USING PRECIPITATION RADAR FOR URBAN HYDROLOGY: A NEW METAGAUSSIAN MODEL

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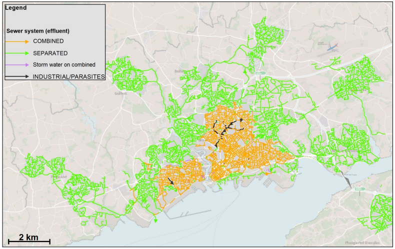
INTRODUCTION

Hydrological model

- Describe the functioning of the sewer system
- Calibrated on the adjustment between rain and network measurements
- Currently using 1 rain gauge for the entire area

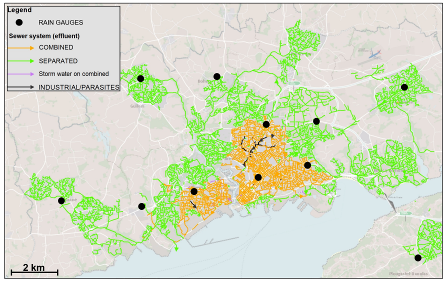


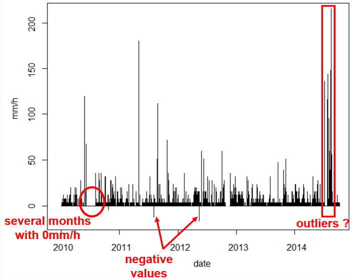
Question

What precipitation data should be used as input?

Especially, can spatialization impact the model outputs?

Rain gauges



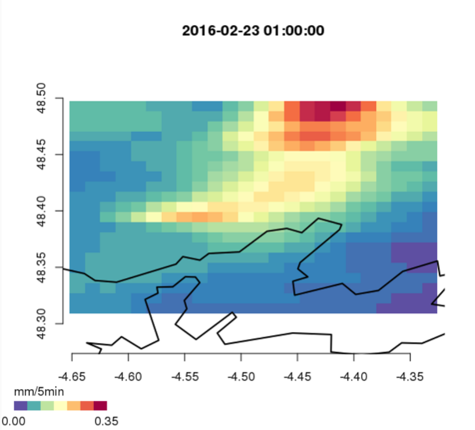


→ 3 time step

→ Measurement errors (raw data)

Radar

- → 1km^2 grid, 5 min time step
- → Sometimes biased measurement
- → Heterogeneous post-processing



MODELLING RAINFALL

Transformed censored Gaussian distribution

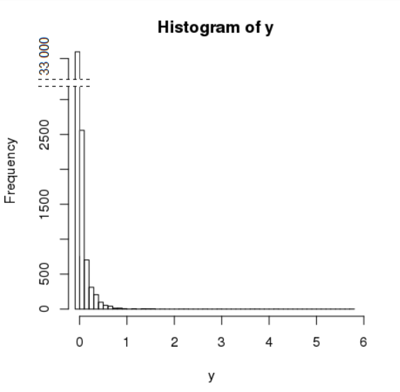
[Benoit et al., 2018, Allcroft and Glasbey, 2003]

Latent normal field

$$X \sim \mathcal{N}(\mu, \Sigma)$$

Precipitation

$$Y = \begin{cases} 0 & \textit{if } X \leq 0 \\ \psi(X) & \textit{otherwise} \end{cases}$$



State of art

$\psi(x) = ax^b$

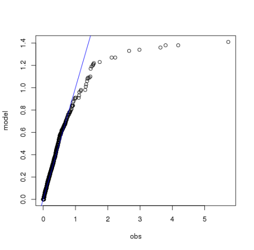
[Ailliot et al., 2009]

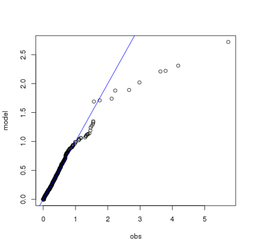
$\psi(x) = ax^r + bx^{2r}$

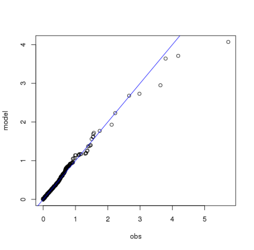
[Allcroft and Glasbey, 2003]

