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The 2nd IMT-Atlantique & RIKEN Joint Workshop

The parameter estimation system in SCALE for reduced-precision floating-point numbers

TSUYOSHI YAMAURA

COMPUTATIONAL CLIMATE SCIENCE RESEARCH TEAM, RIKEN CENTER FOR COMPUTATIONAL SCIENCE (R-CCS)

Outline of this talk

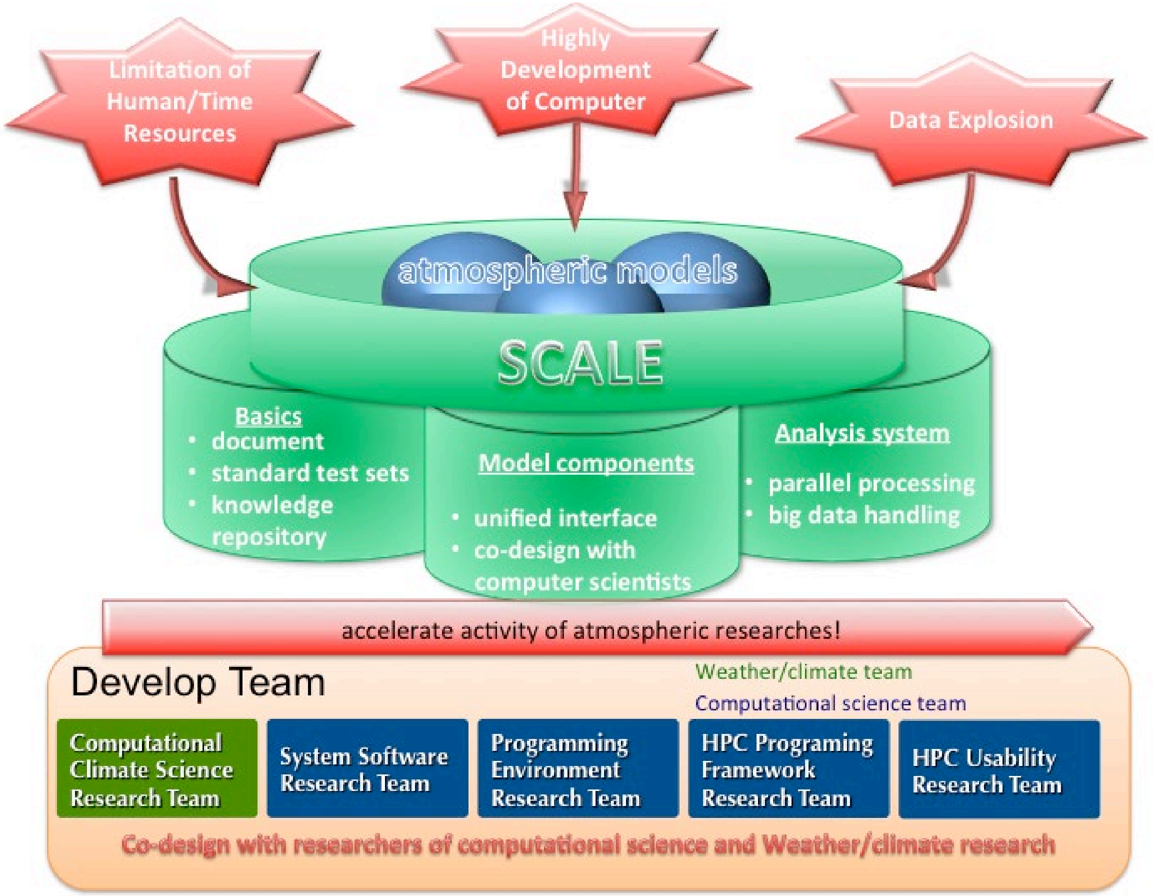
1. Introduction of “SCALE”
   - We need parameter tuning !!
   - Current and future LETKF system with SCALE
2. Using reduced-precision floating-point number (FPN)
   - Why do we need reduced-precision FPN?
   - Theory of FPN errors
   - Numerical experiments
3. Summary

