Accounting for the horizontal observation error correlation of satellite radiances in data assimilation

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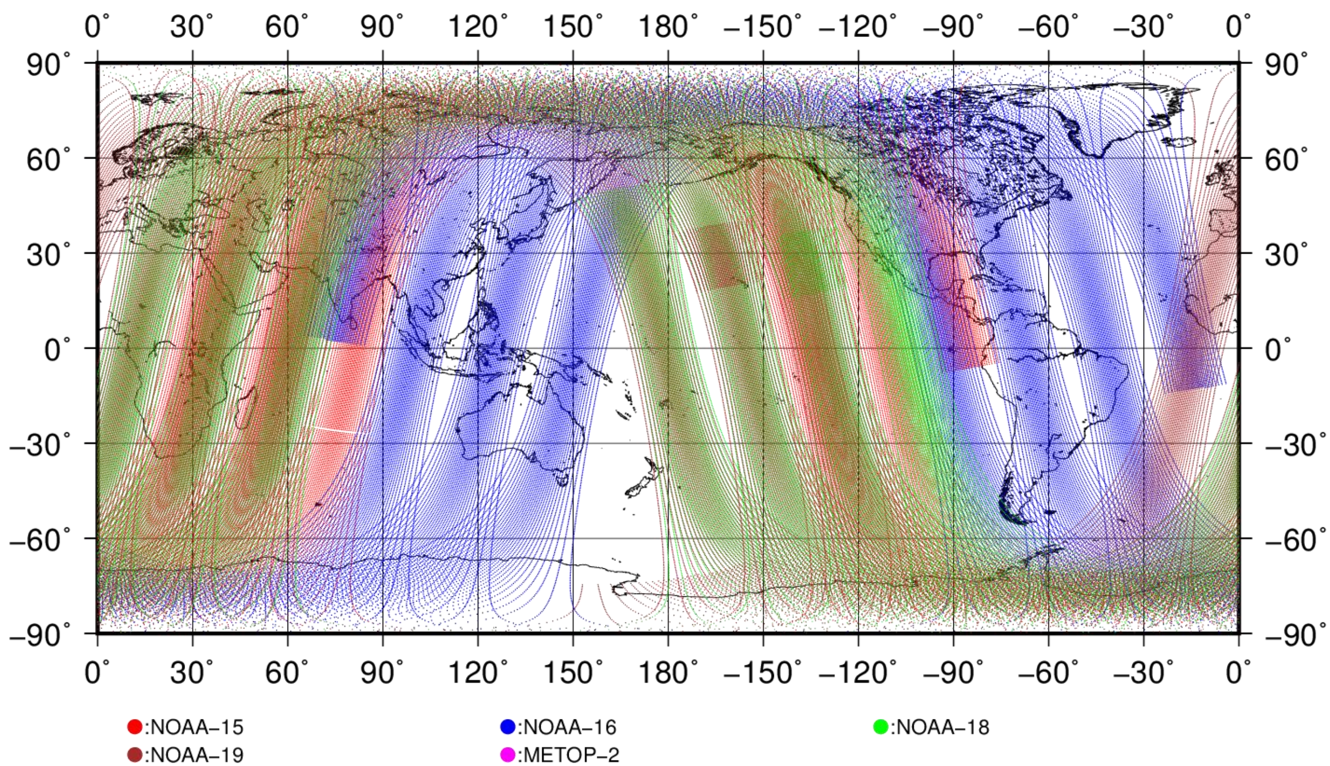
Observation error correlations

- Observations measured with the same instrument are known to have **error correlations**.
  - e.g., Satellite radiances, Atmospheric motion vector, Doppler radar

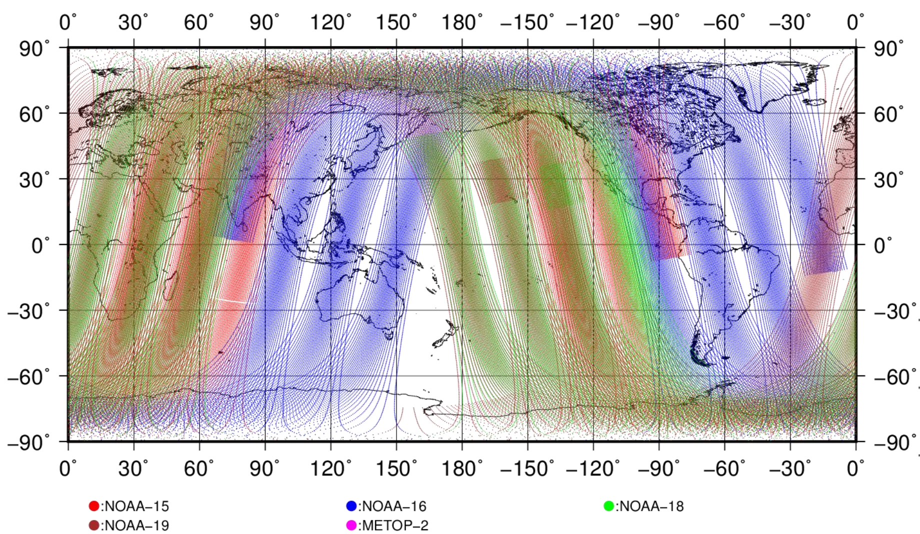
AMSU-A radiances (6 hours)



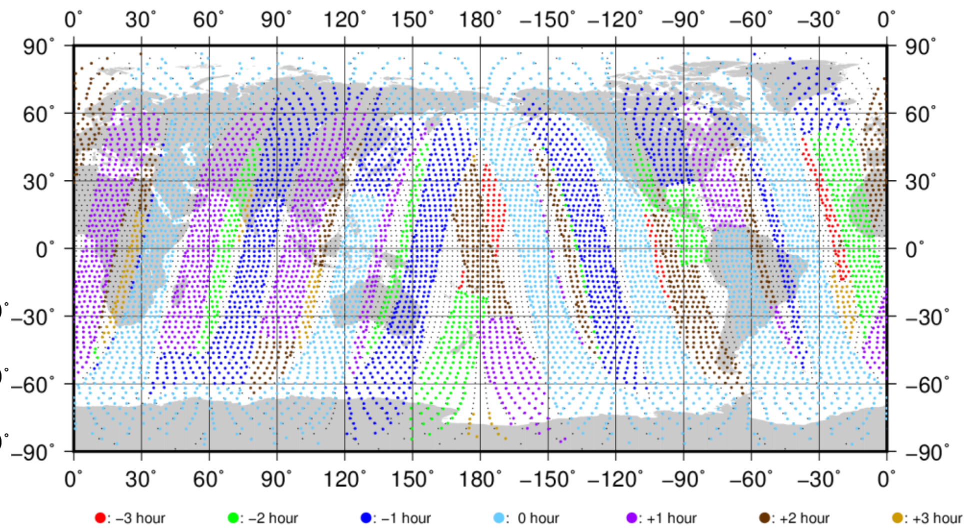
Observation error correlations

- There are some studies to estimate the **horizontal observation error correlations**, but not used in data assimilation (DA). We usually **thin** the horizontally dense observations and assume no-error correlations in DA.

Before thinning ($\approx \mathbf{400,000}$ **points**)



After thinning ($\approx \mathbf{7,000}$ **points**)

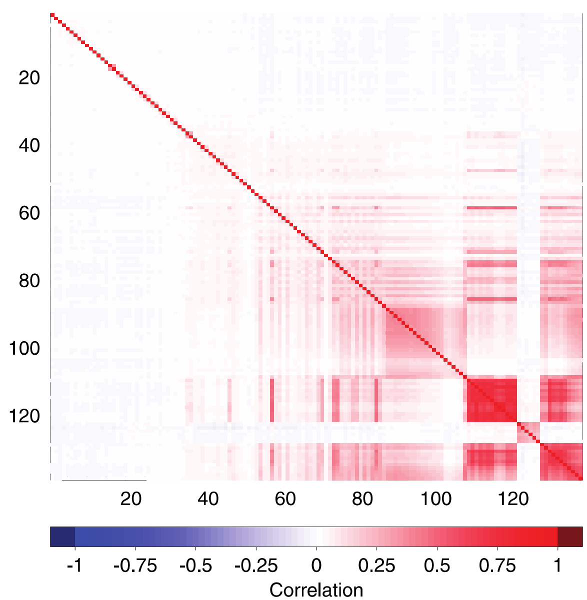


Observation error correlations

- There are some studies to estimate the **<u>horizontal observation error correlations</u>**, but not used in data assimilation (DA). We usually **<u>thin</u>** the horizontally dense observations and assume no-error correlations in DA.