$\psi(x) = b(e^{ax^c} - 1)$

[Allard and Bourotte, 2015]





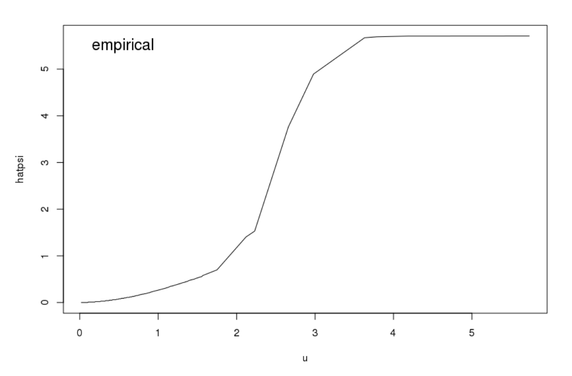


Empirical anamorphosis

We can write

$$\hat{\psi}(u) = F_{emp}^{-1}(\Phi_\mu(u)),$$

with F_{emp} the c.d.f. of Y^+, the strictly positive values of Y, and Φ_μ the Gaussian



c.d.f. with mean μ.

We can write

$$\hat{\psi}(u) = F_{emp}^{-1}(\Phi_\mu(u)),$$

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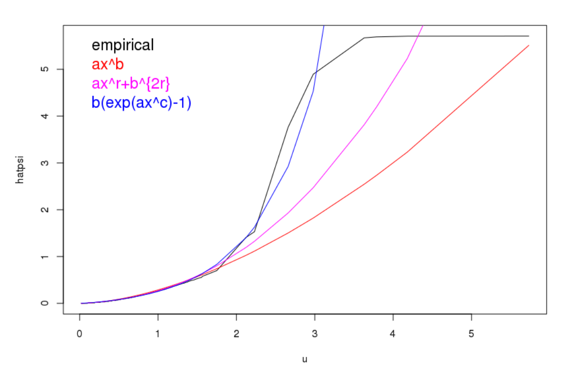


Proposed anamorphosis

We propose

$$\psi(x) = \sigma x^{\frac{1}{\alpha}} e^{\frac{\xi x^2}{2}},$$

with $\sigma > 0, \alpha > 0, \xi \geq 0$.

- Moment of order p is finite if $p < \frac{1}{\xi}$
- Equivalent to a GPD in $+\infty$:

$$P(Y > y + u \mid Y > u) \sim \left(1 + \frac{y}{\sigma_u}\right)^{-\frac{1}{\xi}}$$

as $u \to +\infty$, with $\sigma_u = \frac{u}{\sigma}$.

- Power shape controlled by α as $y \to 0$:

$$f_Y(y) \sim \frac{\alpha}{\sqrt{2\pi}} \left(\frac{y}{\sigma}\right)^{\alpha-1} exp\left(-\frac{\mu^2}{2}\right)$$

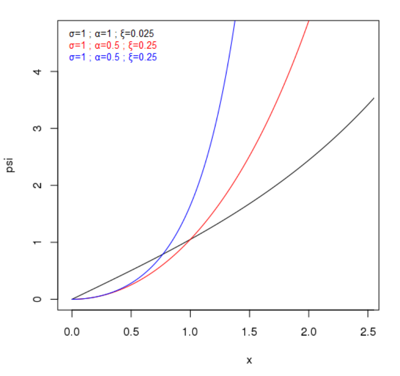
Proposed anamorphosis

We propose

$$\psi(x) = \sigma x^{\frac{1}{\alpha}} e^{\frac{\xi x^2}{2}},$$

with $\sigma > 0, \alpha > 0, \xi \geq 0$.