Open source software ~ SCALE ~

SCALE

Scalable Computing for Advanced Library and Environment

http://r-ccs-climate.riken.jp/scale/



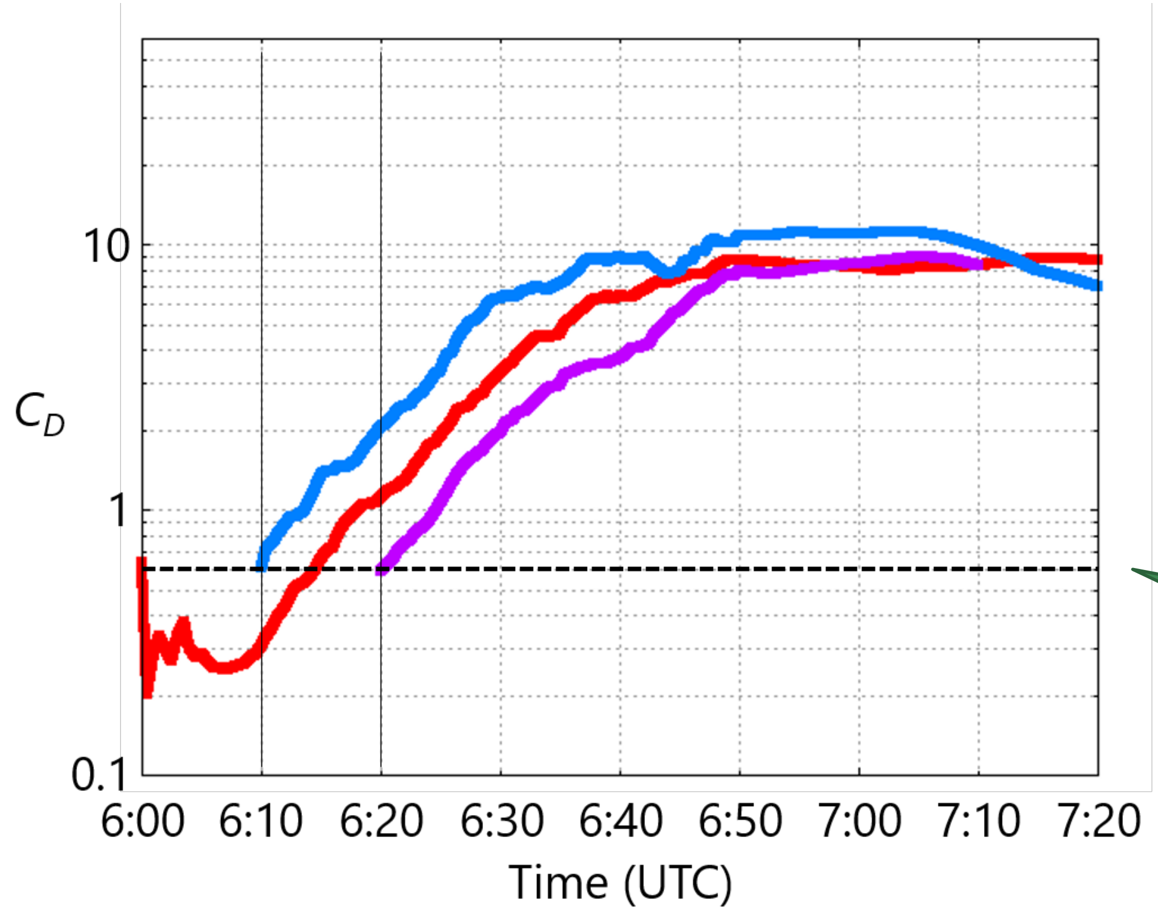
Parameter tuning is strongly needed!!

- Weather forecasting model has a lot of unknown/empirical parameters …
  - Is the parameter correct?
  - We don't know it … but we have to set parameters for running the model.
- Parameter tuning is needed for getting good results.
  - So far, we just checked the parameters and those results manually.

```
&PARAM_ATMOS_PHY_MP_TOMITA08
 Cr            = 78.0,
 Cs            =  0.9,
 drag_g        =  2.5,
 beta_saut     =  0.006,
 gamma_saut    =  0.06,
 gamma_sacr    =  0.02,
/
```

Ex. Microphysics parameters using SCALE model.

Parameter estimation using data assimilation



A example of parameter estimation by using data assimilation. We can estimate the reasonable value of a coefficient from data science.
(Next talk by Dr. Sueki)