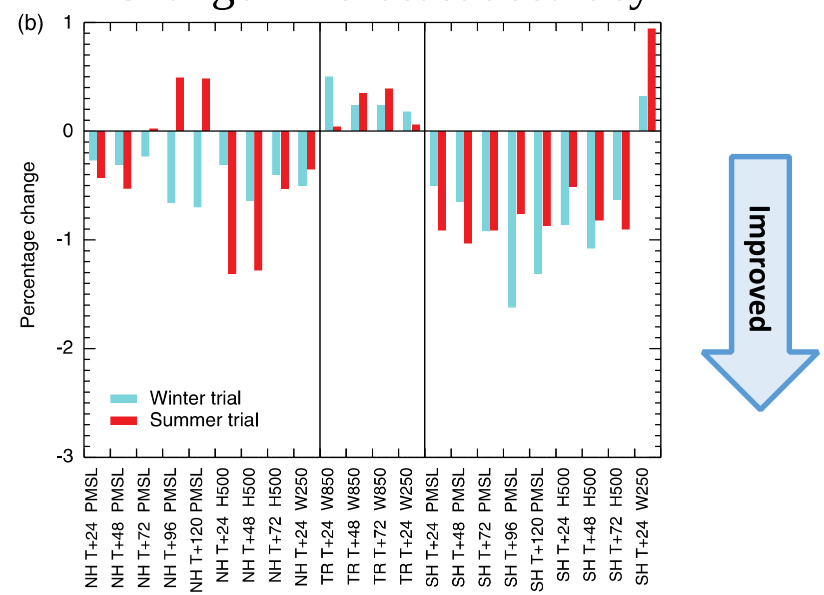
- Accounting for the **<u>inter-channel (vertical)</u>** observation error correlation will improve the analysis and forecast. (e.g., Weston et al. 2014, Bormann et al. 2016)

Estimated error correlation matrix (IASI)



(Fig. 10 of Weston et al. 2014)

Change in forecast accuracy



(Fig. 11 of Weston et al. 2014)

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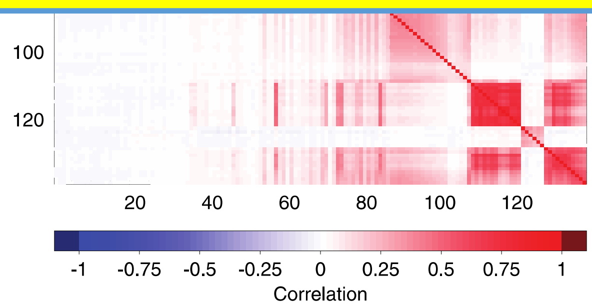
Goal is to investigate how to effectively utilize dense observations in horizontal by accounting for the **horizontal observation error correlations** of AMSU-A radiances in DA and improve the weather forecast.



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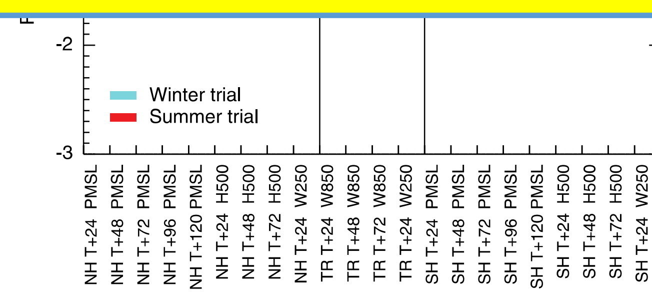


(Fig. 11 of Weston et al. 2014)

Local ensemble transform Kalman filter

Analysis Equation for LETKF

Observation error covariance matrix

$$\boxed{\mathbf{x}_a} = \boxed{\bar{\mathbf{x}}_f} + \boxed{d\mathbf{x}_f \left\{ \mathbf{U}\mathbf{D}^{-1}\mathbf{U}^{\mathrm{T}} (\mathbf{H} d\mathbf{x}_f)^{\mathrm{T}} \boxed{\mathbf{R}^{-1}} (\mathbf{y} - \overline{\mathbf{H}\mathbf{x}_f}) + \sqrt{m-1}\,\mathbf{U}\mathbf{D}^{-1/2}\mathbf{U}^{\mathrm{T}} \right\}}$$

Analysis　Ens. mean （FG)　Analysis Increment

y: observation, **x**: state variable,
H: observation operator,
a: analysis, f: forecast

Eigenvalue decomposition

$$\mathbf{U}\mathbf{D}\mathbf{U}^{\mathrm{T}} = (m-1)\mathbf{I} + (\mathbf{H} d\mathbf{x}_f)^T \mathbf{R}^{-1} (\mathbf{H} d\mathbf{x}_f)$$

- Assuming diagonal **R** matrix

<Merit>
- Low computational cost for inverting **R** matrix

<Demerit>
- Need to thin the observations
  (in spatial and between channels)

$$\mathbf{R} = \begin{pmatrix} 2 & 0.8 & 0.6 & 0.1 & 0.05 & 0.4 \\ 0.8 & 1.5 & 0.4 & 0.2 & 0.1 & 0.7 \\ 0.6 & 0.4 & 3 & 0.5 & 0.3 & 0.4 \\ 0.1 & 0.2 & 0.5 & 2 & 0.2 & 0.1 \\ 0.05 & 0.1 & 0.3 & 0.2 & 1.2 & 0.7 \\ 0.4 & 0.7 & 0.4 & 0.1 & 0.7 & 4 \end{pmatrix}$$

Observation error
Error correlations

Accounting for the OECs in LETKF

Require the inversion the **R** matrix
High computational cost
Destabilize due to the **high condition number** → Stabilize by **Recondition**
（Condition number: ratio between the largest and smallest eigenvalues）

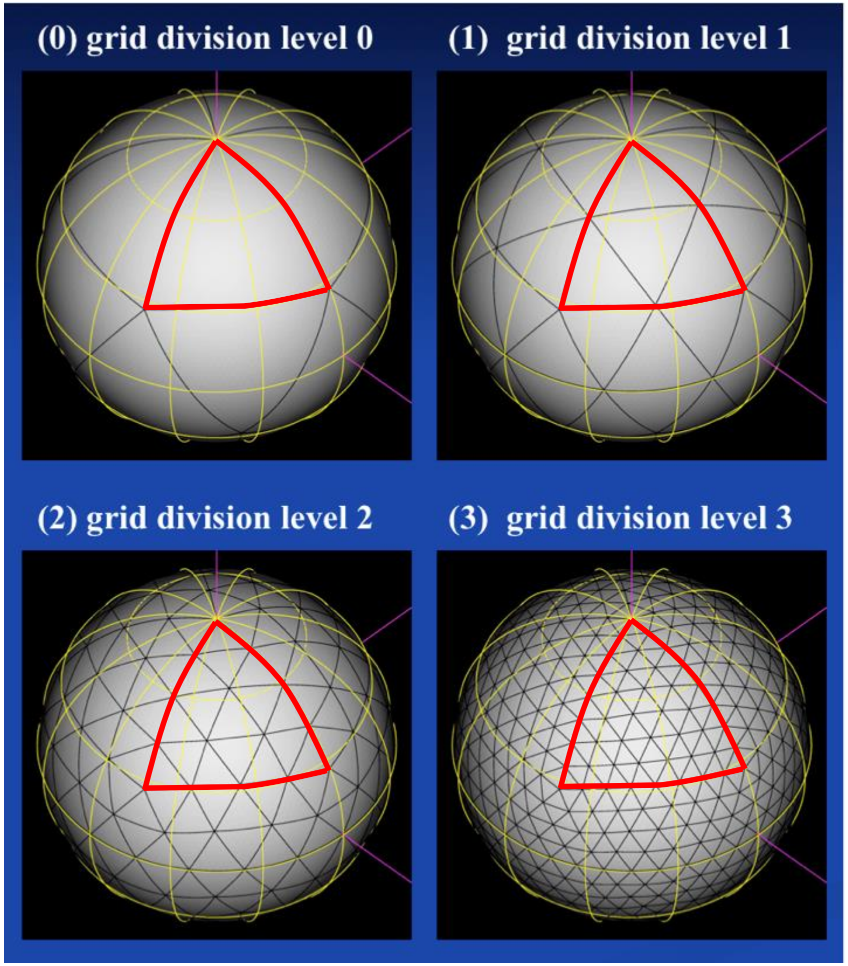
Idealized experiment

NICAM: Nonhydrostatic Icosahedral Atmospheric Model

Computer simulations create the future

Grid division level 0 is the original Icosahedron.
The horizontal resolution can be increased by splitting one triangle into four triangles.

