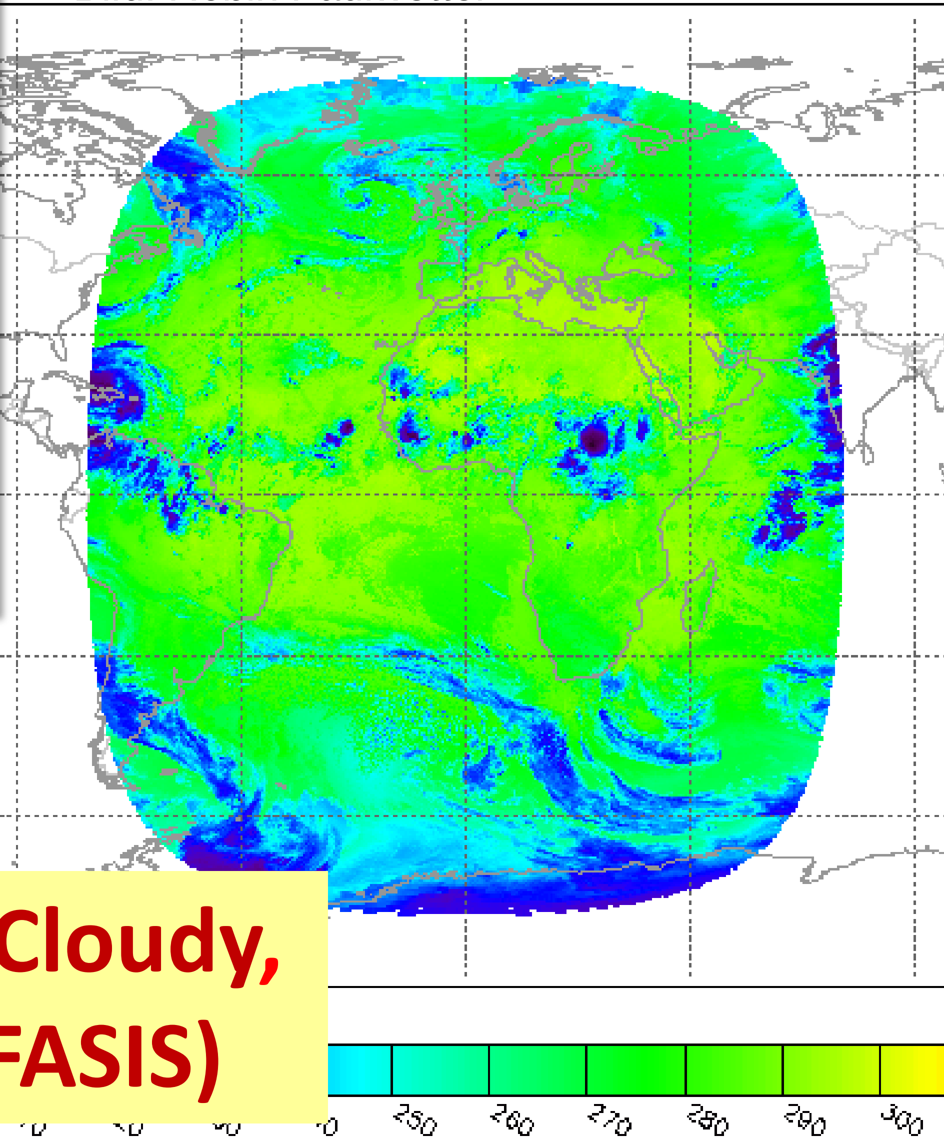
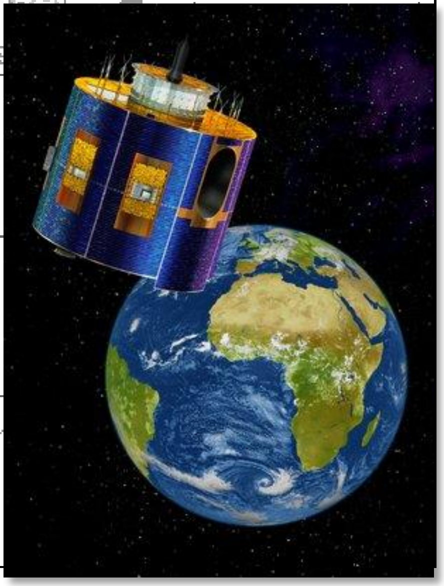


Bild: Robin Faulwetter



SEVIRI
Visible,
Near Infrared,
Infrared



**Clear and Cloudy,
IR, VIS (MFASIS)**

Observations: Polar Orbiting Satellites

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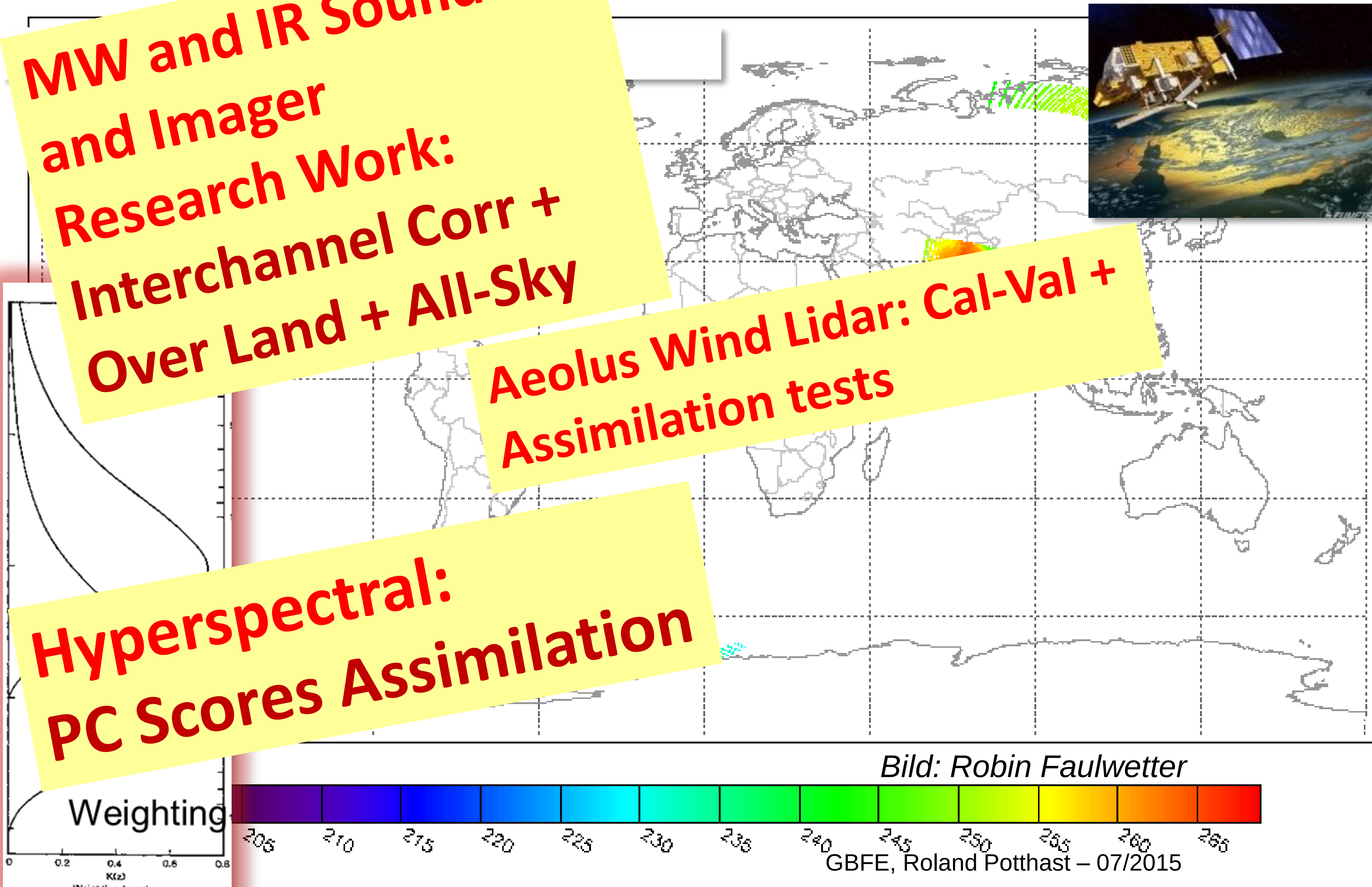
Wetter und Klima aus einer Hand

MW and IR Sounder and Imager Research Work: Interchannel Corr + Over Land + All-Sky



Aeolus Wind Lidar: Cal-Val + Assimilation tests

Hyperspectral: PC Scores Assimilation

Bild: Robin Faulwetter

Ensemble Datenassimilation EnVar

Operational since January 2016

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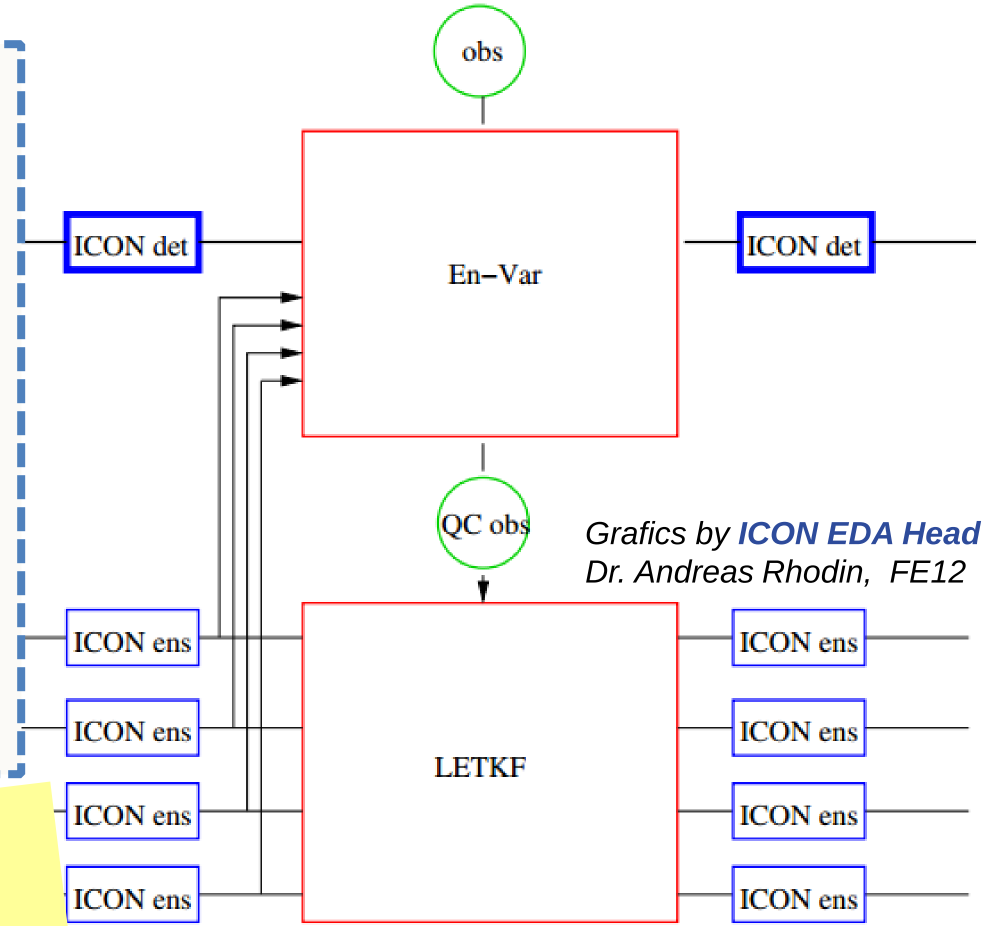
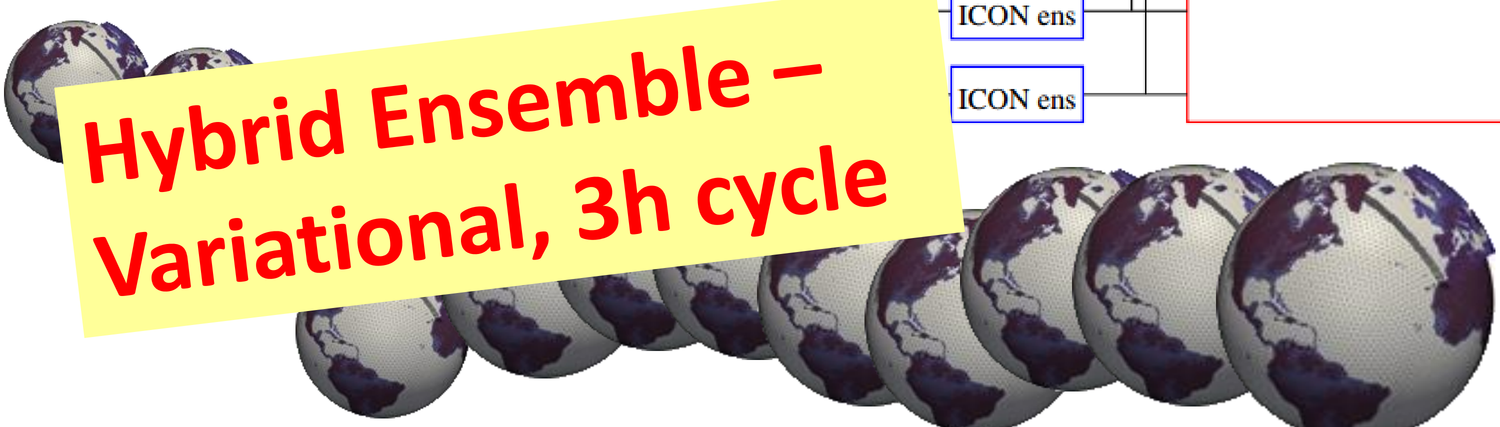
Wetter und Klima aus einer Hand

We are running **ICON EDA** in our Routine since *Jan 2016*

- 40 Members each with 40km global resolution and 20km NEST over Europe
- 1 deterministic 13km/6.5km
- **EPS forecasts** 40 Members 7 Days + 1 Deterministic
- Output for convective-scale EDA/EPS
- **Hybrid System**



*Grafics by **ICON EDA Head** Dr. Andreas Rhodin, FE12*



Hybrid Ensemble – Variational, 3h cycle

Hybrid Methods: EnVAR Scores

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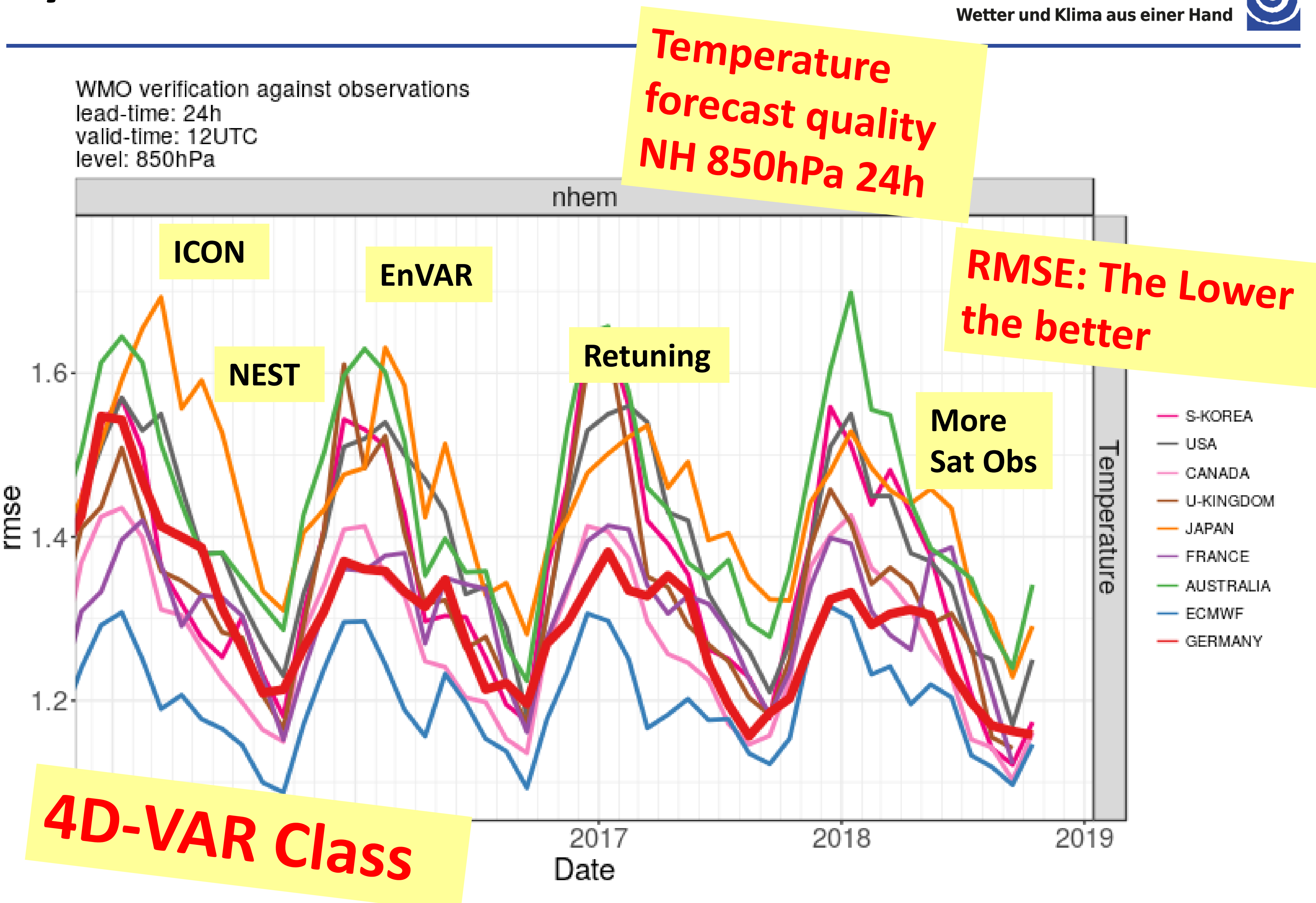