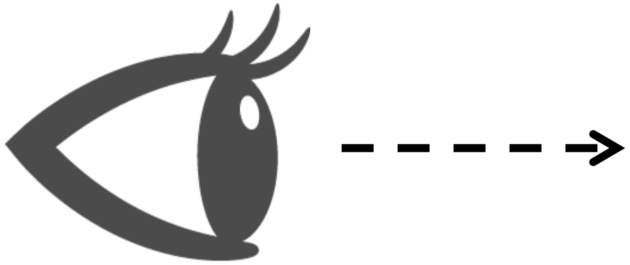
Challenging points

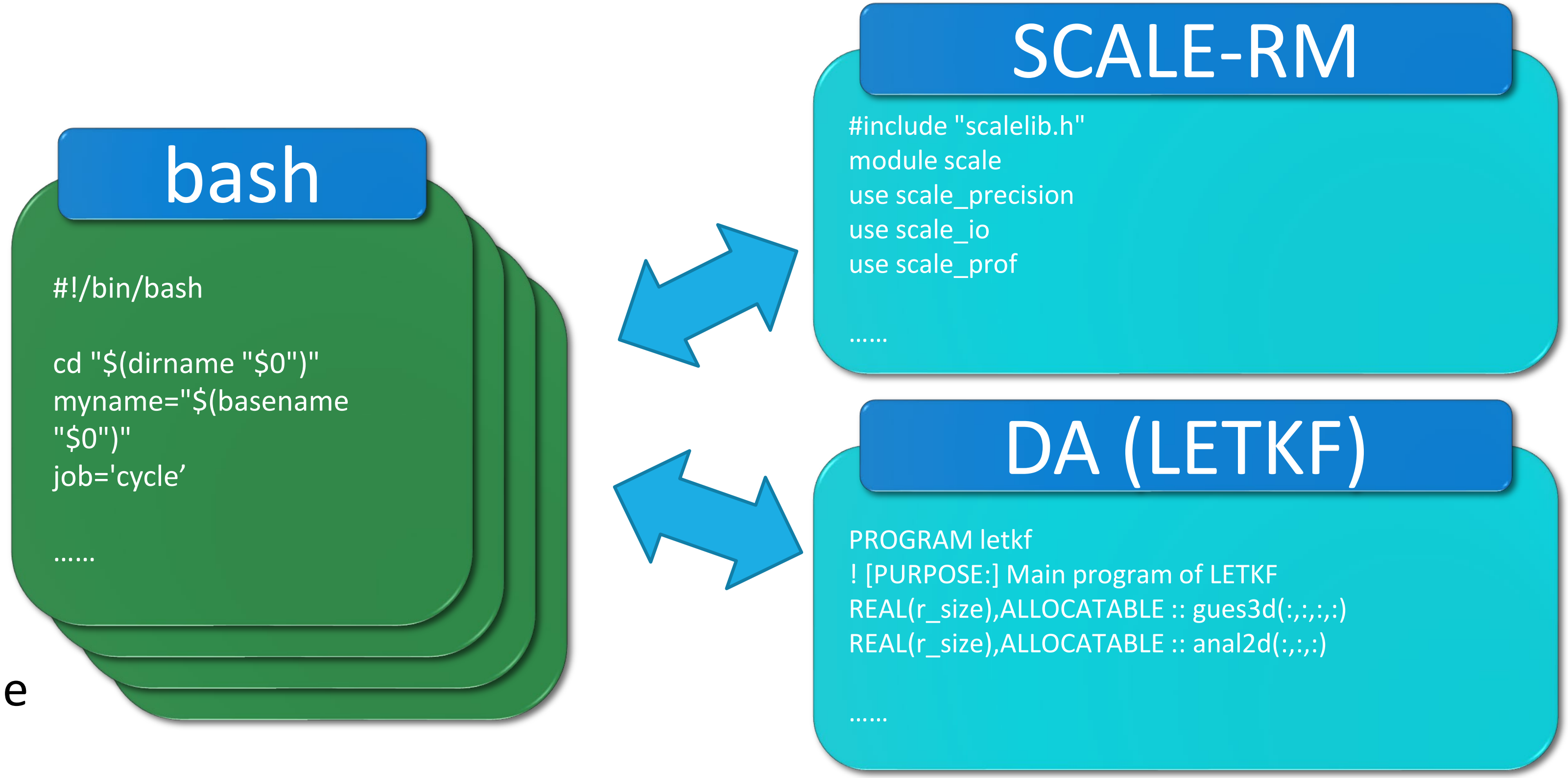
We can search the most superior parameter combinations by using DA!