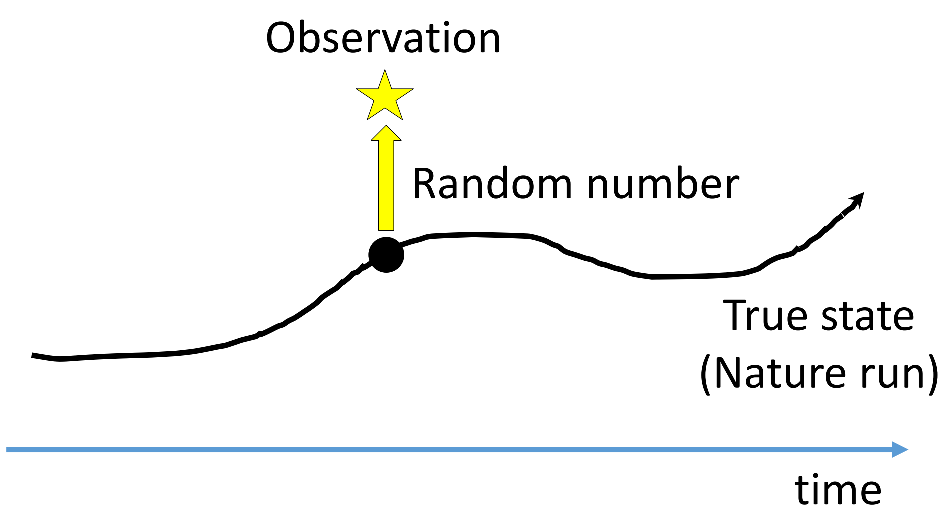
Grid division level	Horizontal resolution
6	112 km
7	56 km
8	28 km
9	14 km
10	7 km
11	3.5 km
12	1.7 km
13	0.87 km



Idealized experiment with NICAM-LETKF

Horizontal resolution:	Glevel-6 (112km)
Vertical resolution:	38 layers (model top = 40km)
Ensemble size:	40
Period:	2 months (From 2012/1/1/00Z - 2012/2/29/18Z)

Observing System Simulation Experiments (OSSE)



- Error-correlated random number

$$\mathbf{R} = \mathbf{C}\mathbf{C}^T$$

$$\boldsymbol{\varepsilon} = \mathbf{C}\boldsymbol{\mu}$$

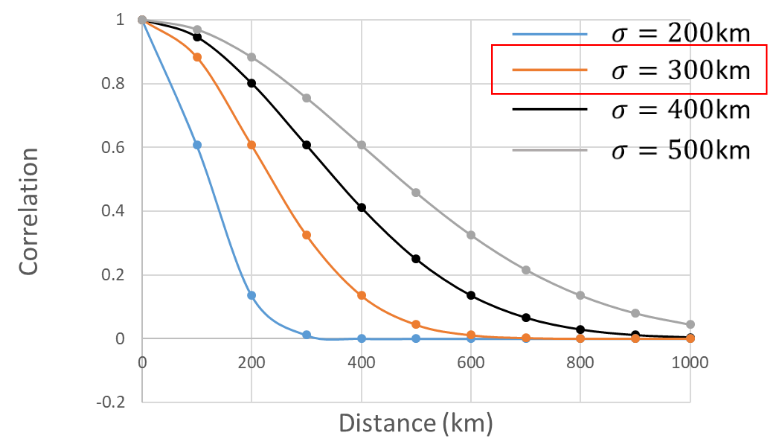
R: Observation error covariance matrix
C: Cholesky decomposition of **R** matrix
μ: Normal random number
ε: Error-correlated random number

Idealized experiment with NICAM-LETKF

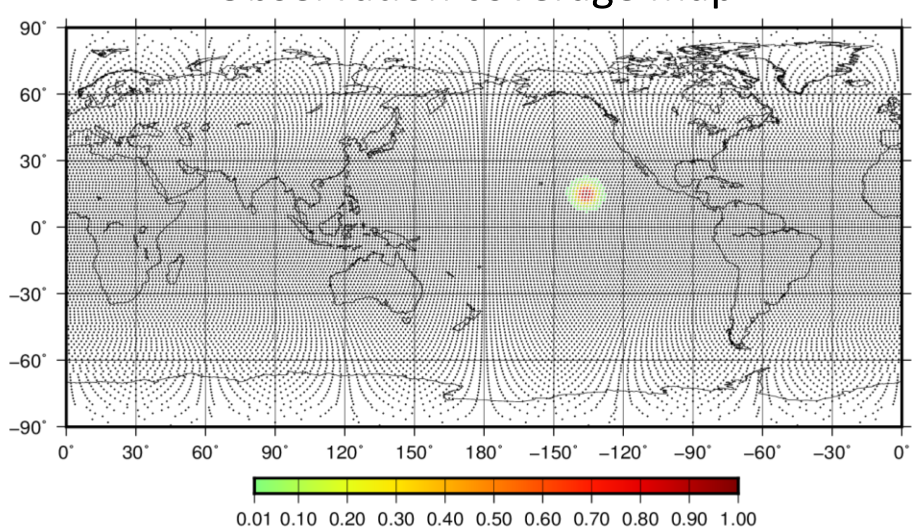
- **Simulated observations with dx=150km**
  - Error standard deviations
    - T = 2 (K), U & V = 4 (m/s)
  - Error correlations
    - 15 pressure levels
    - No error-correlation in different levels
    - **Condition number > 10^{10}**