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Hybrid Methods: EnVAR Scores

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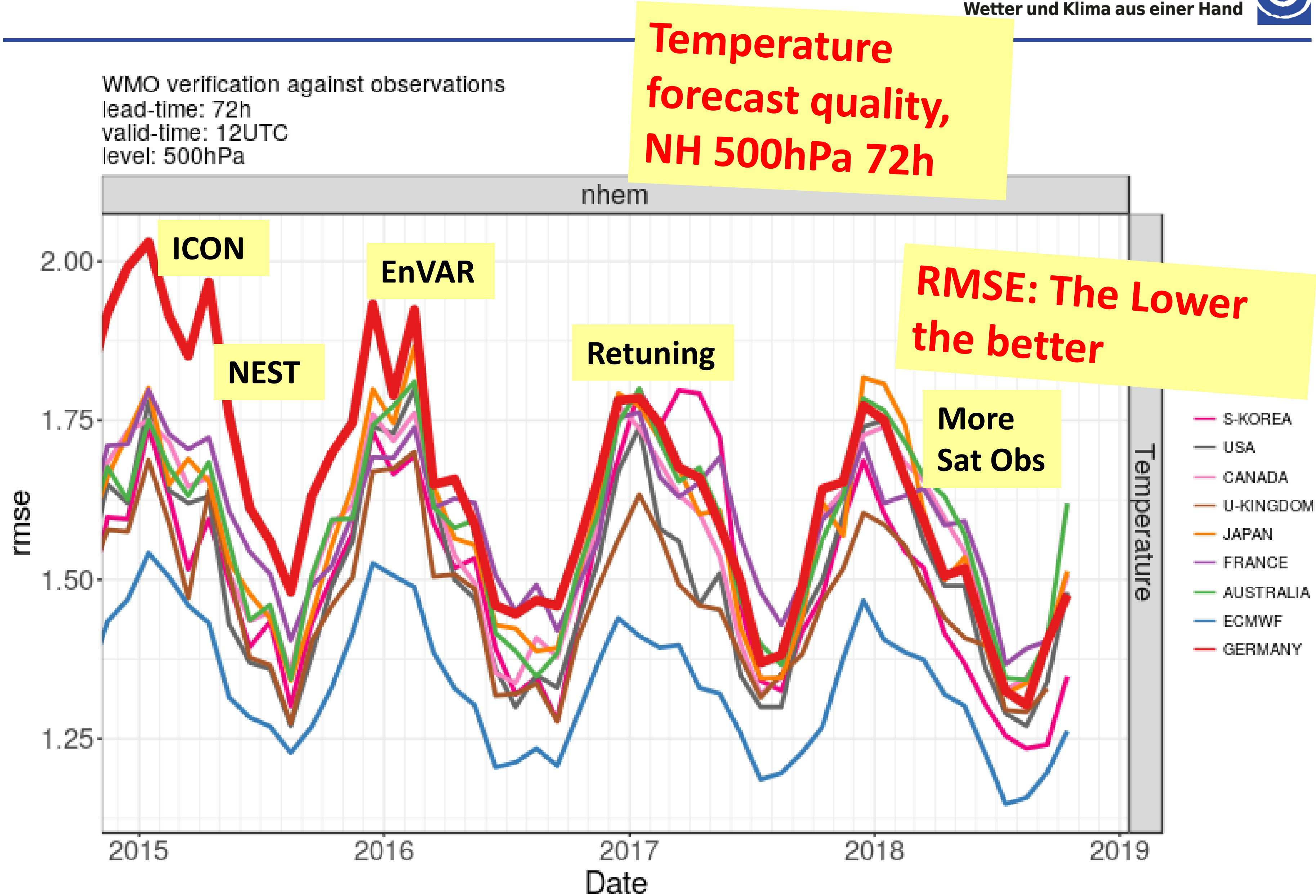

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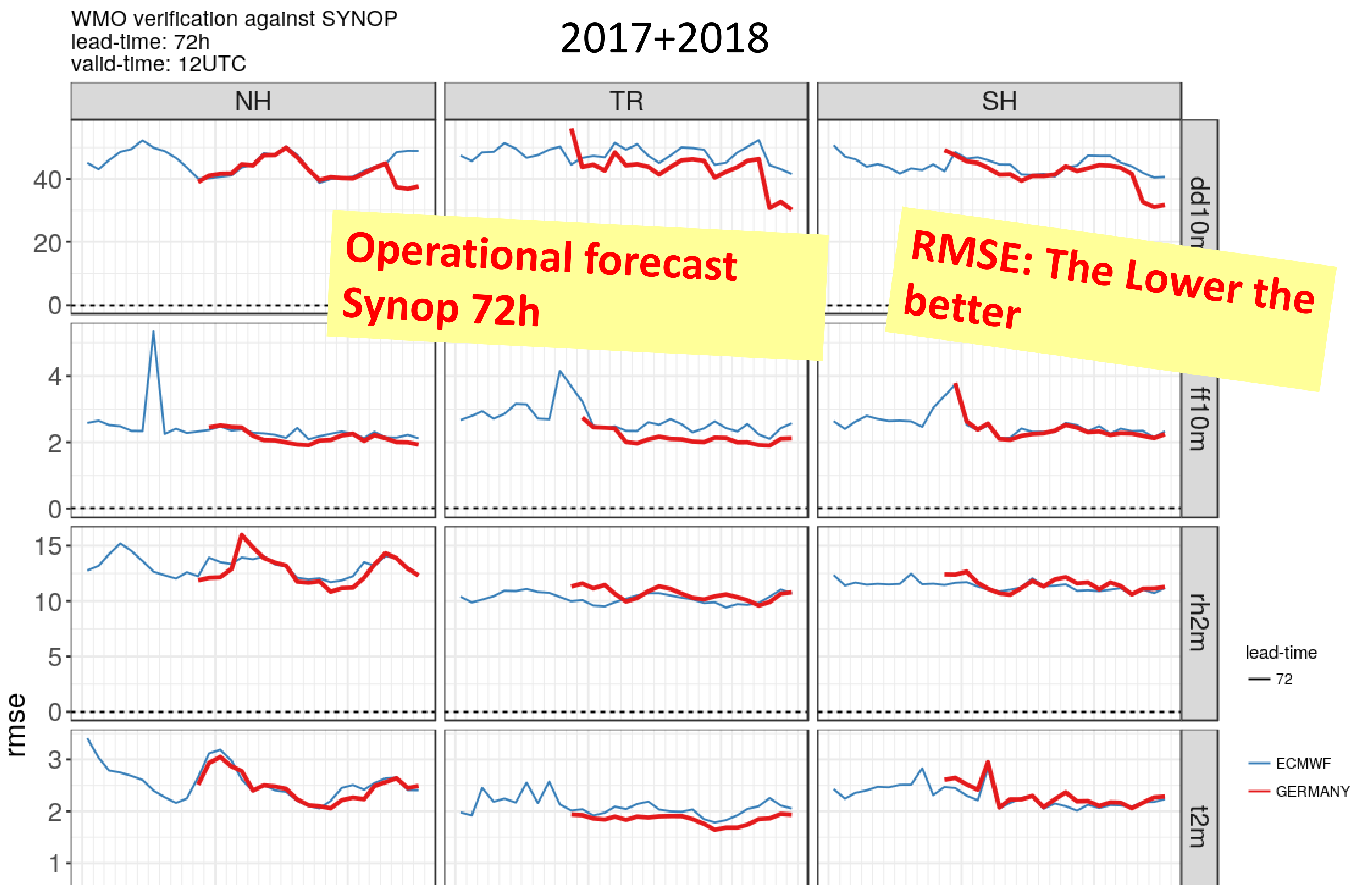

Hybrid Methods: EnVAR Scores

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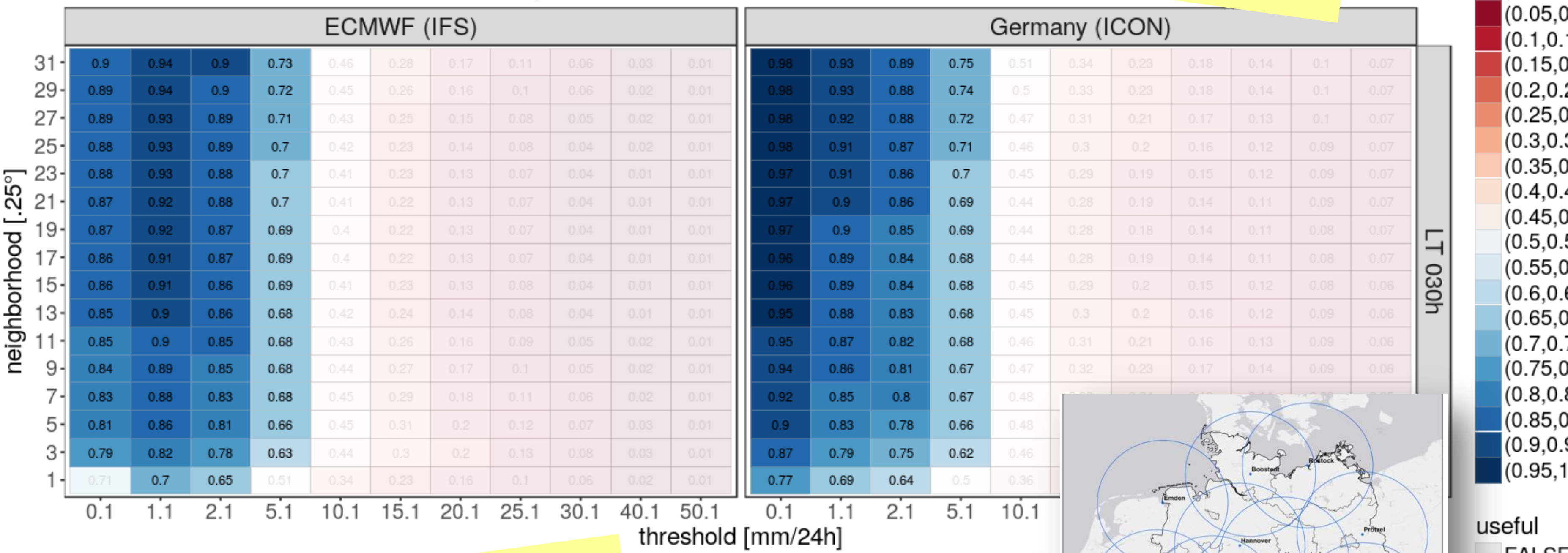
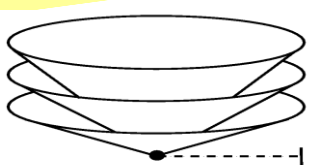
Wetter und Klima aus einer Hand

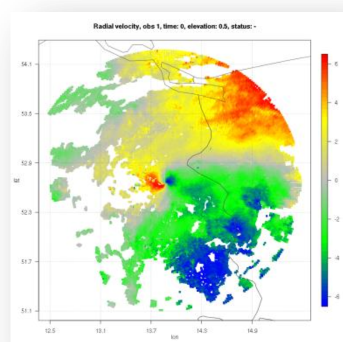
FSS: the higher the better

OBS: german radar (rw) 06UTC-06UTC
PRED: rr_24h (0.25°)
INI: 00UTC
CASES: ECMWF (IFS) 90, Germany (ICON) 90



Operational forecast Precipitation vs RADAR





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BAYES Data Assimilation

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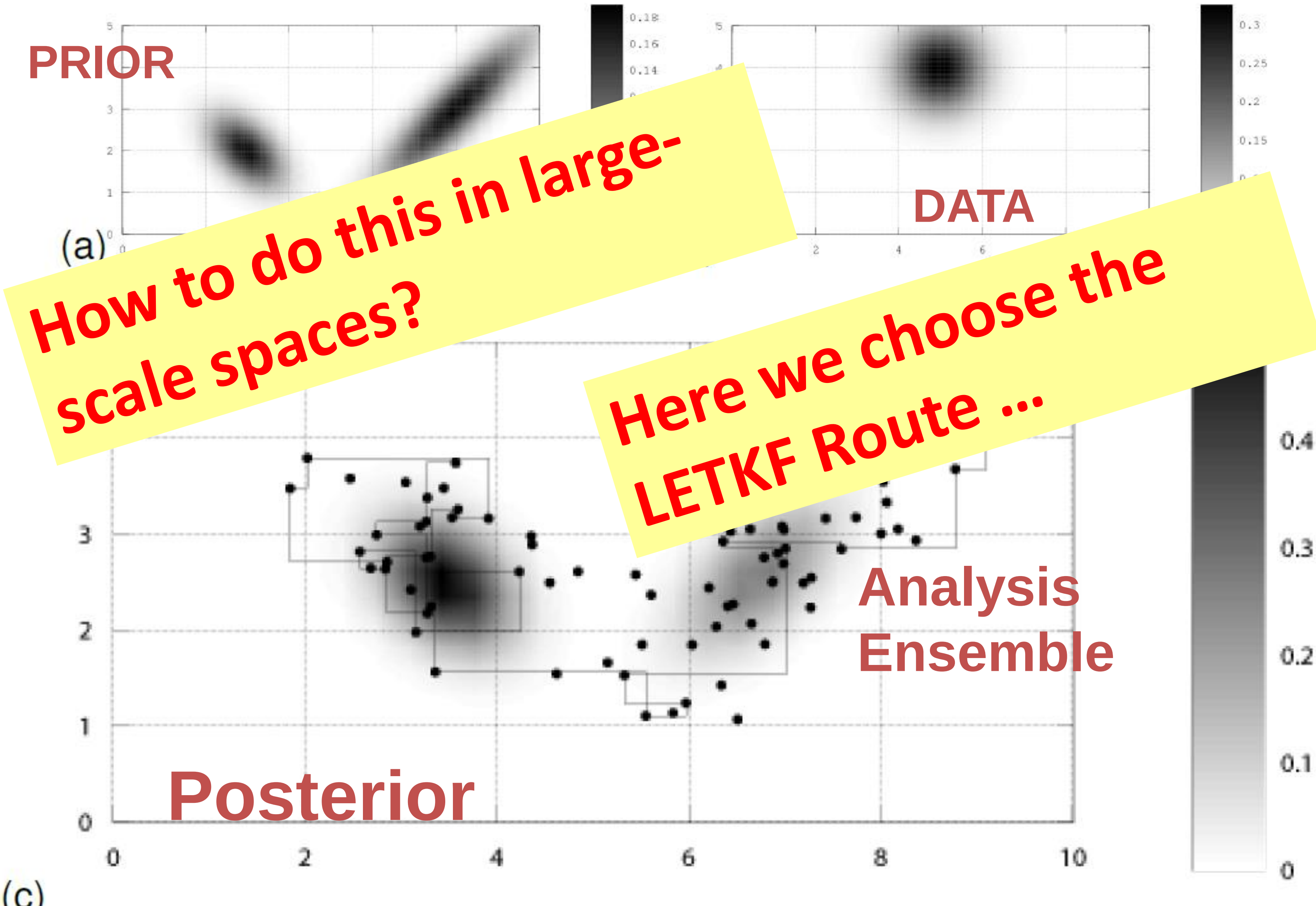