- the lower α, the more we produce low values
- the higher ξ, the heavier the tail is



PRELIMINARY RESULTS

Adjusting margins for radar data

$$X \sim N(\mu, 1) \text{ and } Y = \begin{cases} \sigma X^{\frac{1}{\alpha}} e^{\frac{\xi X^2}{2}} & \text{if } X > 0 \\ 0 & \text{otherwise} \end{cases}$$

Estimation

$$log(\mathcal{L}(Y, \theta)) = N_{dry} log(\Phi_\mu(0)) + \sum_{y>0} log\left[\Phi_\mu\left(\psi^{-1}(y)\right) - \Phi_\mu\left(\psi^{-1}(y + step)\right)\right]$$

with $\psi^{-1}(y) = \sqrt{\frac{1}{\alpha\xi} W\left(\alpha\xi\left(\frac{y}{\sigma}\right)^{2\alpha}\right)}$, *step* the discretization, N_{dry} the number of dry observations and the Lambert *W* function.

Data

Radar in November, 2014-2018, 5 minutes time step, 1 point

Adjusting margins for radar data

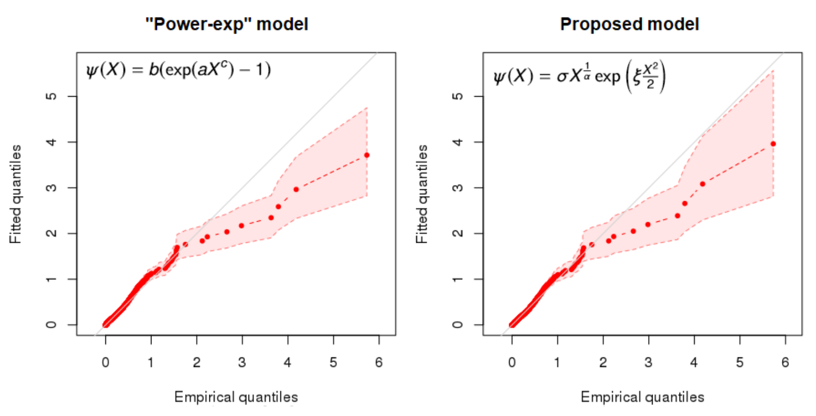


Figure 7: Quantile-quantile plots of the adjusted models, in mm. The light red area gives the 95% intervals, computed with 500 non parametric bootstrap replicates.

Interpretation of the parameters

$$X \sim N(\mu, 1) \;\text{ and }\; Y = \begin{cases} \sigma X^{\frac{1}{\alpha}} e^{\frac{\xi X^2}{2}} & \textit{if } X > 0 \\ 0 & \textit{otherwise} \end{cases}$$

As the time step increases,

- $\mu \nearrow$: less dry weather
- $\sigma \nearrow$
- $\alpha \searrow$: more low and medium rain rates
- $\xi \searrow$: less heavy tails

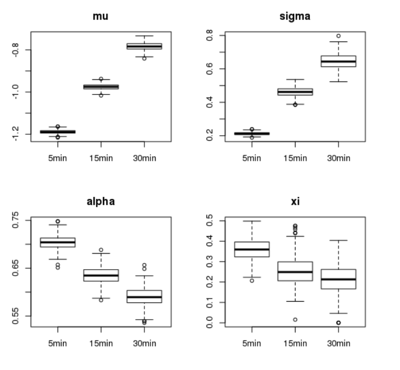


Figure 8: Boxplot of the estimated parameters at different time steps, with 500 non parametric bootstrap replicates.

APPLICATION TO URBAN HYDROLOGY

Context

The hydrological model is calibrated with **1 rain gauge**.

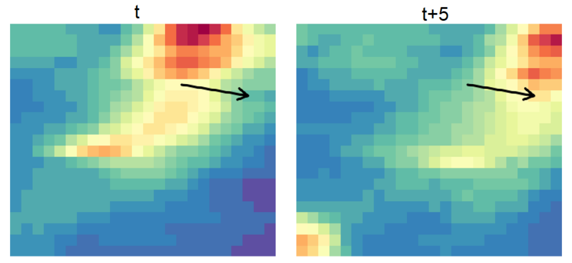
Question: Can the spatial pattern of rainfall improve the hydrological model outputs?