✓Gaussian Function

$$(R_{ij}) = \exp(-\frac{1}{2}(\frac{d(i,j)}{\sigma})^2)$$

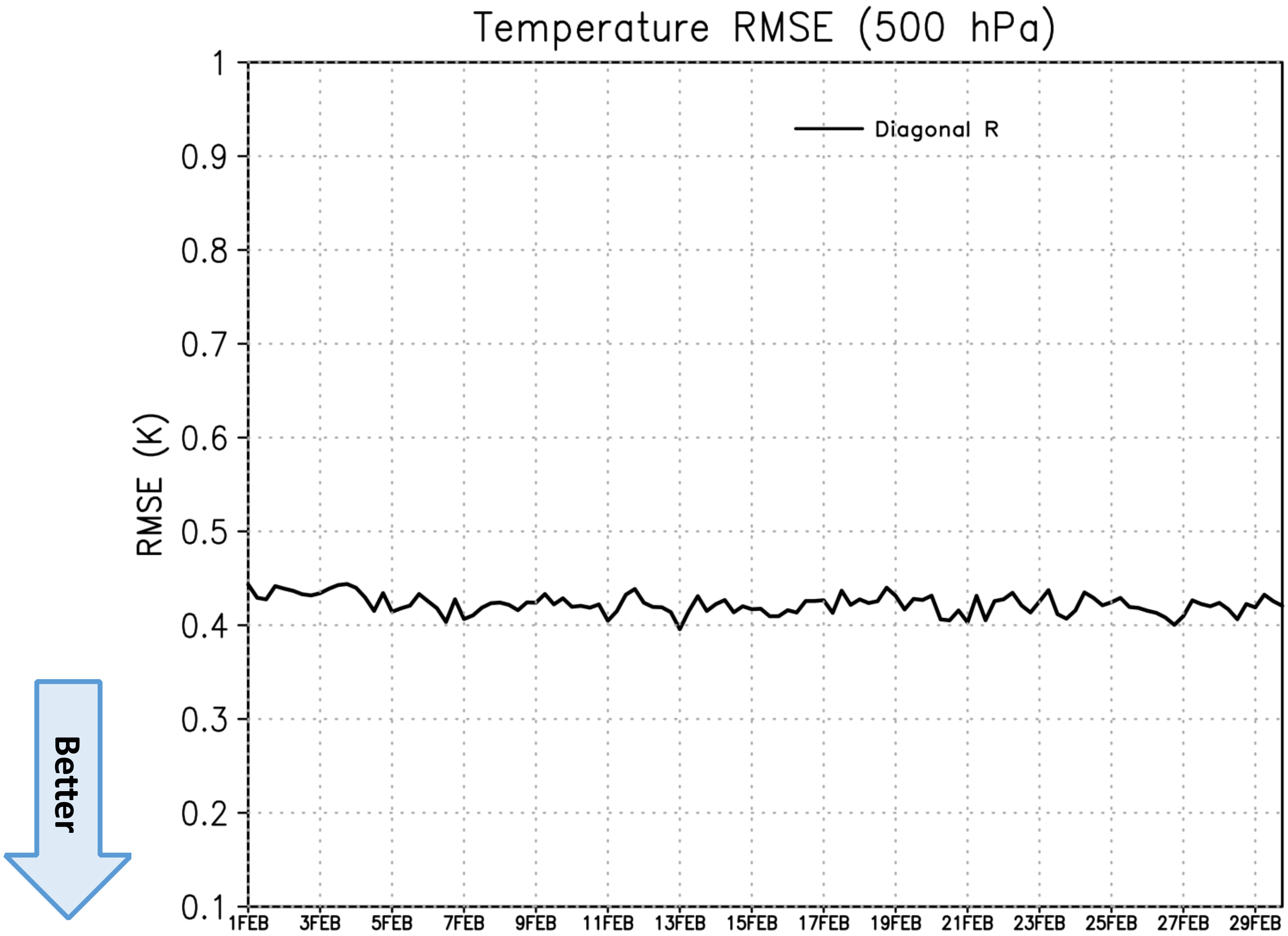


Observation coverage map



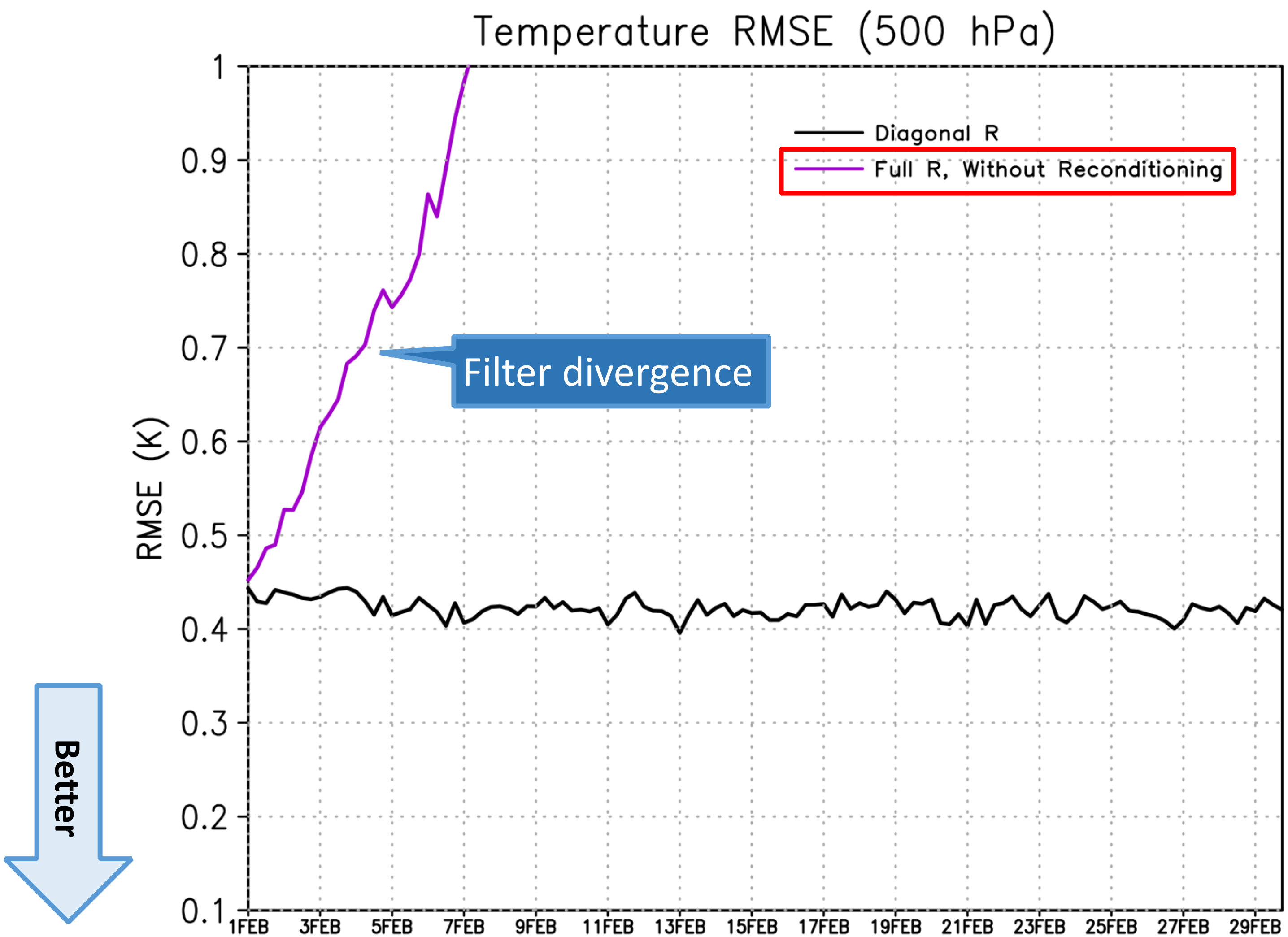
Error correlation for the observation located at 136.047°W and 14.887°N

Analysis RMSE (Temperature)



Analysis RMSE (Temperature)

Computer simulations create the future



Using extremely ill-conditioned **R** matrix leads to filter divergence.

Stabilize the LETKF by reconditioning

- **Reconditioning is a method to reduce the condition number of a matrix.**
  - Add a constant value to the diagonal terms of the **R** matrix (Weston et al. 2014).
    - It corresponds to shifting the all eigenvalues by λ_{inc}.

$$\mathbf{R}_{\mathrm{new}} = \lambda_{inc}\mathbf{I} + \mathbf{R}$$

$$\lambda_{inc} = \frac{\lambda_{max} - \lambda_{min}\kappa_{req}}{\kappa_{req} - 1}$$

κ_{req} is a condition number after reconditioning.