Bayesian Filtering via PF

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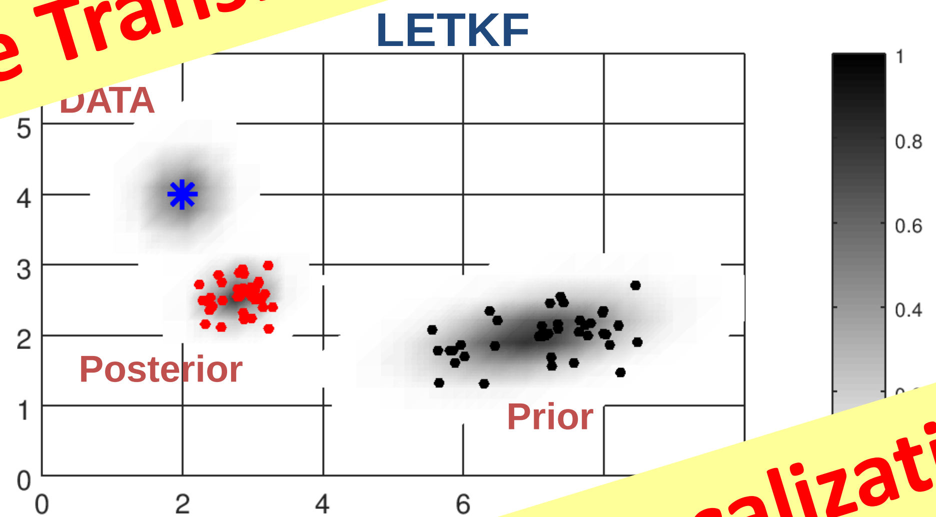
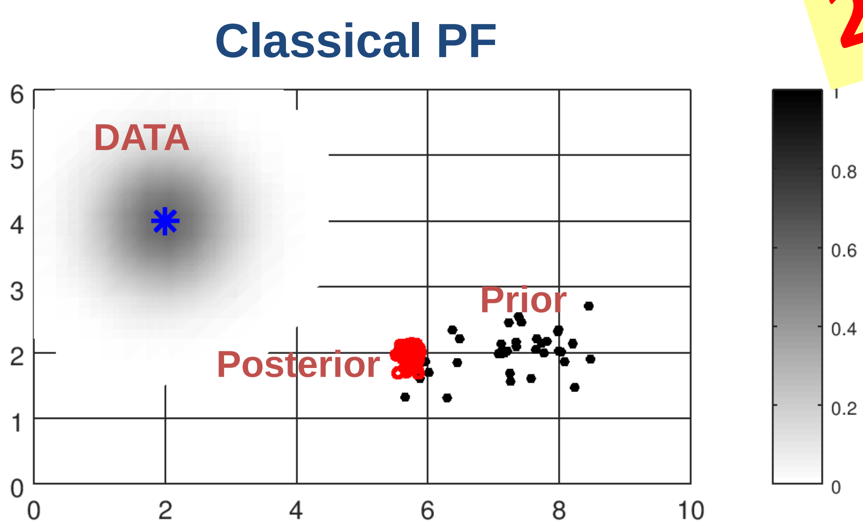
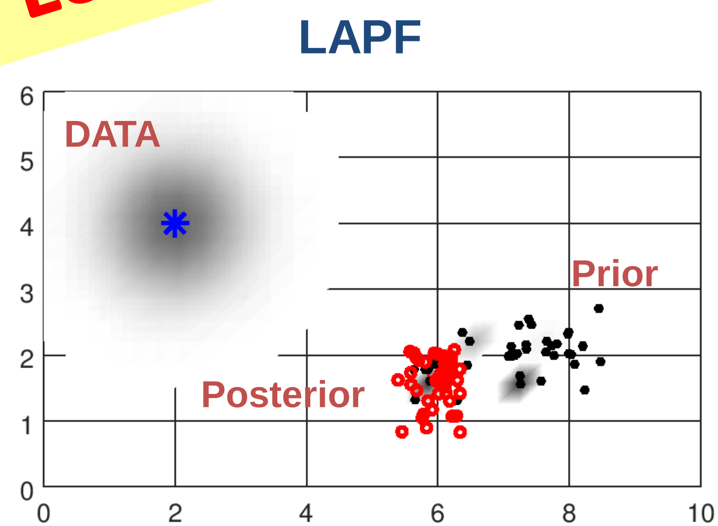
1) Ensemble Transform



2) Localization





Bayesian Filtering via PF

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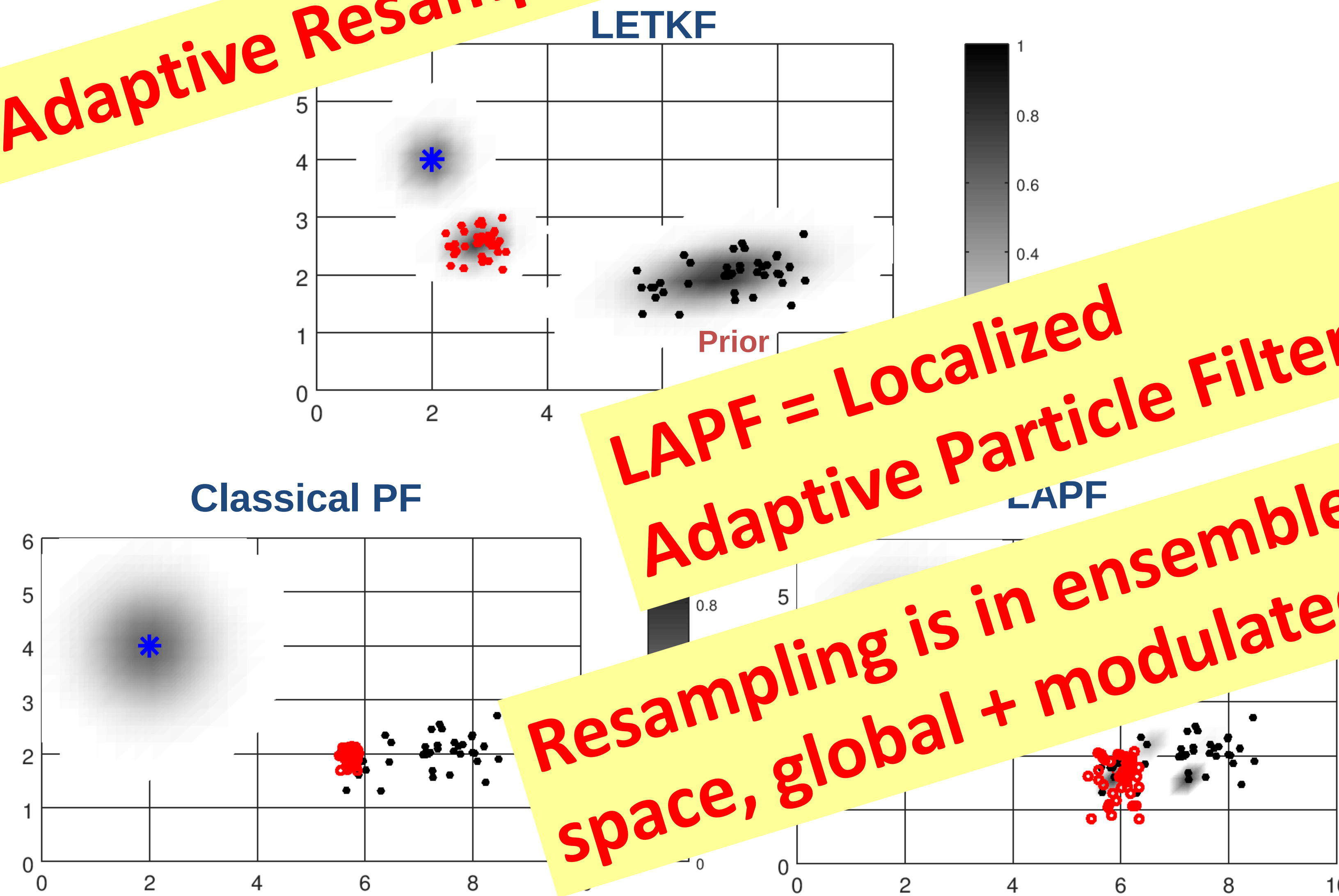
3) Adaptive Resampling



LAPF = Localized Adaptive Particle Filter

Resampling is in ensemble space, global + modulated

LAPF = Transform, Localization, Adaptivity with **global modulated Resampling**

First Step: The Classical Particle Filter

- **Bayes formula** to calculate new analysis distribution

$$p_k^{(a)}(x) := p(x|y_k) = c\, p(y_k|x) p_k^{(b)}(x), \qquad x \in \mathbb{R}^n$$

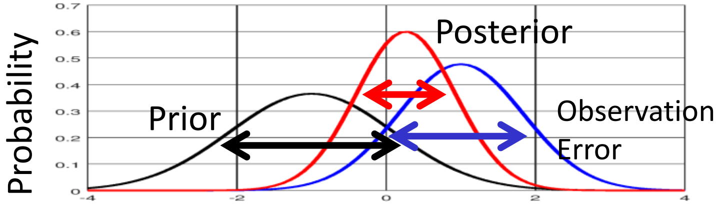
c is a normalization factor: $\int_X p_k^{(a)}(x) dx = 1$

Classical PF Approach

- To carry out the analysis step at time t_k **aposteriori weights** $p_k^{(a)}$ are calculated

$$p_{k,l}^{(a)} = c\, e^{-\frac{1}{2}\left(y - Hx^{(l)}\right)^T R^{-1}\left(y - Hx^{(l)}\right)}$$

c is chosen such that $\sum_{l=1}^{L} p_{k,l}^{(a)} = L$



Second Step: Classical Resampling

- **Accumulated weights** w_{ac} are defined:

$$w_{ac_0} = 0$$

$$w_{ac_i} = w_{ac_{i-1}} + p_i^a, \qquad i = 1, \dots, L$$

where L denotes the ensemble size

- Drawing $r_j \sim U([0,1]), j = 1, \dots, L$, set $R_j = j - 1 + r_j$ and define **transform matrix W** for the particles by:

$$W_{i,j} = \begin{cases} 1 & if\ R_j \in \left(w_{ac_{i-1}}, w_{ac_i}\right], \\ 0 & otherwise, \end{cases}$$

$i, j = 1, \dots, L$ with $W \in \mathbb{R}^{LxL}$, $(s, t]$ denotes the interval of values $s < \eta \leq t$.

Resampling

Adaptivity based on o-b statistics

- Based on the adaptive multiplicative **inflation factor** $\boldsymbol{\rho}$ determined by the LETKF

$$\rho = \frac{\mathrm{E}\left[\boldsymbol{d}_{o-b}^{T}\boldsymbol{d}_{o-b}\right] - \mathrm{Tr}(\mathbf{R})}{\mathrm{Tr}(\boldsymbol{H}\boldsymbol{P}^{b}\boldsymbol{H}^{T})}$$

from $\mathrm{E}\left[\boldsymbol{d}_{o-b}^{T}\boldsymbol{d}_{o-b}\right] = \mathrm{Tr}(\mathbf{R}) + \rho\,\mathrm{Tr}(\boldsymbol{H}\boldsymbol{P}^{b}\boldsymbol{H}^{T})$

- **Weighting factor** $\boldsymbol{\alpha}$ has been chosen, due to the small ensemble size ($L = 40$)

$$\rho_k = \alpha\tilde{\rho}_k + (1-\alpha)\rho_{k-1}$$

Third Step: Spread Control

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

- **Pertubation factor σ** is used to add spread to the system

$$\sigma = \begin{cases} c_0, & \rho < \rho^{(0)} \\ c_0 + (c_1 - c_0) * \dfrac{\rho - \rho^{(0)}}{\rho^{(1)} - \rho^{(0)}}, & \rho^{(0)} \leq \rho \leq \rho^{(1)} \\ c_1, & \rho > \rho^{(1)} \end{cases}$$

where $c_0 = 0.02$, $c_1 = 0.2$,

$\rho^{(0)} = 1.0$ and $\rho^{(1)} = 1.4$, with

$\sigma = c_1$ if $\rho \geq \rho^{(1)}$ and

$\sigma = c_0$ if $\rho \leq \rho^{(0)}$

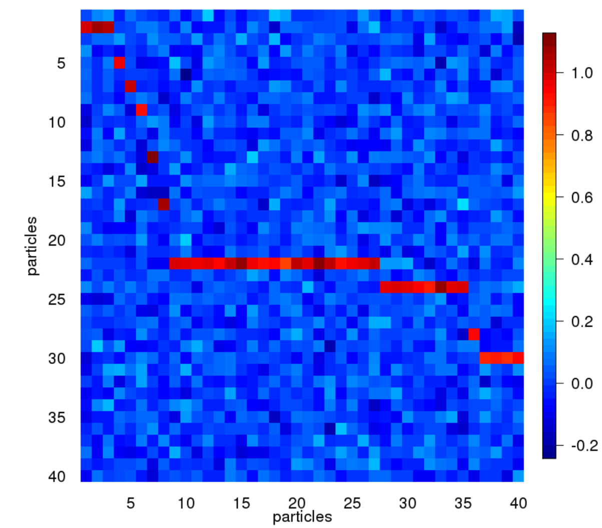
Enforce the desired spread!

Fourth Step: Gaussian Resampling

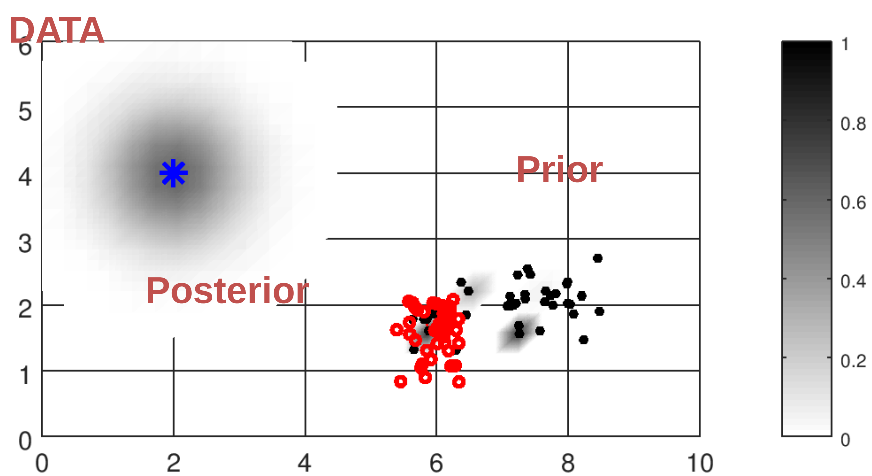
- **Weights *W*** are modified by applying the **pertubation factor σ**