Need of :

- Spatialized data $\rightarrow$ radar
- 3 minute time step data $\rightarrow$ interpolation
- Gauge-like distribution $\rightarrow$ correction

Motion based interpolation

Motion estimation: maximum of correlation between lagged images

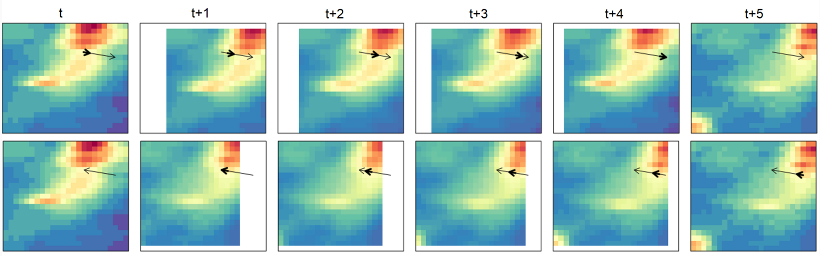


Results

- Better prediction than persitance and optical flow
- Consistent with wind (direction and speed)

Motion based interpolation

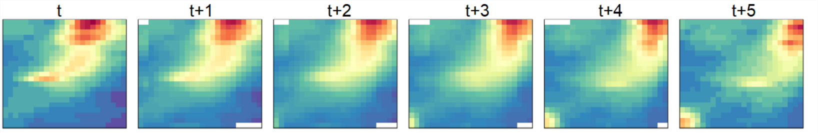
Interpolation



→ Weighted mean of the 2 ways, weights are proportional to the distance with the initial frame.

Motion based interpolation

Interpolation



Results

- We can reproduce 5min radar with 10min radar
- Interpolated images are smoother
  → **Aggregate** to keep the intensity peaks
  (instead of taking t, t+3, t+6 etc.)

Correcting rainfall distribution

Quantile-Quantile mapping

General outline:

$$Y_{corr} = F_{mod}^{-1}(F_{obs}(Y))$$

Where

- F_{obs} is the cdf of $Y \rightarrow$ radar cdf
- F_{mod} is the goal cdf $\rightarrow$ gauge cdf

2 options:

Empirical cdf

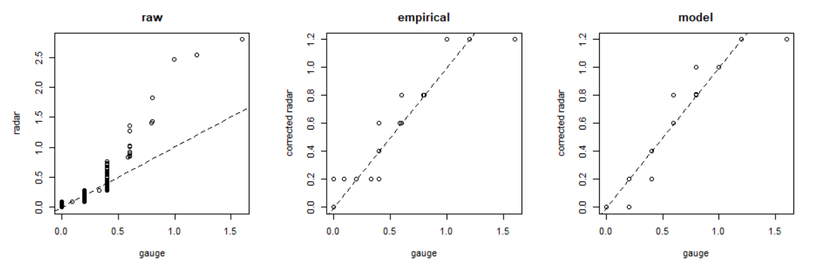
F_{obs}: empirical radar cdf
F_{mod}: empirical gauge cdf

Model cdf

F_{obs}: cdf with radar parameters
F_{mod}: cdf with gauge parameters

Some results

Example on a point where we have both rain gauge data, at a 3 min time step.



Half of the data was used for learning, and the other half was used for the qqplots shown.

Conclusion

The proposed anamorphosis for metagaussian model

- shows good adjustment on both radar and gauges data,
- gives easily interpretable parameters.

Future work

- Compare our model with a framework based on GPD: $Y = \sigma H^{-1}(G^{-1}(U))$ with H the c.d.f. of a GPD and $G(u) = u^{\alpha}$ [Naveau et al., 2016]

Example of application

- Using radar data for a hydrological model

Thank you

Reference

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