Example:

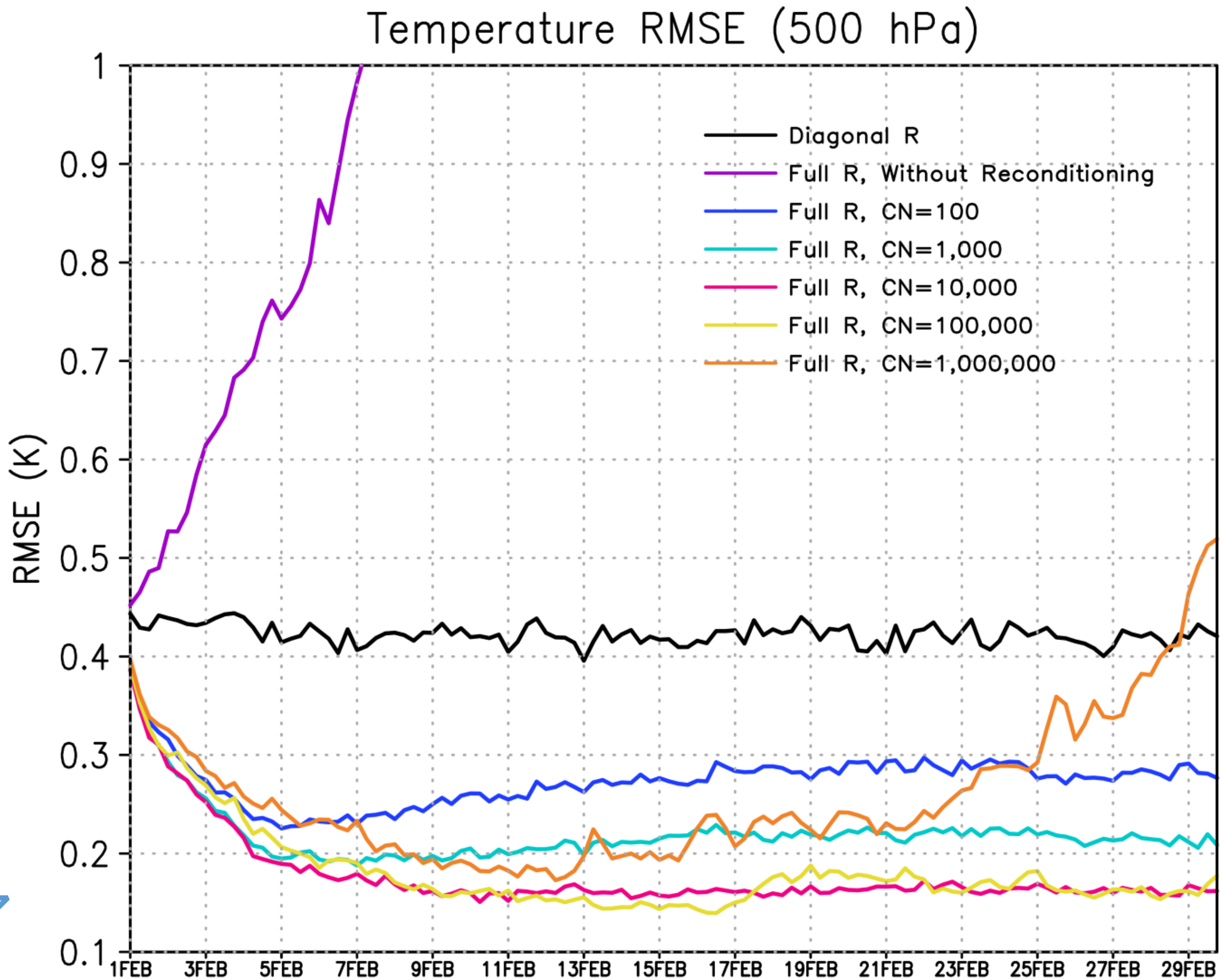
$\lambda_{max} = 10^0$ and $\lambda_{min} = 10^{-8}$ → the condition number is 10^8.

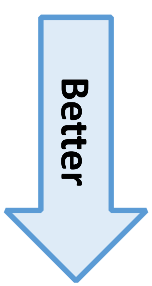
$\kappa_{req} = 10^3$ → $\lambda_{inc} \approx 0.001001$

Adding λ_{inc} does not change the structure of the original **R** matrix.

Analysis RMSE (Temperature)

Computer simulations create the future





The analysis is improved and the best with condition number 10,000 or 100,000.

Experiment with real observations (AMSU-A)

Experimental setting

Computer simulations create the future

Horizontal resolution: Glevel-6 (112km)

Vertical resolution: 38 layers (model top = 40km)

Ensemble size: 32

Period: From 2018/6/10/00UTC 2018/9/1/00UTC

Observations: Conventional observations, **AMSU-A**

	Observation error correlation	Thinning distance of AMSU-A
DIAG250 (Control experiment)		**250 km**
DIAG125		**125 km**
FULL125	✓	**125 km**

More observation

Account for full **R**

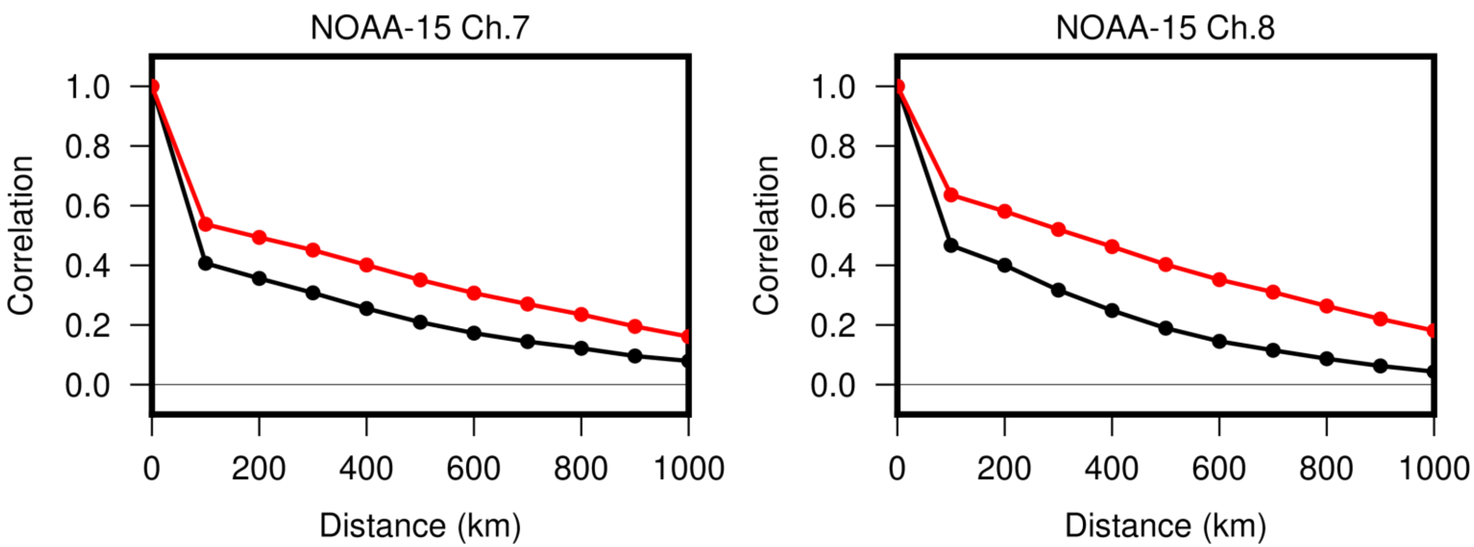
Estimation of R matrix

- **R** matrix is estimated using innovation statistics (Desroziers et al. 2005)
  - $<\mathbf{d}_a \mathbf{d}_b^{\mathrm{T}}> = <(\mathbf{y}-\mathbf{H}\mathbf{x}_a)(\mathbf{y}-\mathbf{H}\mathbf{x}_b)^{\mathrm{T}}> = \mathbf{R}$

a: analysis, b: forecast, < >: statistical expectation

- This estimation assumes that appropriate **R** matrix is used in DA.

1. **Estimate R matrix using DIAG125 experiment (Black line)**
2. **Estimate R matrix using FULL125 experiment (Red line)**



The condition number of R matrix is not so large.

Computational Cost

- Inverting the **R** matrix will increase when the non-diagonal components are considered.

- The **R** matrix becomes block diagonal because the error correlation between satellites and channels is not considered.