Current parameter estimation of SCALE-LETKF

- The bash scripts should call SCALE and LETKF programs

User sees many bash scripts



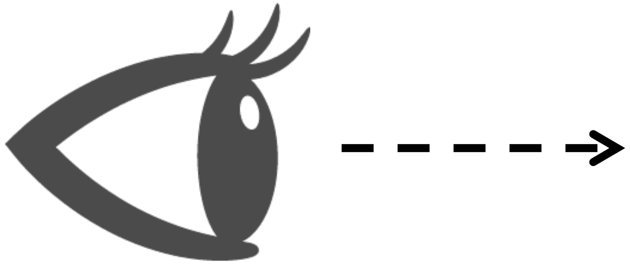


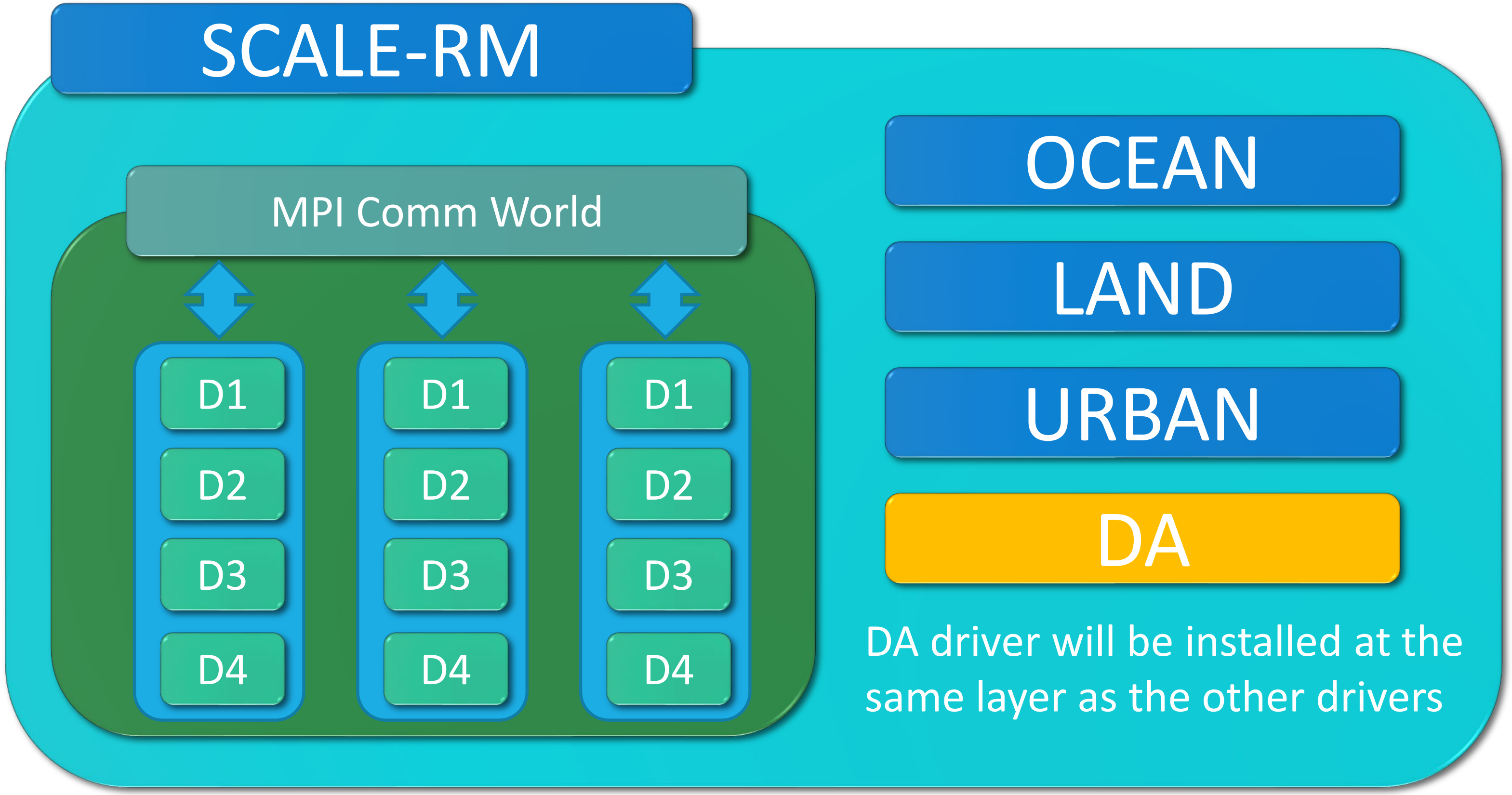
It’s too complex to understand the structure of the whole system …