$$W = W + R_{nd} * \sigma$$

with R_{nd} normally distributed random numbers



An example for a W-Matrix after applying σ determined wit… for 60… … are chosen

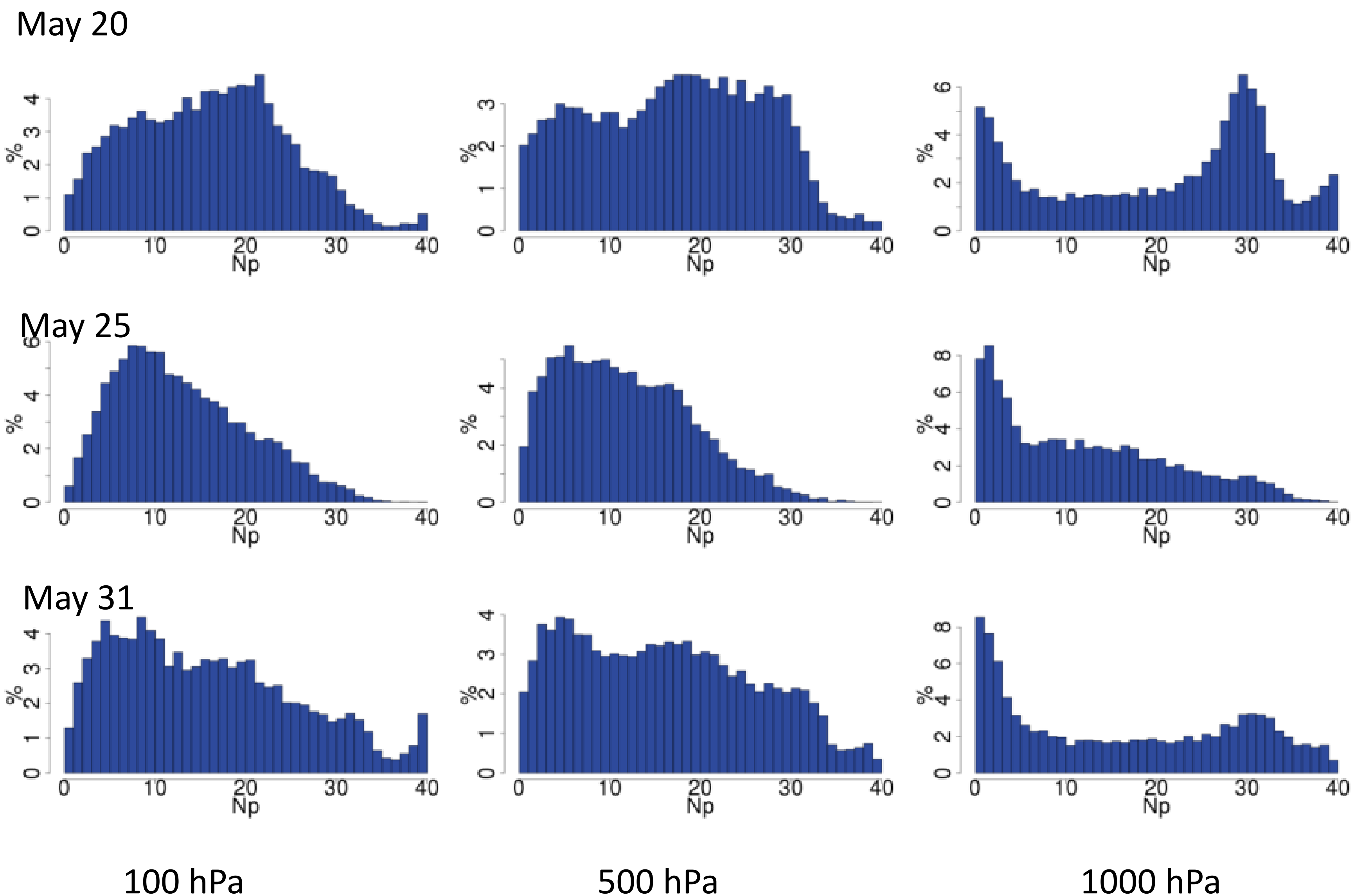


Enforce the desired spread!

Effective Ensemble Size Distributions

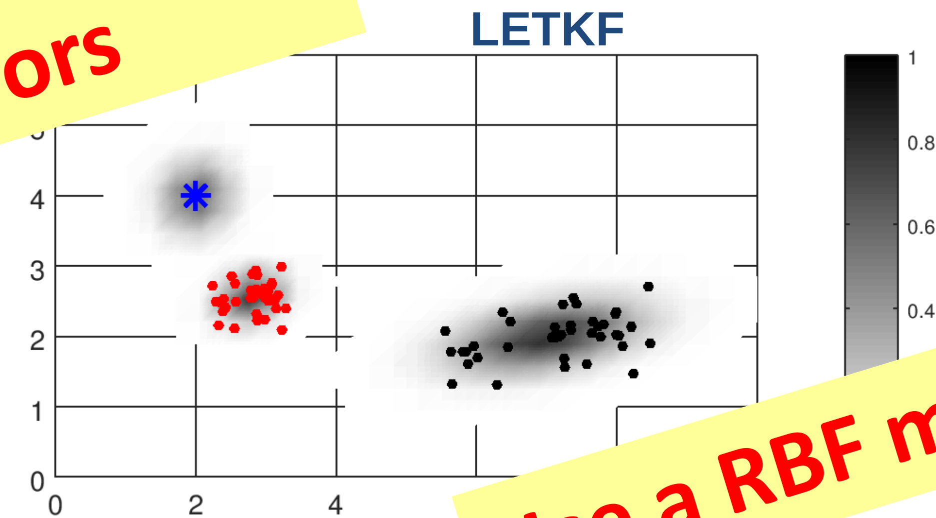
Deutscher Wetterdienst
Wetter und Klima aus einer Hand

DWD

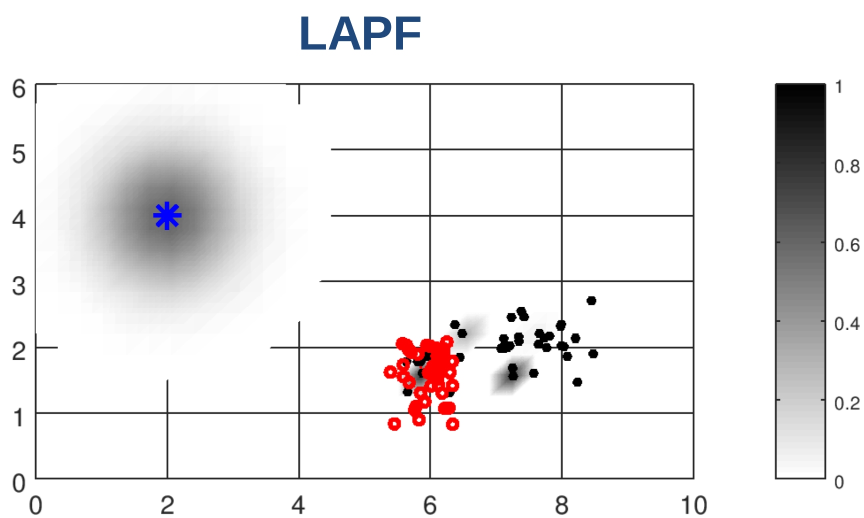


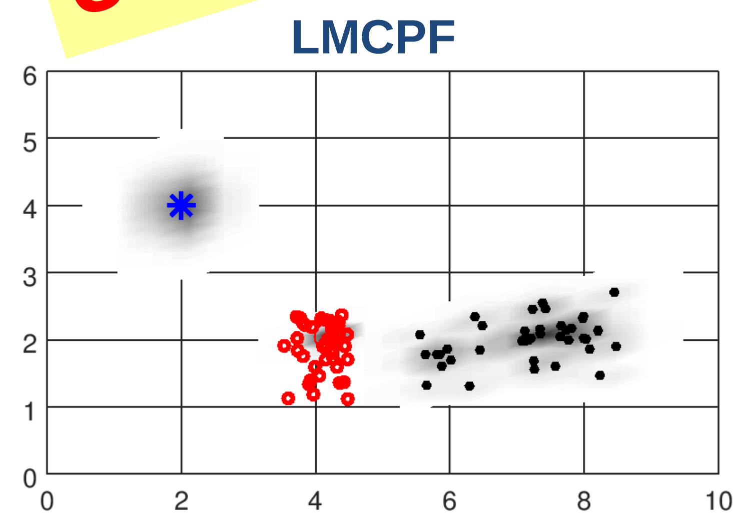
LAPF versus LMCPF

LAPF has problems with model errors



Use a RBF mixture





LMCPF = Transform, Localization, RBF mixture, Adaptivity

LMCPF Basics

- **Kalman Filter**

$$x^{(a)} = x^{(b)} + BH^T(R + HBH^T)^{-1}(y - Hx^{(b)})$$

$$K = BH^T(R + HBH^T)^{-1} \qquad \tilde{B} = (I - KH)B$$

- **Ensemble B Estimator**

$$\bar{x} := \frac{1}{L}\sum_{\ell=1}^{L} x^{(\ell)}$$

$$B = \frac{1}{L-1}XX^T$$

$$X = \left(x^{(1)} - \bar{x}, \ldots, x^{(L)} - \bar{x}\right) \in \mathbb{R}^{n \times L}.$$

LETKF/LMCPF Basics: B Posterior

$$Y := HX$$

$$\begin{aligned}
\tilde{B} &= (I - KH)B \\
&= (I - BH^T(R + HBH^T)^{-1}H)B \\
&= \left(I - \gamma X X^T H^T (R + \gamma H X X^T H^T)^{-1} H\right)\gamma X X^T \\
&= X\left(I - \gamma Y^T (R + \gamma Y Y^T)^{-1} Y\right)\gamma X^T \\
&= X\left(I - \gamma (I + \gamma Y^T R^{-1} Y)^{-1} Y^T R^{-1} Y\right)\gamma X^T \\
&= X\left((I + \gamma Y^T R^{-1} Y)^{-1}(I + \gamma Y^T R^{-1} Y - \gamma Y^T R^{-1} Y\right)\gamma X^T \\
&= X(I + \gamma Y^T R^{-1} Y)^{-1}\gamma X^T \\
&= X(\frac{1}{\gamma} I + Y^T R^{-1} Y)^{-1} X^T
\end{aligned}$$

$$Y^T(R + \gamma Y Y^T)^{-1} = (I + \gamma Y^T R^{-1} Y)^{-1} Y^T R^{-1}$$

RBF Basis Function in Ensemble Space

LMCPF Basics: Bayes Formula

Gaussian Mixture Case

$$
\begin{aligned}
p(x|y) &= c p(y|x) \cdot p(x) \\
&= c e^{-\frac{1}{2}(y-Hx)^T R^{-1}(y-Hx)} \sum_{\ell=1}^{L} p_G(x - x^{(\ell)}) \\
&= c \sum_{\ell=1}^{L} e^{-\frac{1}{2}(y-Hx)^T R^{-1}(y-Hx)} p_G(x - x^{(\ell)}),
\end{aligned}
$$

$$
p_G(x - x^{(\ell)}) = \tilde{c} e^{-\frac{1}{2}(x-x^{(\ell)})^T G^{-1}(x-x^{(\ell)})}
$$

Explicit Calculations possible for each term

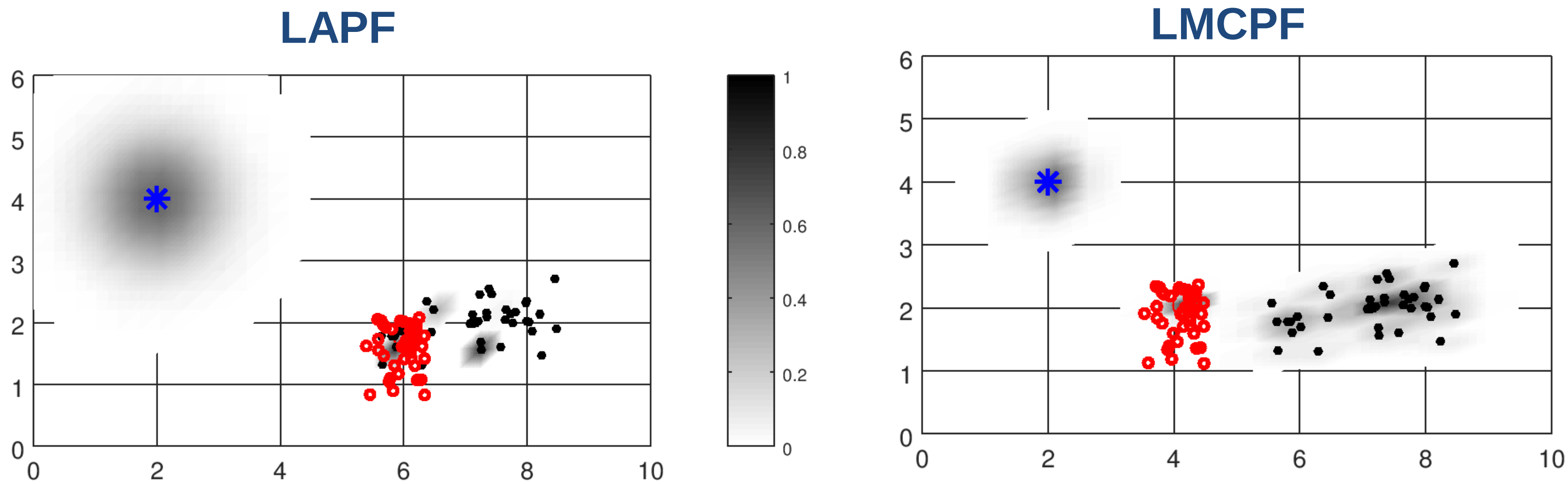
We need a selection based on relative weights!

LMCPF Basics: Relative Weights

$$w_\ell := e^{-\frac{1}{2}(y - Hx^{(\ell)})^T R^{-1} (y - Hx^{(\ell)})}, \quad \ell = 1, ..., L$$

$$w_{tot} := \sum_{\ell=1}^{L} w_\ell .$$

We need a selection based on relative weights!



LAPF and LMCPF Weights

Projection onto Ensemble Space

Abbreviating $A := \mathbf{Y}^T \mathbf{R}^{-1} \mathbf{Y}$ and $C := A^{-1} \mathbf{Y}^T \mathbf{R}^{-1} (\mathbf{y}^o - \overline{\mathbf{y}}^b)$

Projection Operator

$$P(\mathbf{y}^o - \overline{\mathbf{y}}^b) = \mathbf{Y}(\mathbf{Y}^T \mathbf{R}^{-1} \mathbf{Y})^{-1} \mathbf{Y}^T \mathbf{R}^{-1} (\mathbf{y}^o - \overline{\mathbf{y}}^b),$$

Projected discrepancy

$$P(\mathbf{y}^o - H\mathbf{x}^{(\ell)}) = \mathbf{Y} A^{-1} \mathbf{Y}^T \mathbf{R}^{-1} ((\mathbf{y}^o - \overline{\mathbf{y}}^b) - \mathbf{Y} e_\ell)$$

$$= \mathbf{Y}(C - e_\ell), \quad \ell = 1, \ldots, L.$$

Exponent

$$P(\mathbf{y}^o - H\mathbf{x}^{(\ell)})]^T \mathbf{R}^{-1} P(\mathbf{y}^o - H\mathbf{x}^{(\ell)}) = [C - e_\ell]^T A [C - e_\ell], \quad \ell = 1, \ldots, L,$$

Weight

$$w_{k,\ell} = c e^{-\frac{1}{2}[C - e_\ell]^T A [C - e_\ell]}, \quad \ell = 1, \ldots, L.$$

LAPF and LMCPF Weights

Classical versus projected weights

$$\begin{aligned} w_{k,\ell}^{classical} &= e^{-\frac{1}{2}[(\mathbf{y}^o - H\mathbf{x}^{(\ell)})]^T \mathbf{R}^{-1} [(\mathbf{y}^o - H\mathbf{x}^{(\ell)})]} \\ &= e^{-\frac{1}{2}[\left(P+(I-P)\right)(\mathbf{y}^o - H\mathbf{x}^{(\ell)})]^T \mathbf{R}^{-1} [\left(P+(I-P)\right)(\mathbf{y}^o - H\mathbf{x}^{(\ell)})]} \\ &= e^{-\frac{1}{2}[P(\mathbf{y}^o - H\mathbf{x}^{(\ell)})]^T \mathbf{R}^{-1} [P(\mathbf{y}^o - H\mathbf{x}^{(\ell)})]} \cdot \underbrace{e^{-\frac{1}{2}[(I-P)(\mathbf{y}^o - H\mathbf{x}^{(\ell)})]^T \mathbf{R}^{-1} [(I-P)(\mathbf{y}^o - H\mathbf{x}^{(\ell)})]}}_{=\tilde{c}}, \end{aligned}$$

Factor is a constant term, since we have

$$\begin{aligned} (I-P)(\mathbf{y}^o - Hx^{(\ell)}) &= (I-P)(\mathbf{y}^o - \overline{\mathbf{y}}^b + \mathbf{Y}e_\ell) \\ &= (I-P)(\mathbf{y}^o - \overline{\mathbf{y}}^b) - \underbrace{(I-P)\mathbf{Y}e_\ell}_{=0}. \end{aligned}$$

Projected particle filter weights and classical particle filter weights are <u>equivalent theoretically</u>, but numerically <u>remove a very small common factor</u>