- Inverting the small block diagonal matrix suppress the increase in computational cost. (Up to 13 %)

$$\mathbf{R} = \begin{pmatrix} \boxed{R_1} & & & \mathbf{0} \\ & \boxed{R_2} & & \\ & & \boxed{R_3} & \\ \mathbf{0} & & & \boxed{R_4} \end{pmatrix} \Rightarrow \mathbf{R}^{-1} = \begin{pmatrix} \boxed{\mathbf{R}_1^{-1}} & & & \mathbf{0} \\ & \boxed{\mathbf{R}_2^{-1}} & & \\ & & \boxed{\mathbf{R}_3^{-1}} & \\ \mathbf{0} & & & \boxed{\mathbf{R}_4^{-1}} \end{pmatrix}$$

Computational Cost

Using 32 nodes of supercomputer FX100

	Obs Cor	Thinning		
			Obs. Ope.	**LETKF**
DIAG250		250 km	**24.54 (s)**	**70.89 (s)**
DIAG125		125 km	**32.36 (s)**	**75.18 (s)**
FULL125	✓	125 km	**32.11 (s)**	**84.89 (s)**

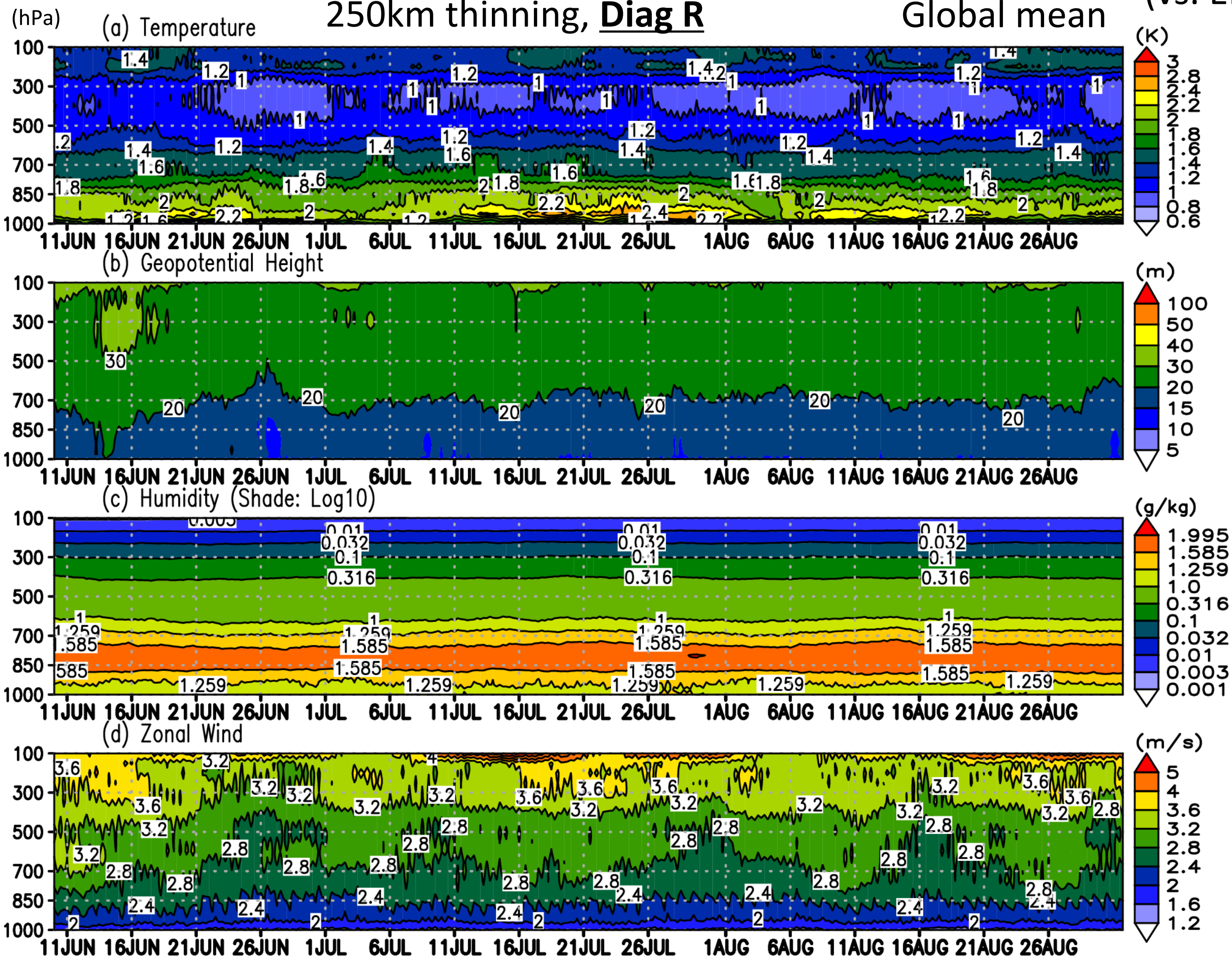
Only 13% increase

Analysis RMSE (DIAG250 : Control experiment)

Computer simulations create the future

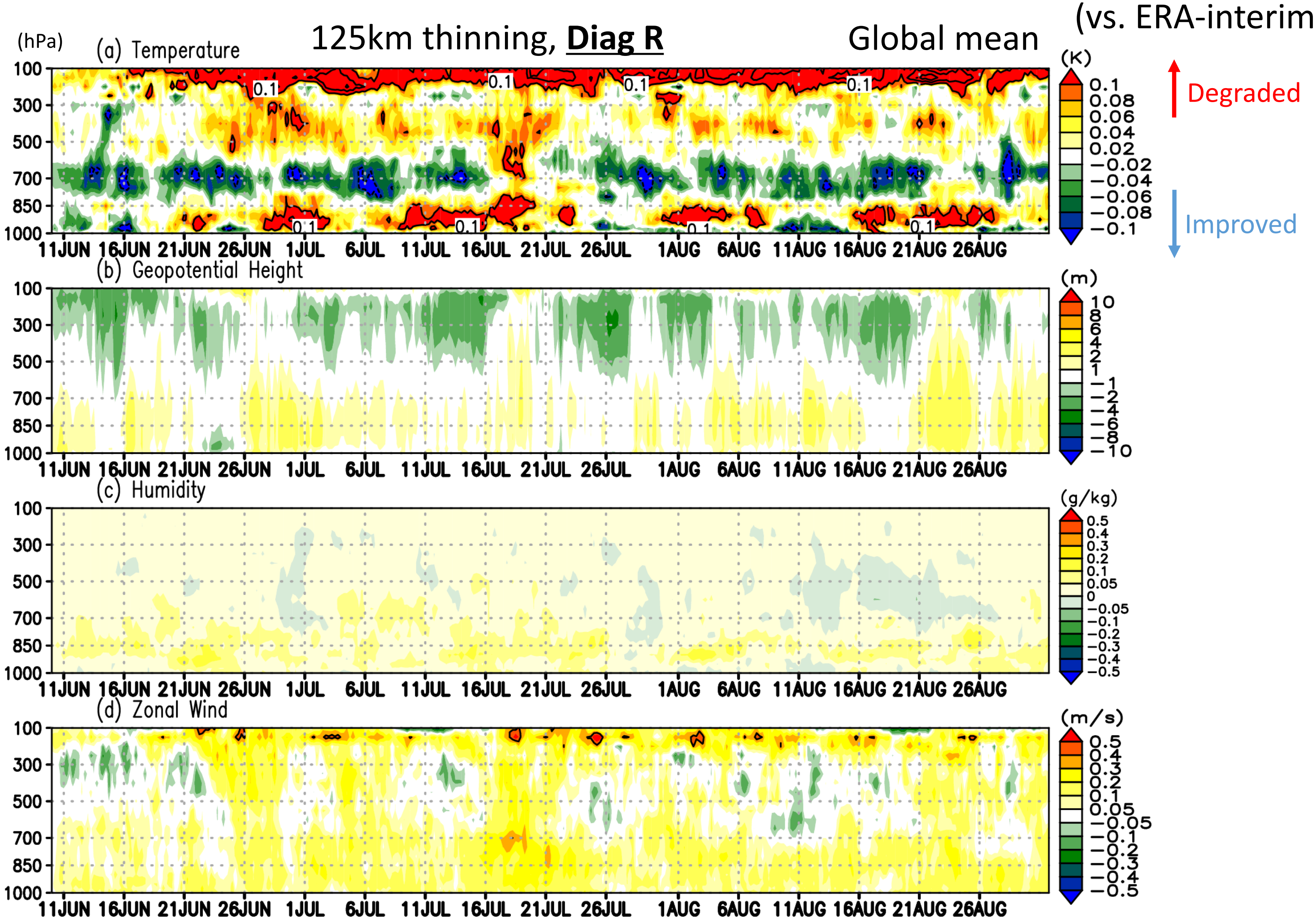
(vs. ERA-interim)

