The impact of different model error covariance matrices on the performance of a particle filter applied to a coupled ocean-atmosphere climate model

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Proposal densities

$$p(x \mid y) = \frac{p(x)p(y \mid x)}{p(y)}$$

The freedom in the choice of $q$ is how we design methods to avoid the curse of dimensionality.
Proposal densities

\[ p(x \mid y) = \frac{p(x)p(y \mid x)q(x, y, z)}{p(y)q(x, y, z)} \]

The freedom in the choice of \( q \) is how we design methods to avoid the curse of dimensionality.
Basic Particle Filter
Equivalent weights particle filter
A fully nonlinear particle filter which has not been found to suffer from filter degeneracy. See van Leeuwen [2010] and Ades and van Leeuwen [2012].

**Required:**
- Observations $y$ with error covariance matrix $R$
- (linear) observation operator $H$
- 2 tuning parameters
- Model *error* covariance matrix $Q$
Coupled Ocean-Atmosphere GCM [Gordon et al., 2000].
Total of 9 prognostic variables:

<table>
<thead>
<tr>
<th>Atmosphere</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zonal and meridional winds, temperature, humidity and surface pressure</td>
</tr>
<tr>
<td>resolution $2.5,^\circ$ with 19 levels.</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Ocean</th>
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<tbody>
<tr>
<td>Zonal and meridional flow, temperature and salinity</td>
</tr>
<tr>
<td>Resolution $1.25,^\circ$ with 20 levels.</td>
</tr>
</tbody>
</table>

State vector dimension $2,314,430$. 
Twin experiment

- Assimilate daily SST data over a period of 6 months
- Full coverage of SSTs – 27,370 variables
- 72 model timesteps between observations
- $R$ diagonal
\[ dx = f(x)dt + d\beta \]

where

\[ d\beta \sim \mathcal{N}(0, Q) \]

- Can always make improvements here
The big question – modelling $Q$.

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- No control variable transform known for the model error covariance matrix of this coupled system
- We estimate $Q$ by looking at a long model run
- 3D, staggered grids, multiple variables, bathymetry, real data
  $\implies$ no elegant way of doing matrix-vector multiplication with $Q$
- Need for sparse BLAS routines to make efficient
Figure: Stencil for localisation matrix
Figure: Stencil for localisation matrix
Matrix properties

- Symmetric so stored in upper triangular form
- $N = 2,314,430$
- $\text{NNZ} = 411,266,537$
- up to 1000 non-zeros per row.
- average of 354 non-zeros per row.
- 16Gb on disk.
- load $Q$ took 228.75 seconds
Matrix-vector multiplication timings

- Serial coord_matmul_t: 1.05s
- Parallel openmp_matmul_t (24 threads):
- SPARSE BLAS matmul (12 threads):

LIBRSB (Recursive sparse blocks) Martone et al. [2010]
Matrix-vector multiplication timings

- Serial coord_matmul_t: 1.05s
- Parallel openmp_matmul_t (24 threads): 0.56s
- SPARSE BLAS matmul (12 threads):
  
  LIBRSB (Recursive sparse blocks) Martone et al. [2010]
Matrix-vector multiplication timings

- Serial coord_matmul_t: 1.05s
- Parallel openmp_matmul_t (24 threads): 0.56s
- SPARSE BLAS matmul (12 threads): 9.44E-002s

LIBRSB (Recursive sparse blocks) Martone et al. [2010]
Two different $Q$ matrices

We estimate $Q$ from a long model run.

\[ \text{Cov}(X_1) = \Lambda_1^{1/2} \Sigma_1 \Lambda_1^{1/2} \]

\[ Q_1 \propto \Lambda_1^{1/2} \Sigma_1^2 \Lambda_1^{1/2}. \]
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\[ X_2 := X_1 - \bar{X}_1 \]

\[ \text{Cov}(X_2) = \Lambda_2^{1/2} \Sigma_2 \Lambda_2^{1/2} \]

\[ Q_2 \propto \Lambda_2^{1/2} \Sigma_2 \Lambda_2^{1/2} \cdot \]
Correlations between atmospheric $U$ and ocean $V$

$Q_1$

$Q_2$
Correlations between atmospheric humidity and ocean $\theta$

$Q_1$

$Q_2$

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$Q$ for a climate model and the EWPF
Atmospheric pressure variables in $\Lambda^{\frac{1}{2}}$

$Q_1$

$Q_2$
Eastward surface winds in $\Lambda^{\frac{1}{2}}$

Atmosphere U, sigma level = 0.996

$Q_1$

$Q_2$

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$Q$ for a climate model and the EWPF
Sea water temperature at the 4th depth level in $\Lambda^{1/2}$
Trajectories

$Q_1$

$Q_2$

for a climate model and the EWPF
Trajectories

$Q_1$

$Q_2$

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$Q$ for a climate model and the EWPF
Rank histogram of observed SSTs

Histogram 22 - Ocean Theta level 1 surface

$Q_1$

Histogram 22 - Ocean Theta level 1 surface

$Q_2$
Rank histogram of atmospheric zonal velocities

$Q_1$

$Q_2$

Histogram 10 - Atmosphere V level 13 200k p*
RMSE errors and EWPF parameters

Observed SSTs

Unobserved Surface Humidity

Stochastic ensemble

- strong nudging, $\kappa = 1.0$

- weak nudging, $\kappa = 1.0$

- strong nudging, $\kappa = 0.8$
Conclusions

- The equivalent weights particle filter continues to show no signs of filter degeneracy when the model has size $2.3 \times 10^6$
- Having a more physically realistic $Q$ matrix has allowed much smaller scaling to be used and appears more robust
- We have only considered twin experiments – what will $Q$ look like in the real system?
- New ideas and lots of research into $Q$ is necessary for both particle filters and Weak Constrained methods.
Thank you for listening