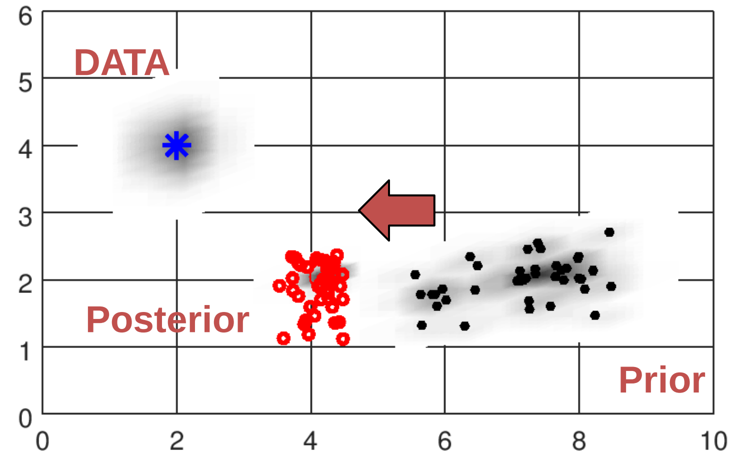
Fourth Step: Gaussian Resampling

LMCPF = Local Markov Chain Particle Filter

- **Weights *W*** are calculated by drawing from the posterior

$$W = W + A_{shift} * W + B_{post} * R_{nd} * \sigma$$

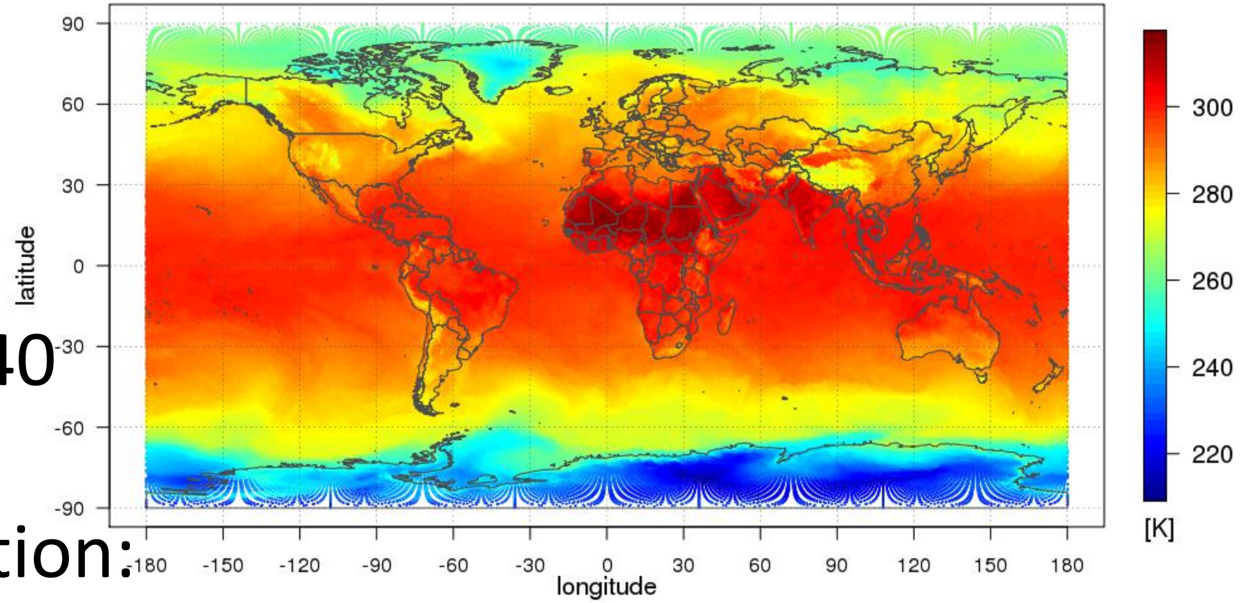
with R_{nd} normally distributed random numbers, A_{shift} and B_{post} calculated with Gaussian radial basis function (rbf) Approximation for prior density and observation error



✓ It is an **explicit calculation** of the **Bayes posterior** based on **radial basis function approximation** of the prior, with subsequent draws from that distribution in the MCMC sense.

Large-Scale Experimental Set-up

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



- Full ensemble: 40 members
- Reduced resolution:
  - 26km deterministic
  - 52km ensembles
- Period: 01.05.2016 – 31.05.2016

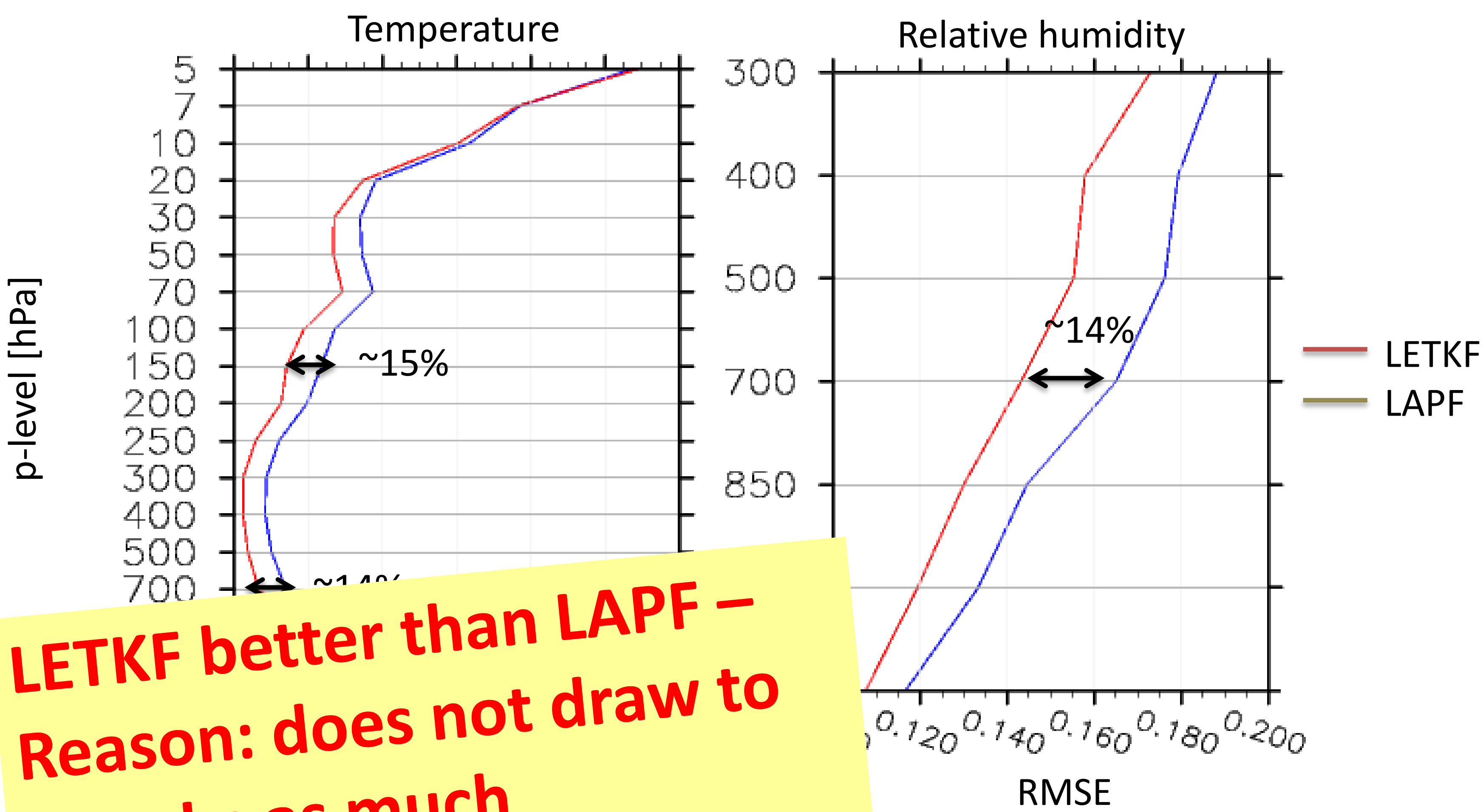
Experiments programmed and carried out **by Anne Walter, DWD& Uni Reading**, and Roland Potthast, DWD& Uni Reading

In Cooperation with Peter-Jan van Leeuwen , Uni Reading

LAPF Scores vs LETKF

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

Global **RMSE** for **obs-fg** statistics (Radiosondes vs. Model)
Period: 08.05.2016 – 31.05.2016

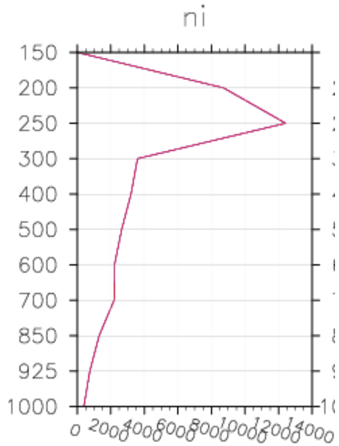


LETKF better than LAPF – Reason: does not draw to the obs as much

Exp… …ned and carried out **by Anne Walter, DWD & Uni Reading**, and Roland Potthast, DWD & Uni Reading

LMCPF Scores vs LETKF

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

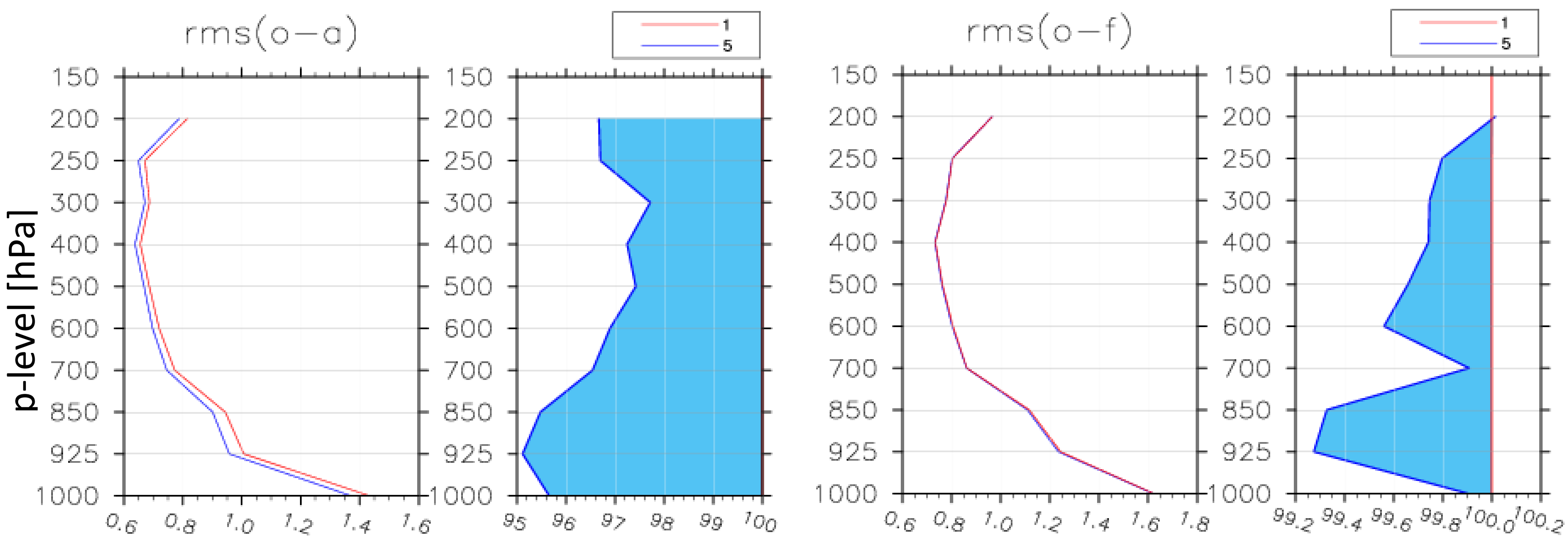


Global **RMSE** for **obs-fg** statistics (AIREP vs. Model)
Period: 08.05.2016 – 22.05.2016

LETKF
LMCPF

Temperature o-a and o-f

LMCPF: Model Error based Shift works

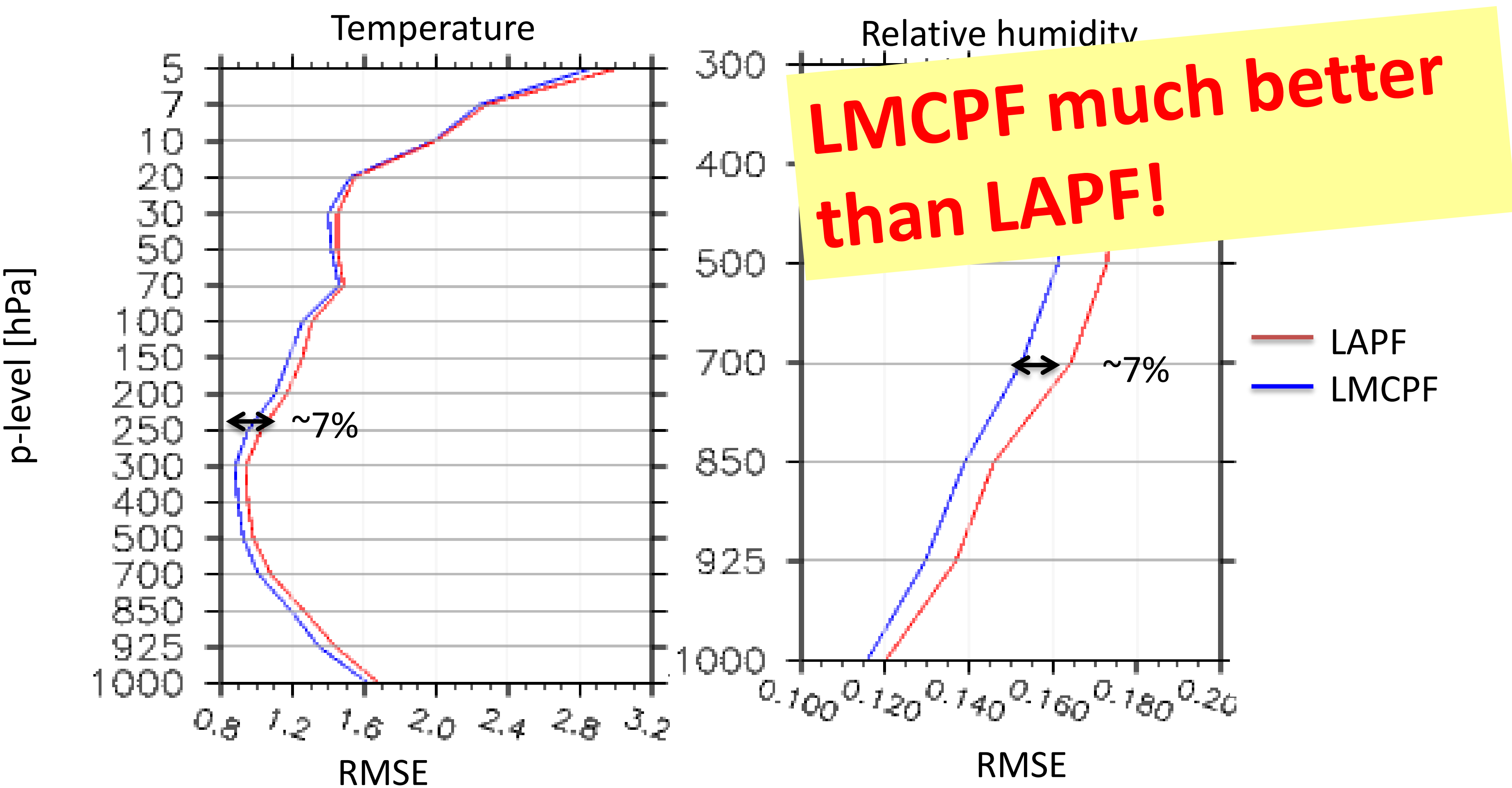


Experiments programmed and carried out by Anne Walter, DWD & Uni Reading, and Roland Potthast, DWD & Uni Reading

LMCPF Scores vs LAPF

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

Global **RMSE** for **obs-fg** statistics (Radiosondes vs. Model)
Period: 08.05.2016 – 22.05.2016