250km thinning, **Diag R** Global mean



Analysis RMSE change (DIAG125 vs DIAG250)

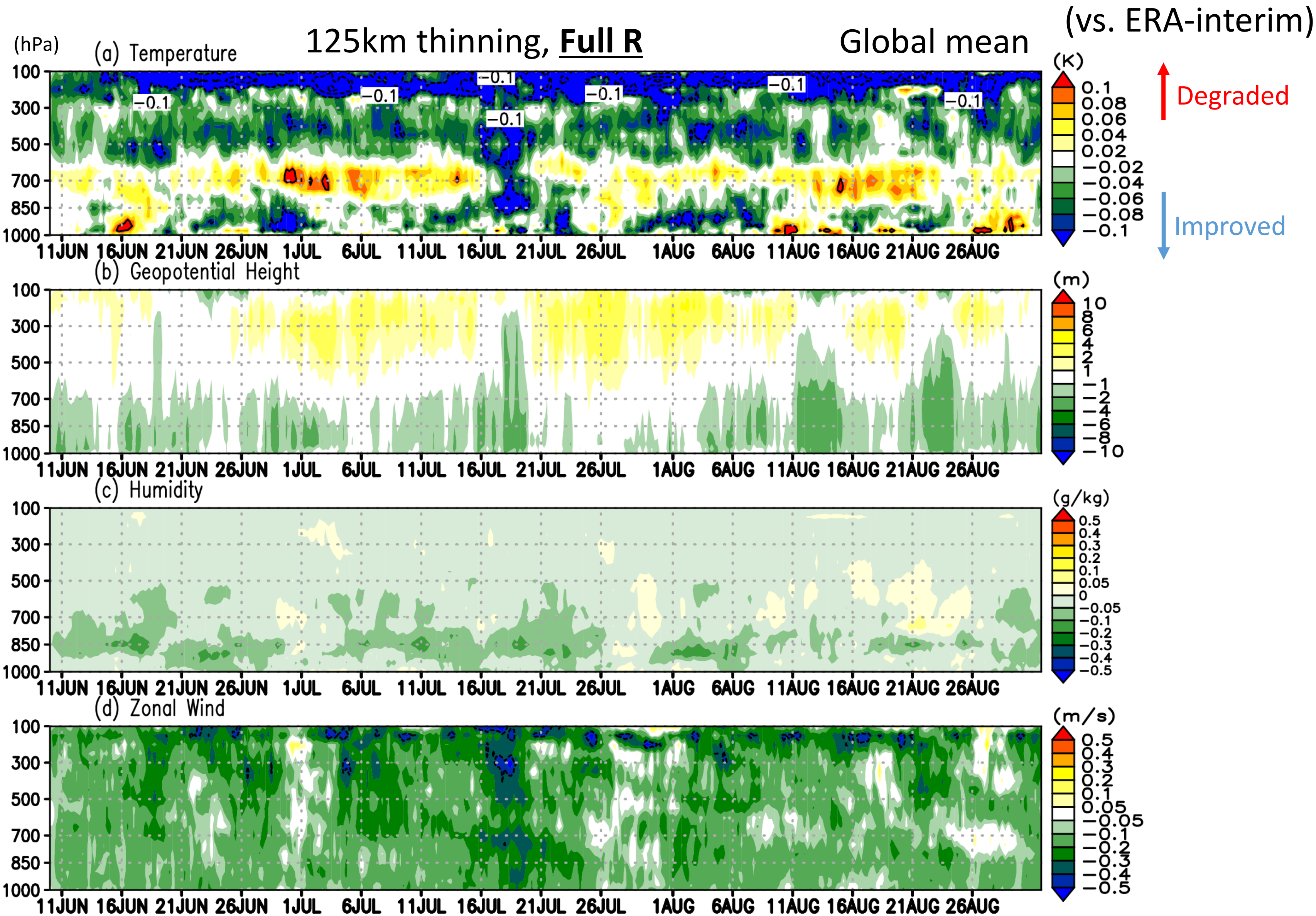
(vs. ERA-interim)



Assimilating dense observations with diagonal **R** matrix makes the analysis worse.

Analysis RMSE change (FULL125 vs DIAG125)

Computer simulations create the future

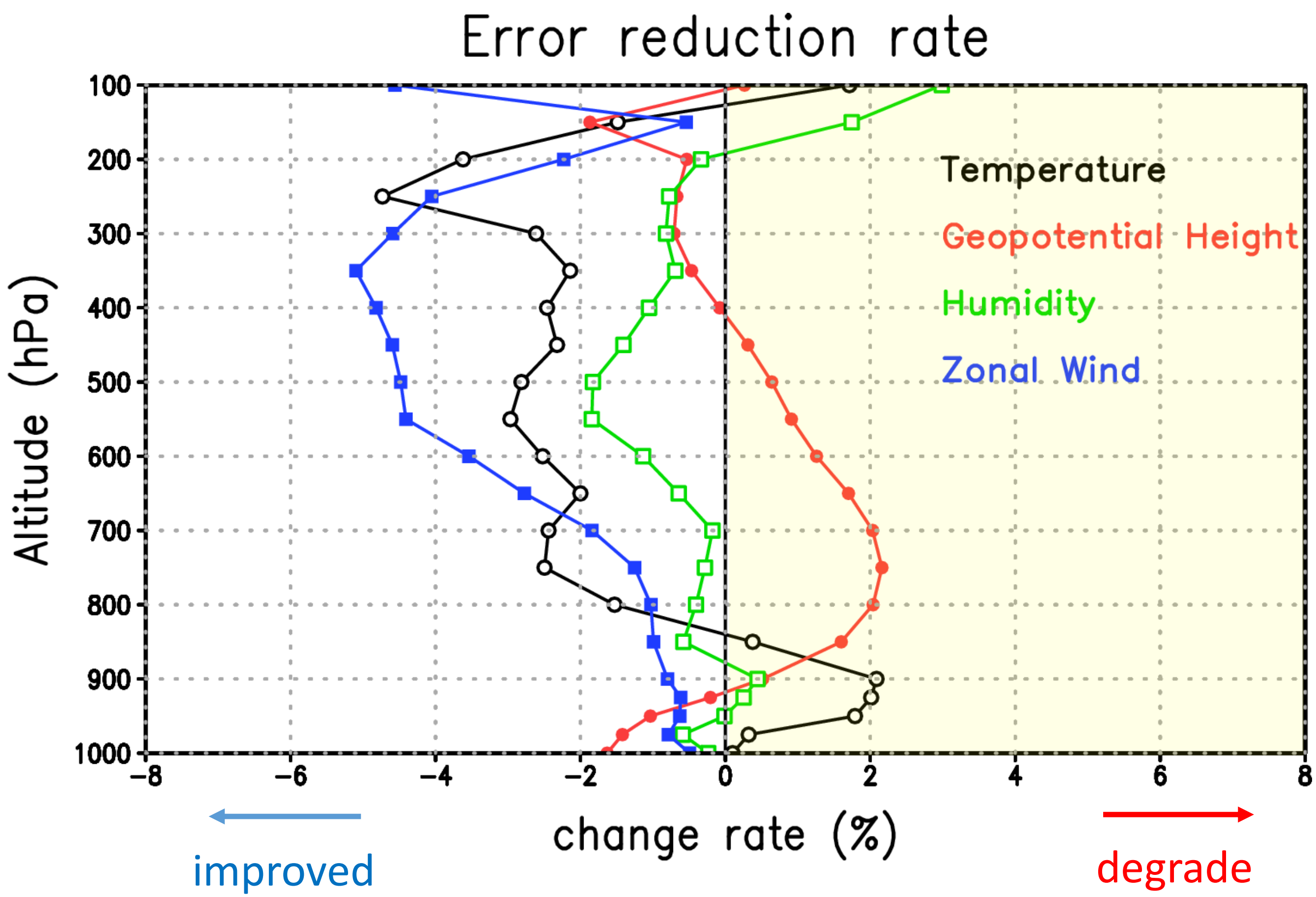


Accounting for full **R** matrix makes the analysis better.

Analysis RMSE change（FULL125 vs DIAG250)

Global mean

2-month average (From 00Z 1 July to 18Z 31 August)



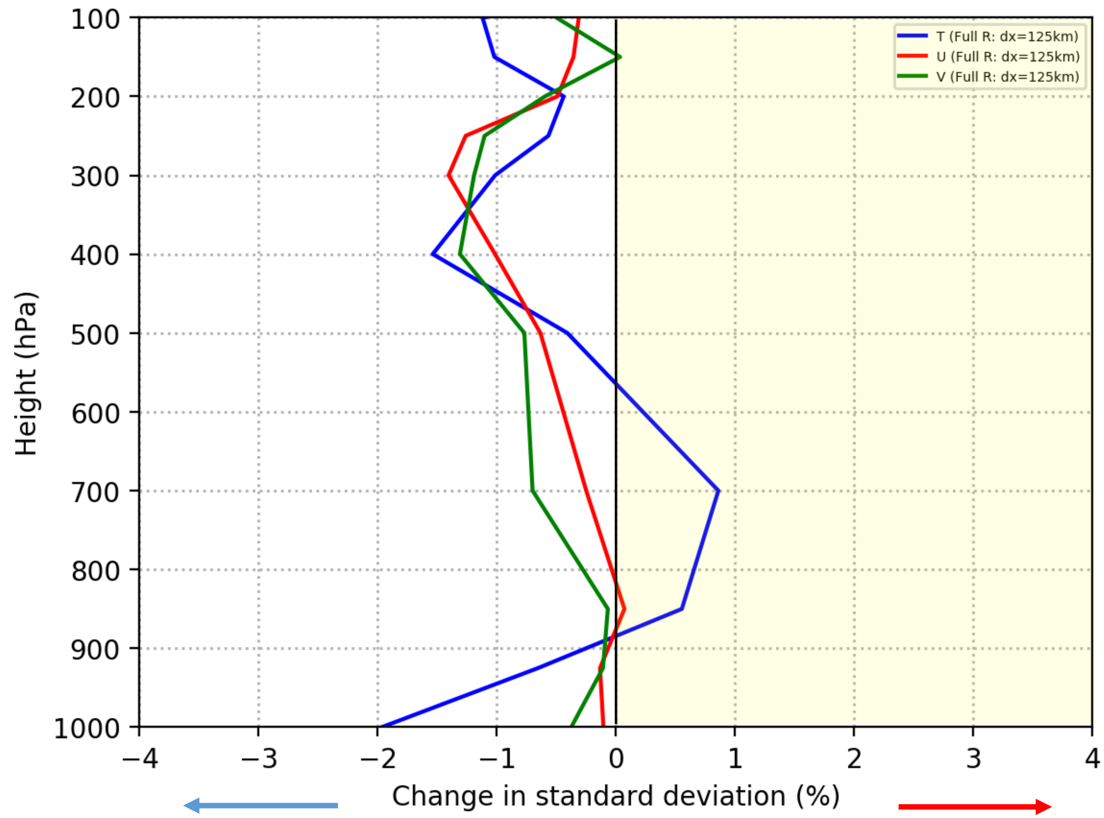
- Positive impact on zonal wind and temperature
- Slightly degrade geopotential height

Verification against observation

Computer simulations create the future

6-hour forecast

FULL125 (vs DIAG250)
250 km (Diag R) → 125 km (Full R)

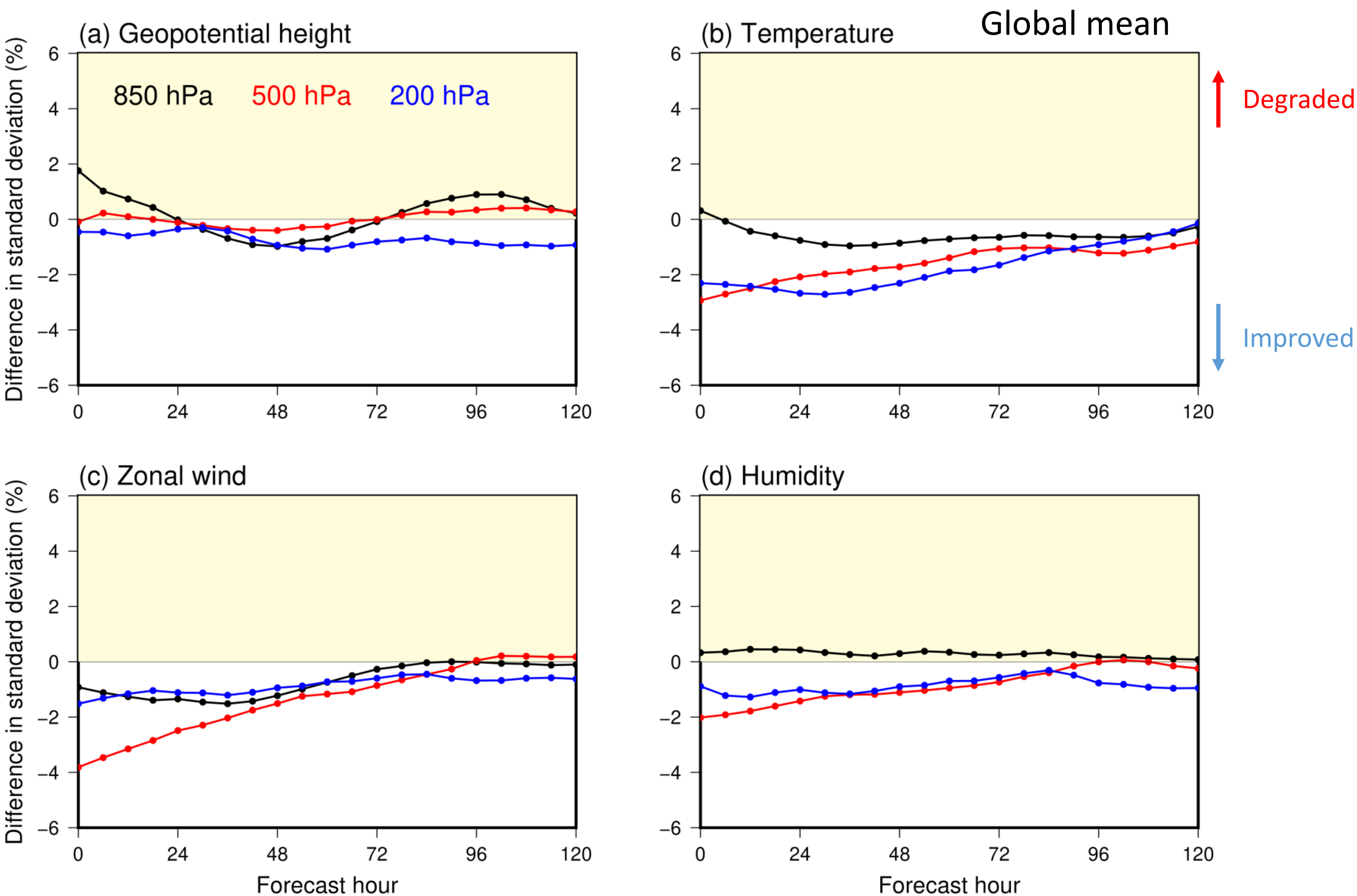


improved

degrade

- Improved except for temperature of lower troposphere

Forecast improvements



Summary

- Accounting for the horizontal observation error correlation in DA
  - Idealized case experiment
    - LETKF computation was unstable when the condition number of **R** matrix was large.
    - Reducing the condition number of **R** matrix by reconditioning stabilized the LETKF computation.
    - The analysis was greatly improved by accounting for the observation error correlation.
  - Experiment with real observations (AMSU-A)
    - **R** matrix was estimated innovation statistics.
    - The analysis was improved by up to 5% by accounting for the observation error correlation.
    - The forecast was also improved especially for temperature and zonal wind.