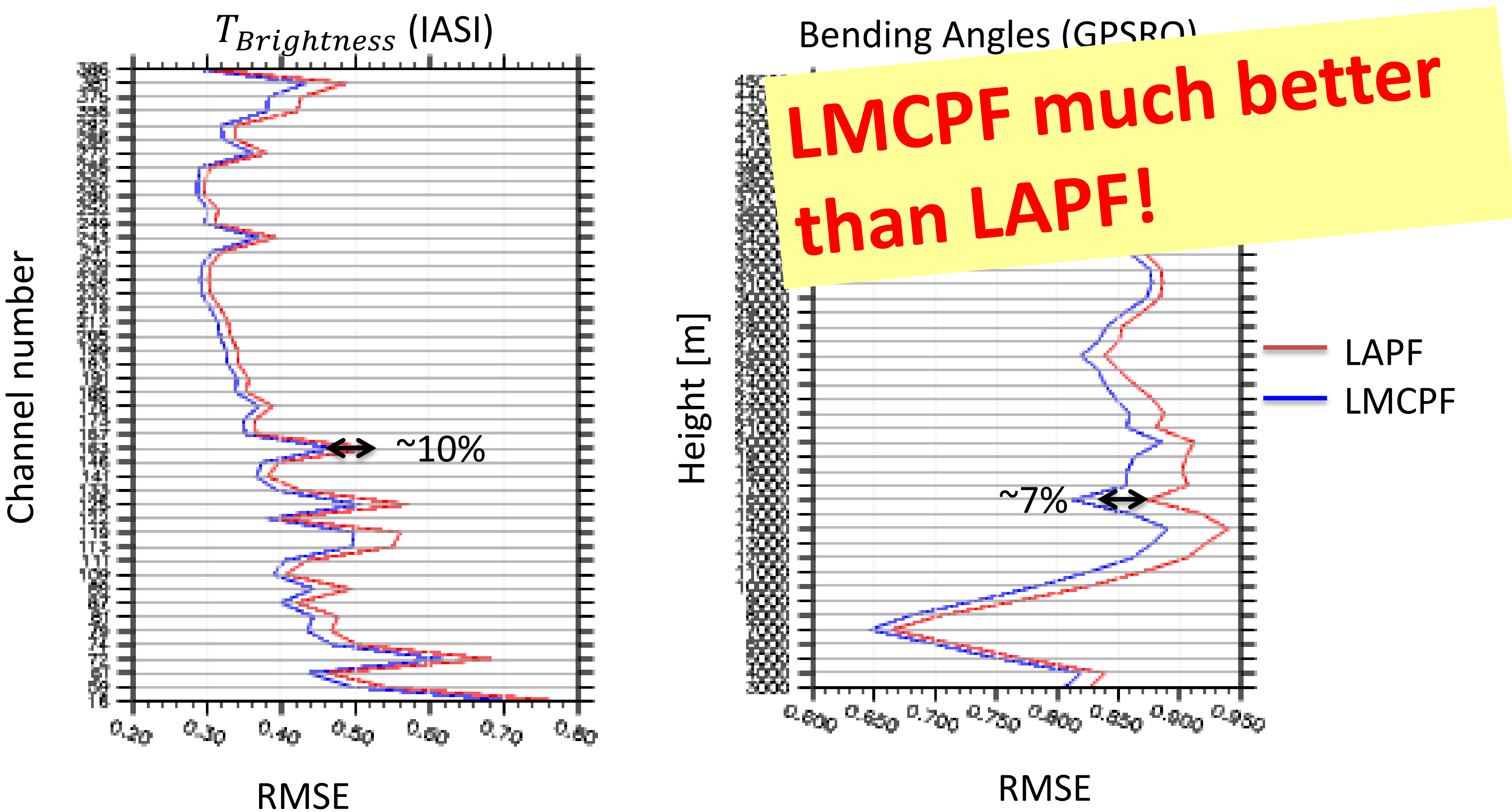


Experiments programmed and carried out **by Anne Walter, DWD & Uni Reading**, and Roland Potthast, DWD & Uni Reading

LMCPF Scores vs LAPF

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

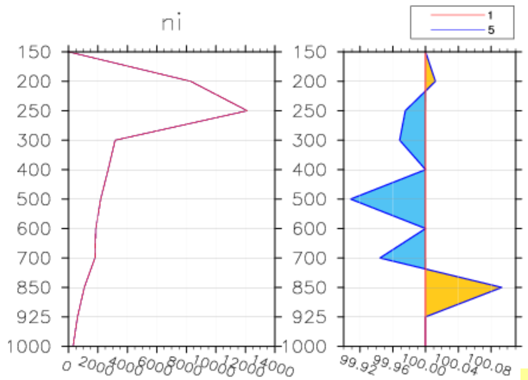
Global **RMSE** for **obs-fg** statistics
Period: 08.05.2016 – 22.05.2016



Experiments programmed and carried out **by Anne Walter, DWD & Uni Reading**, and Roland Potthast, DWD & Uni Reading

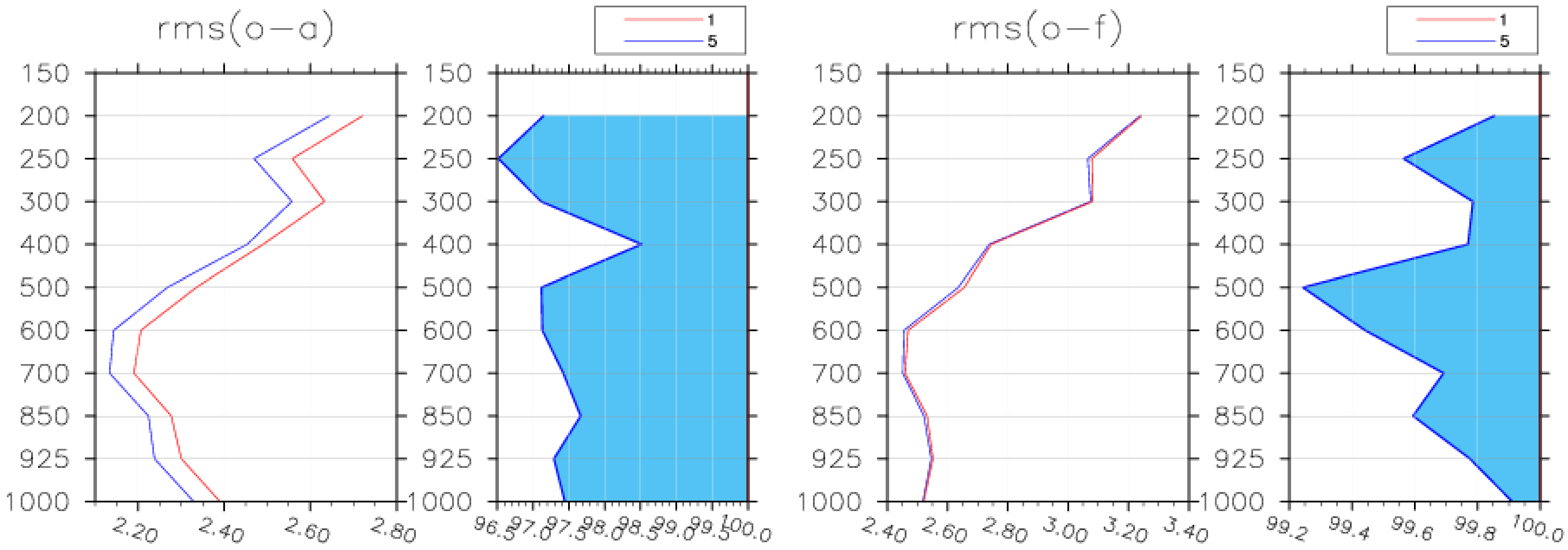
New LMCPF Scores vs LETKF

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



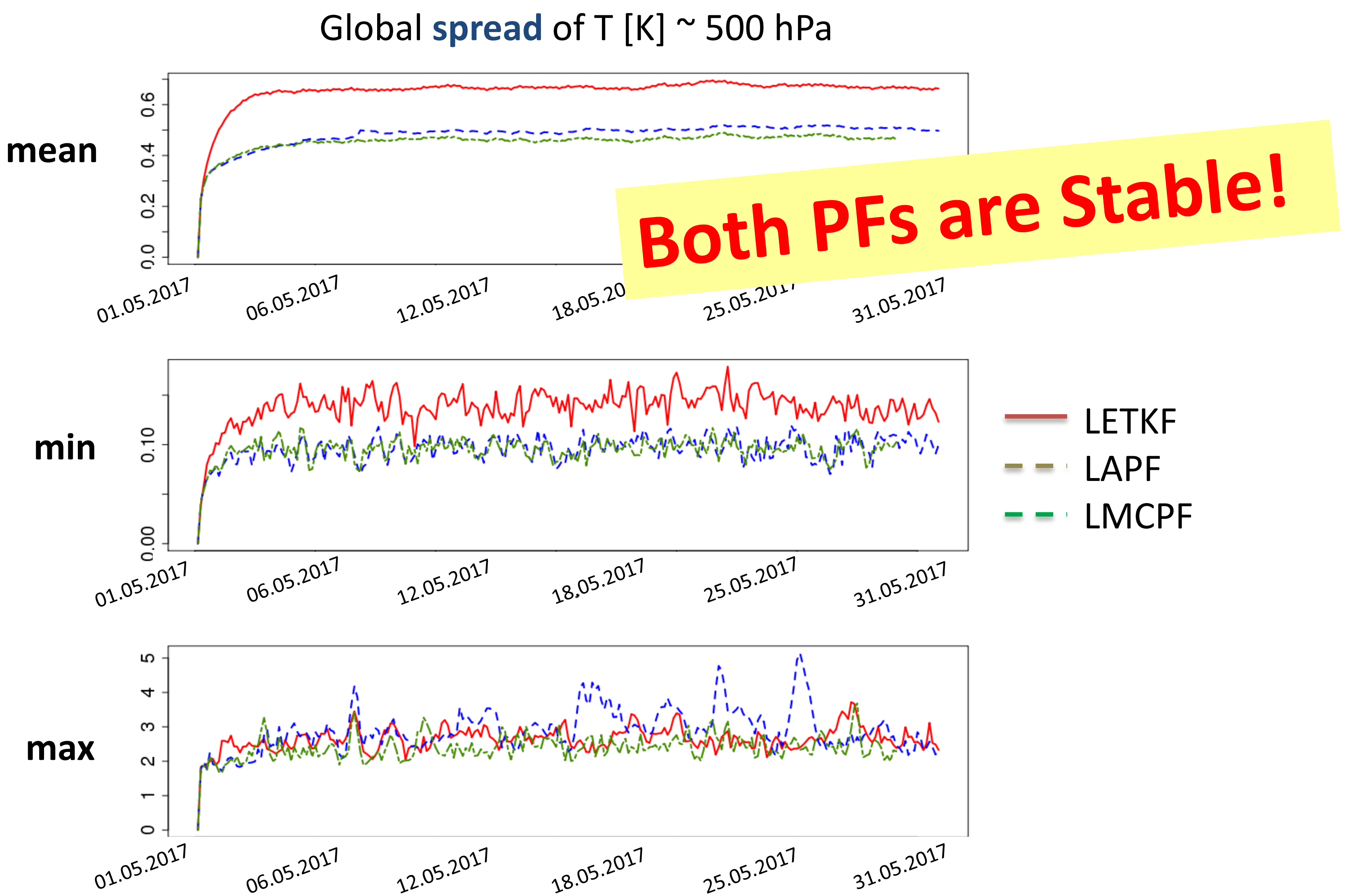
WIND o-a and o-f

LMCPF: Model Error based Shift works

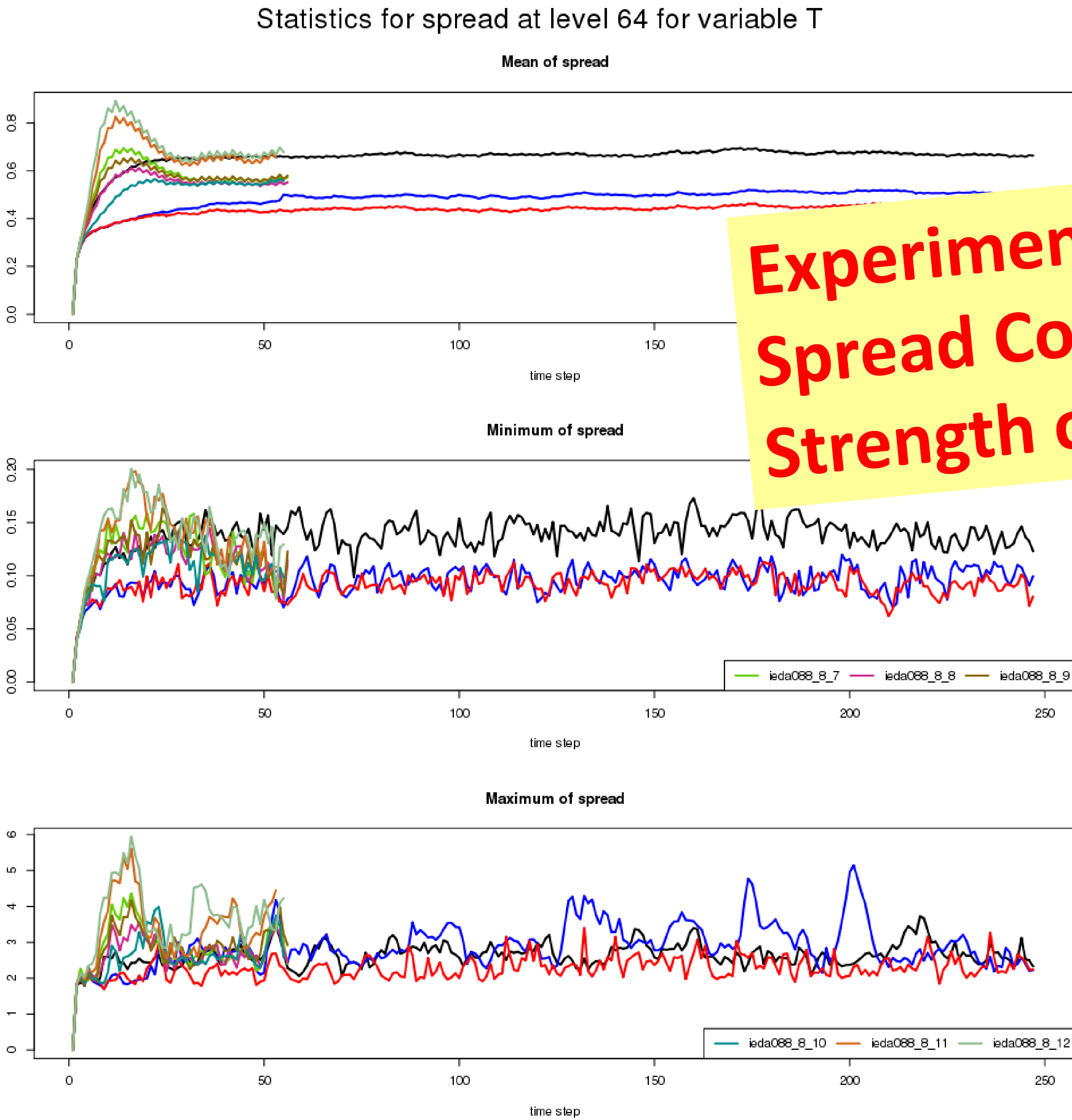


LAPF Spread vs LMCPF & LETKF

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



Experiments programmed and carried out **by Anne Walter, DWD & Uni Reading**, and Roland Potthast, DWD & Uni Reading

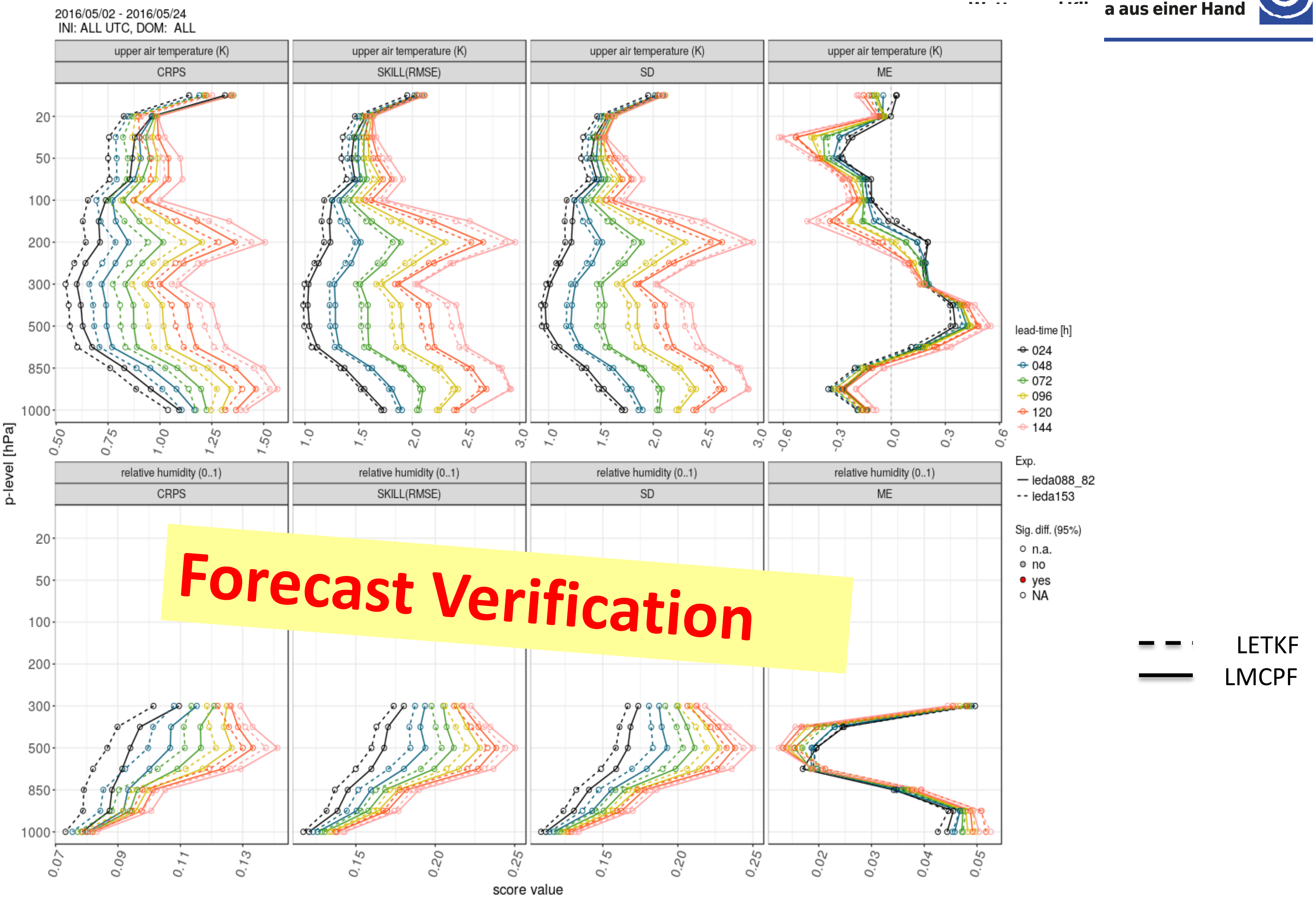


eutscher Wetterdienst
tter und Klima aus einer Hand
DWD

Experiments with Spread Control and Strength of Shifts

LMCPF Scores vs LETKF

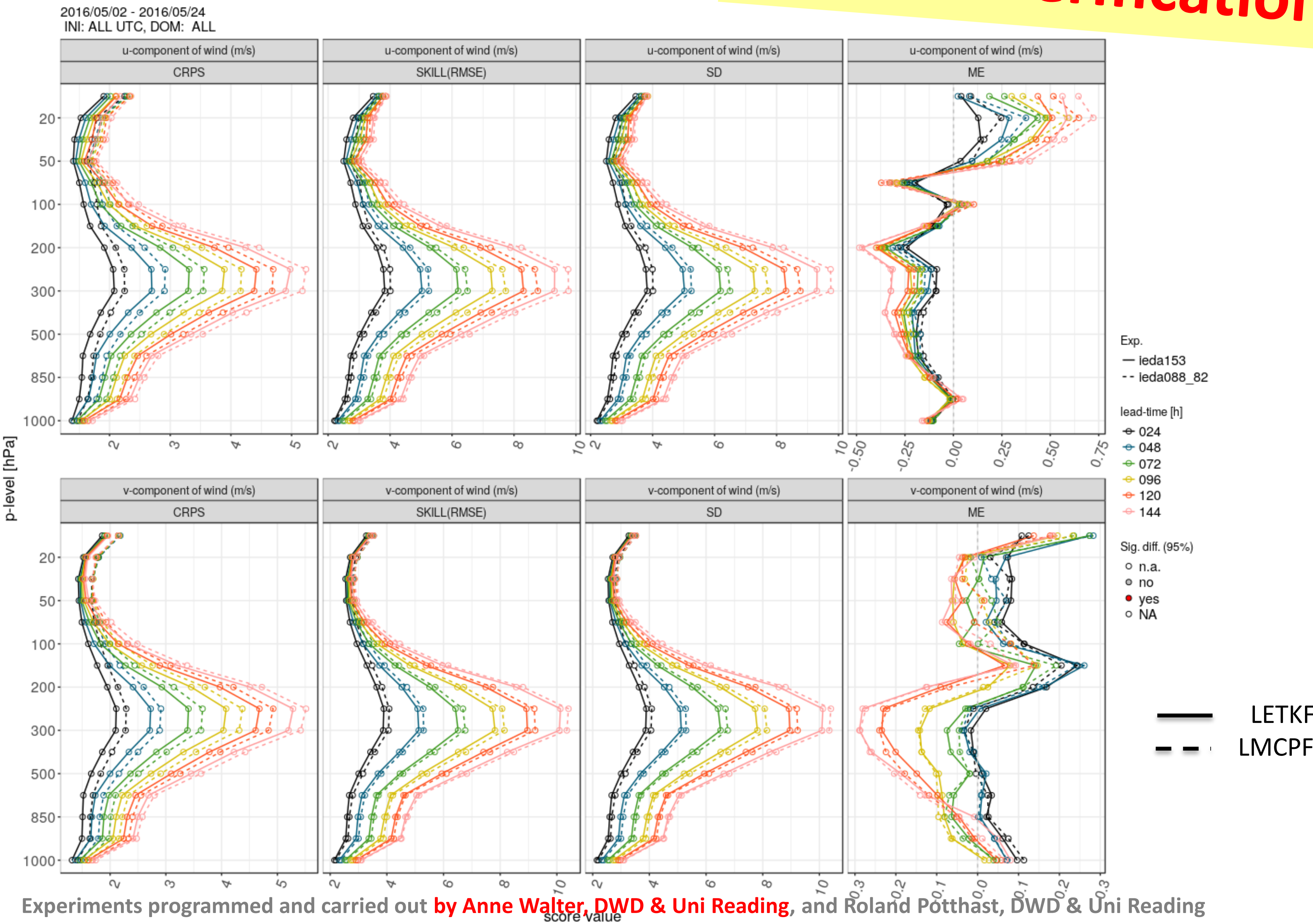
Deutscher Wetterdienst
Wetter und Klima aus einer Hand



Experiments programmed and carried out by Anne Walter, DWD & Uni Reading, and Roland Potthast, DWD & Uni Reading

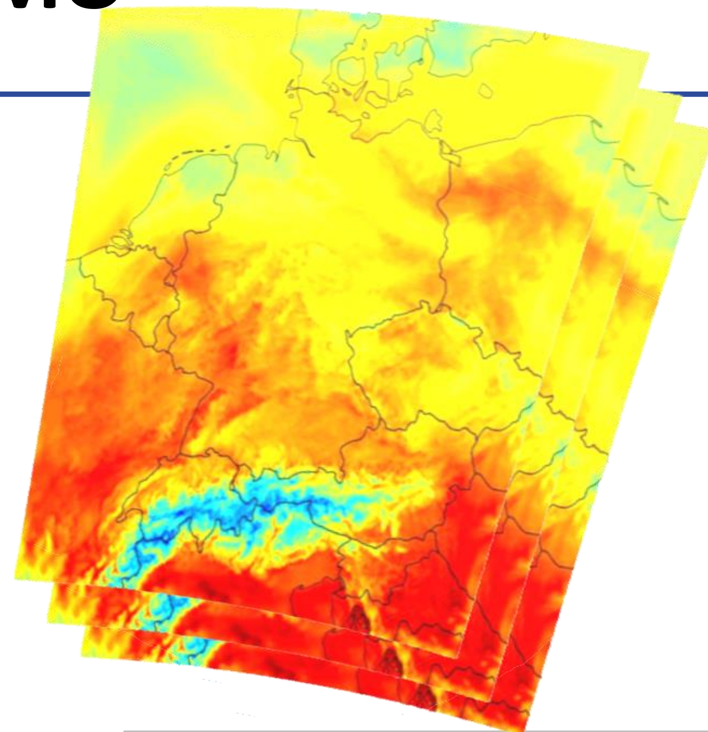
LMCPF Scores vs LETKF

Forecast Verification



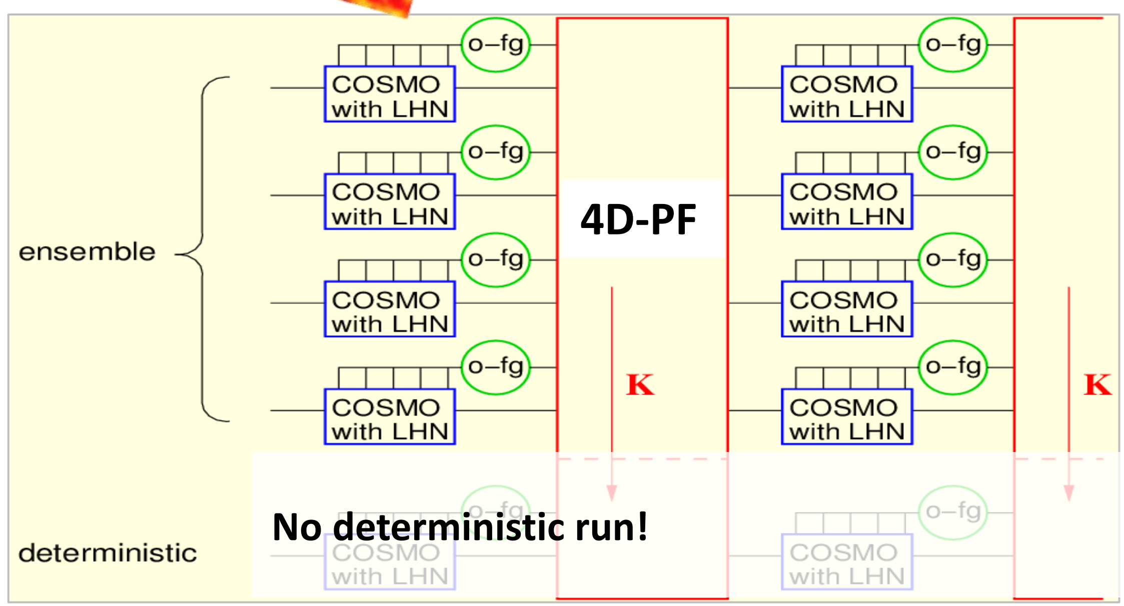
Experiments programmed and carried out **by Anne Walter, DWD & Uni Reading**, and Roland Potthast, DWD & Uni Reading

LMCPF for COSMO



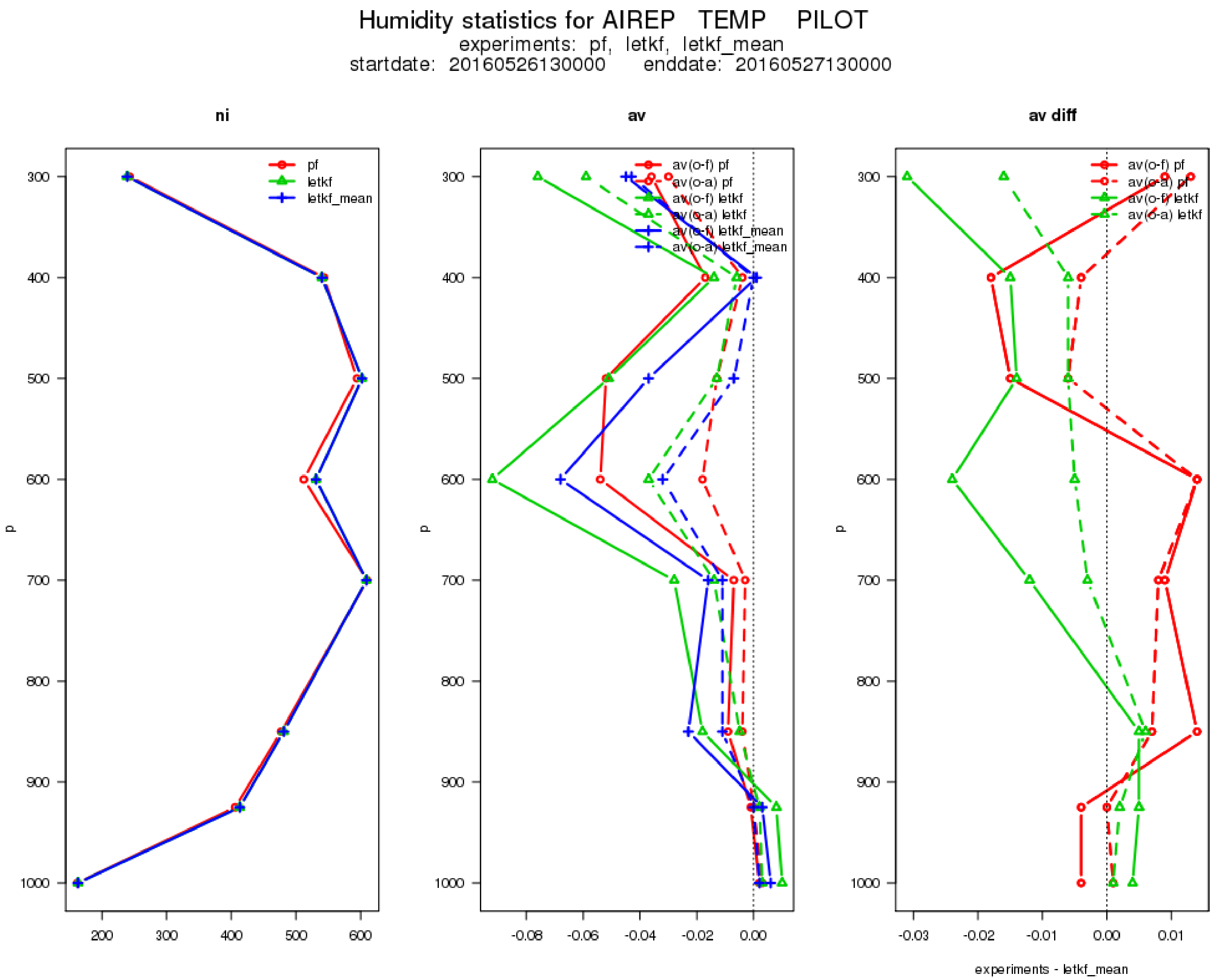
4D-Particle Filter for Convection Permitting Model

COSMO-DE Resolution 2.8km Central Europe



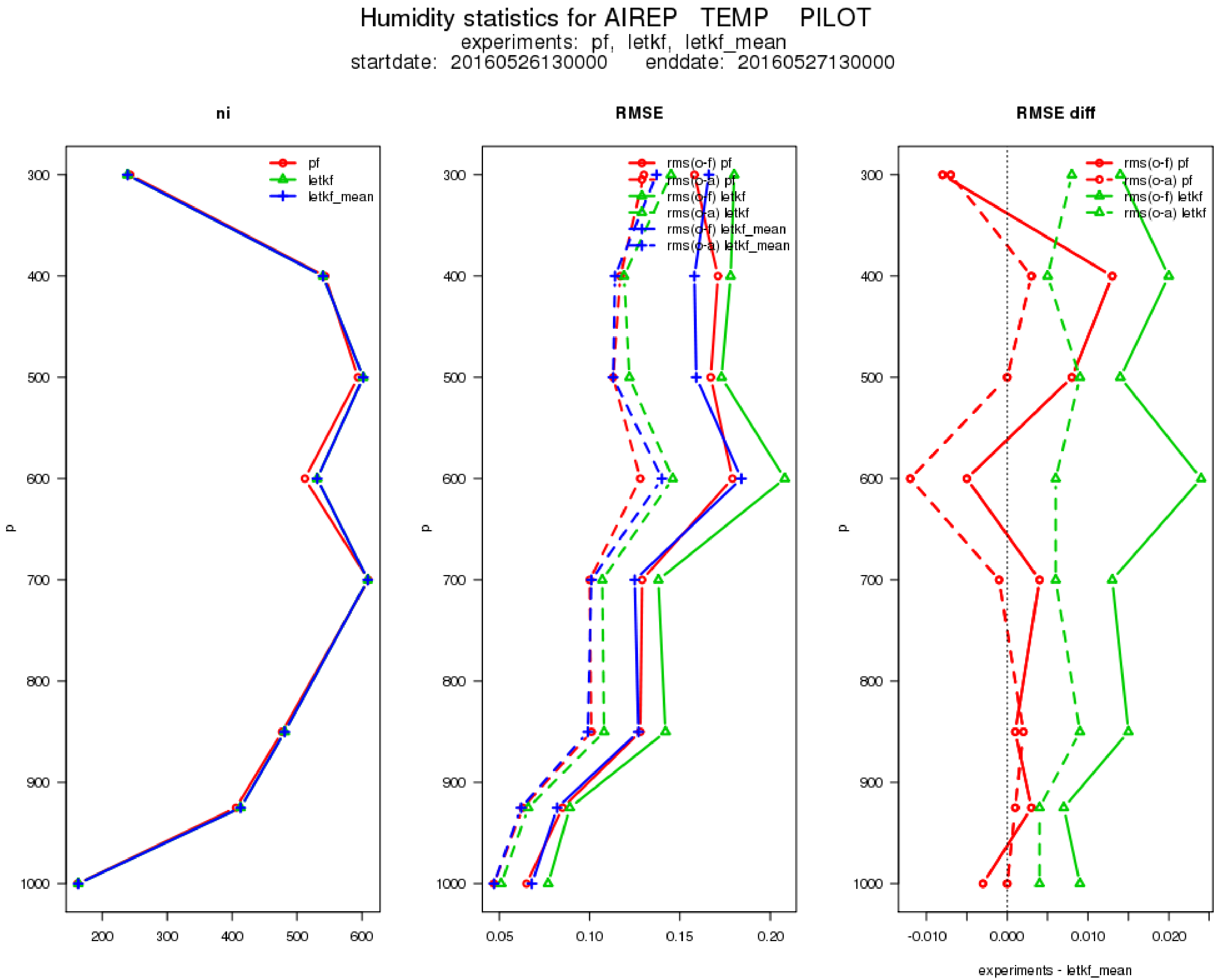
LMCPF for COSMO

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



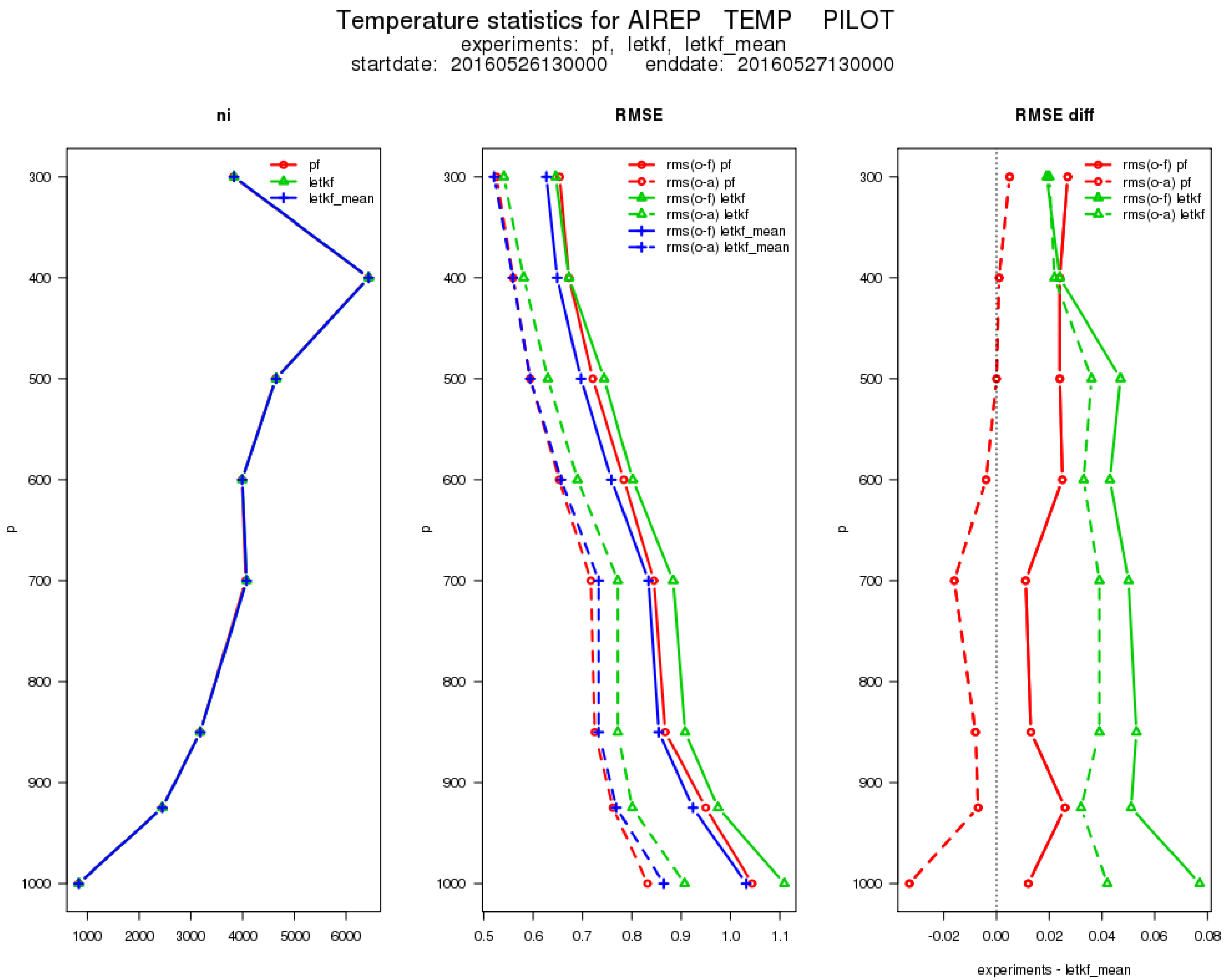
LMCPF for COSMO

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

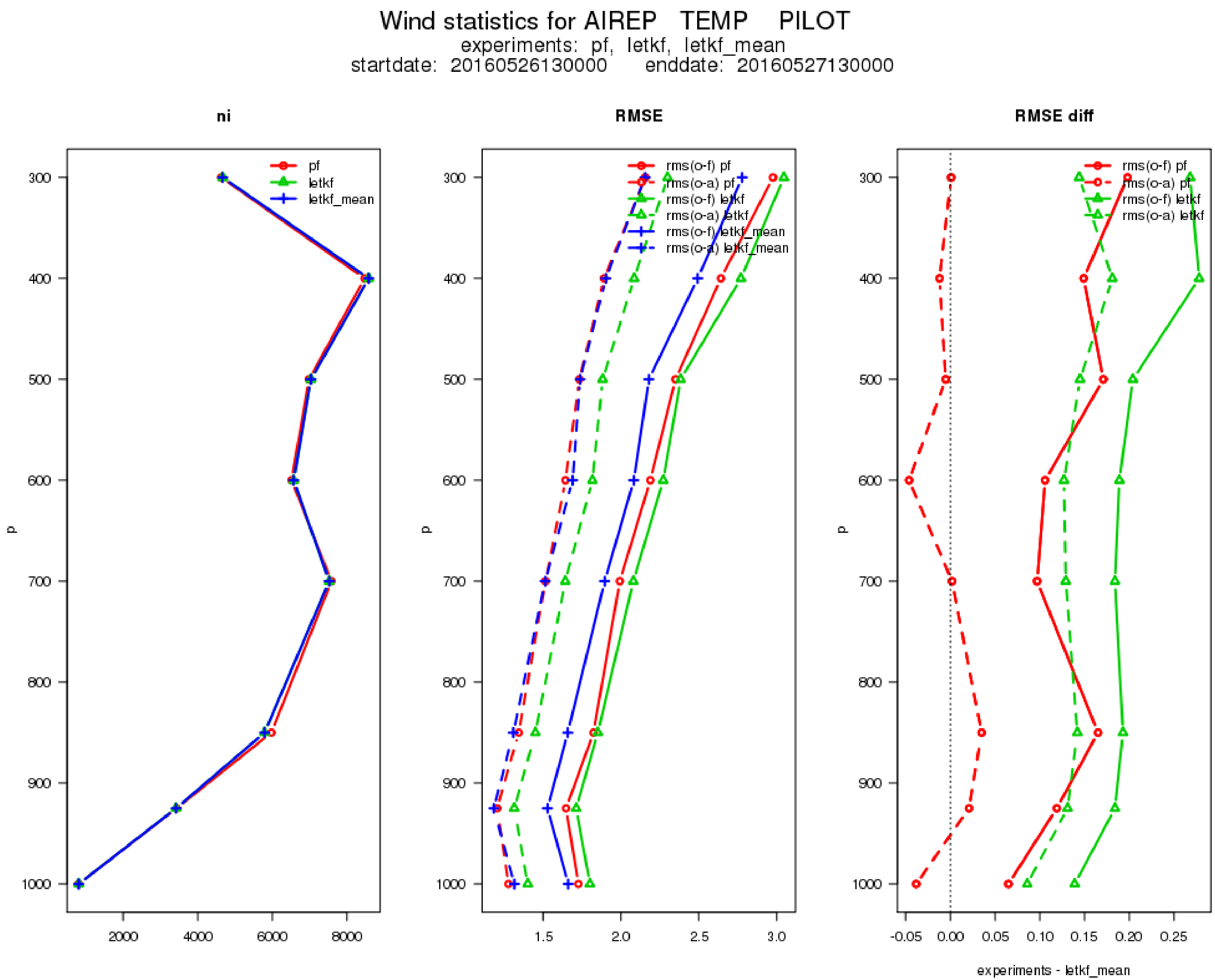


LMCPF for COSMO

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

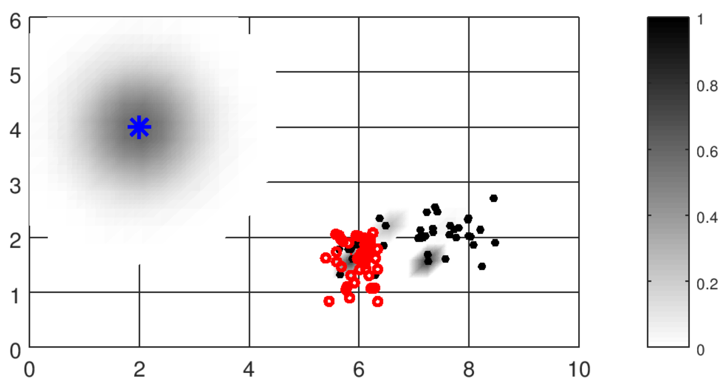


LMCPF for COSMO

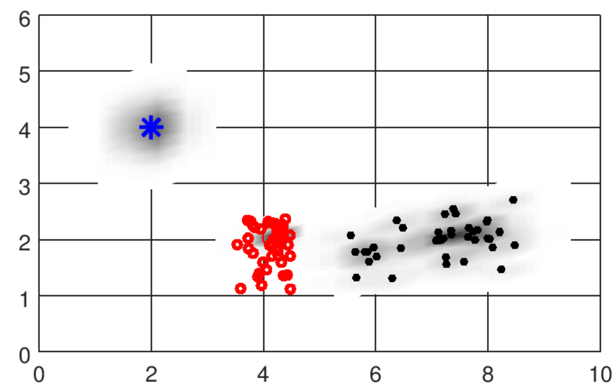


Summary LAPF and LMCPF

LAPF



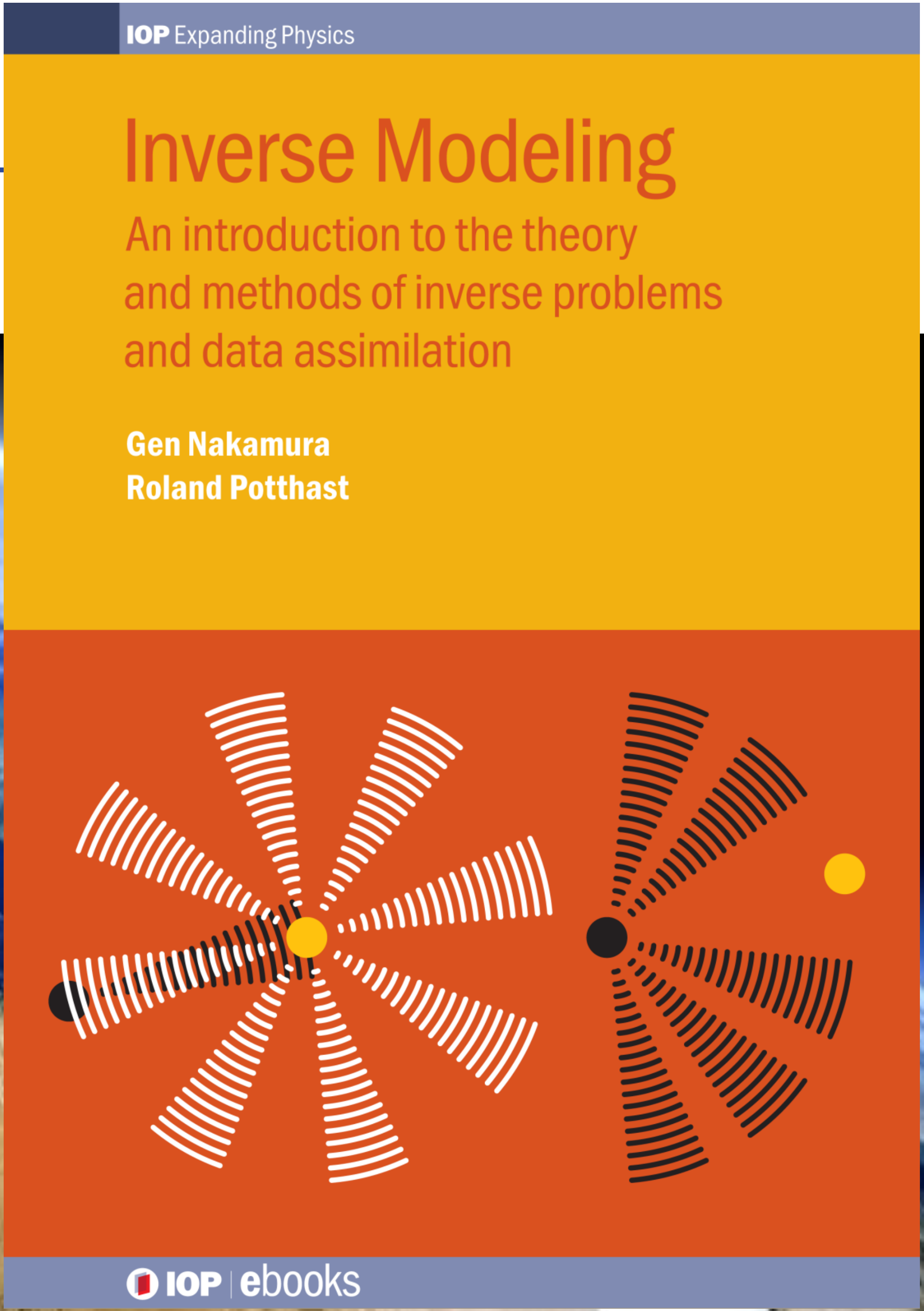
LMCPF



- LAPF and LMCPF are implemented in an **operational NWP system:** **Globally + mesoscale + convective scale (KENDA)**
- Both Particle Filters are able to provide **reasonable atmospheric analysis** in a large-scale (high-dimensional) environment and are running stably over a period of one month
- The LMCPF outperforms the LAPF, both Particle Filters are not far behind the operational LETKF, **LMCPF starting to be comparable**

Both Particle Filters are showing promising results; further tuning and development is in progress.

Many Thanks!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