Implementation of Data Assimilation Driver

- Future: DA (LETKF) driver will be installed in SCALE library

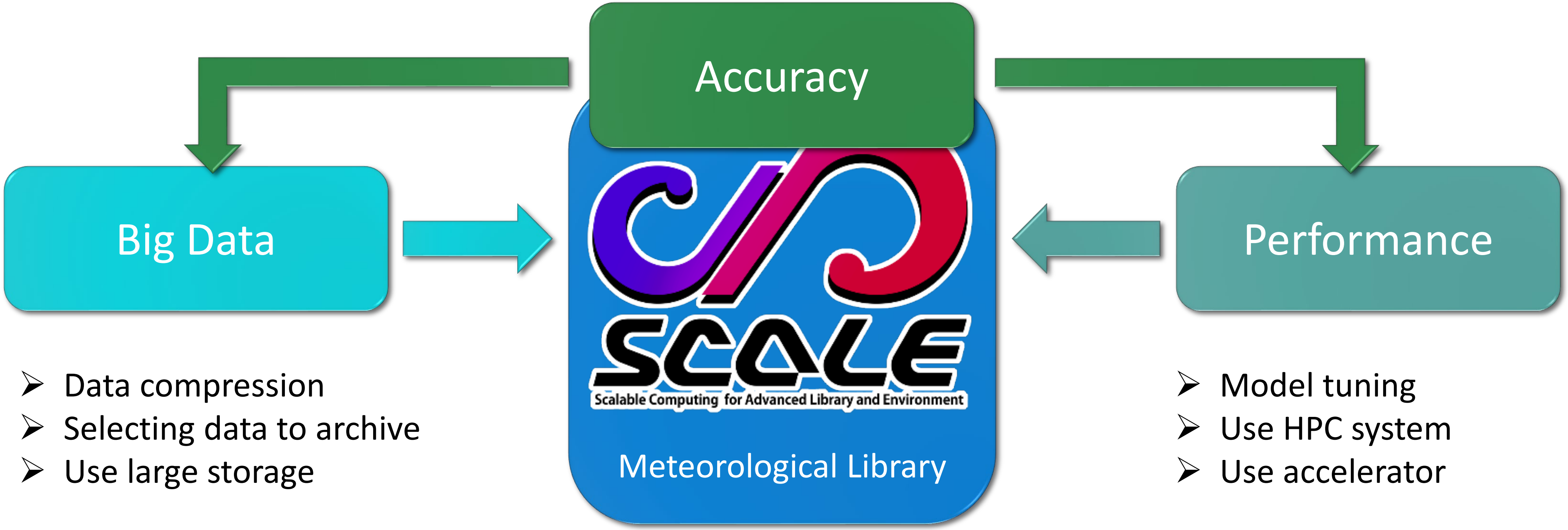
User can see SCALE program only!





DA driver is one of the library in SCALE

Issue of recent weather/climate models

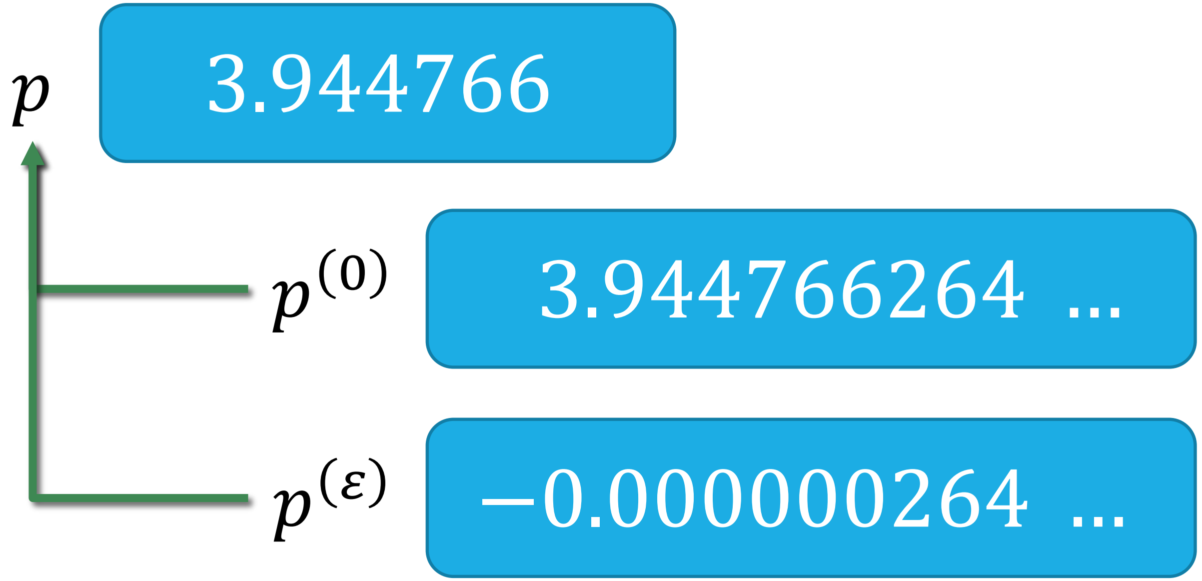


- Data compression
- Selecting data to archive
- Use large storage

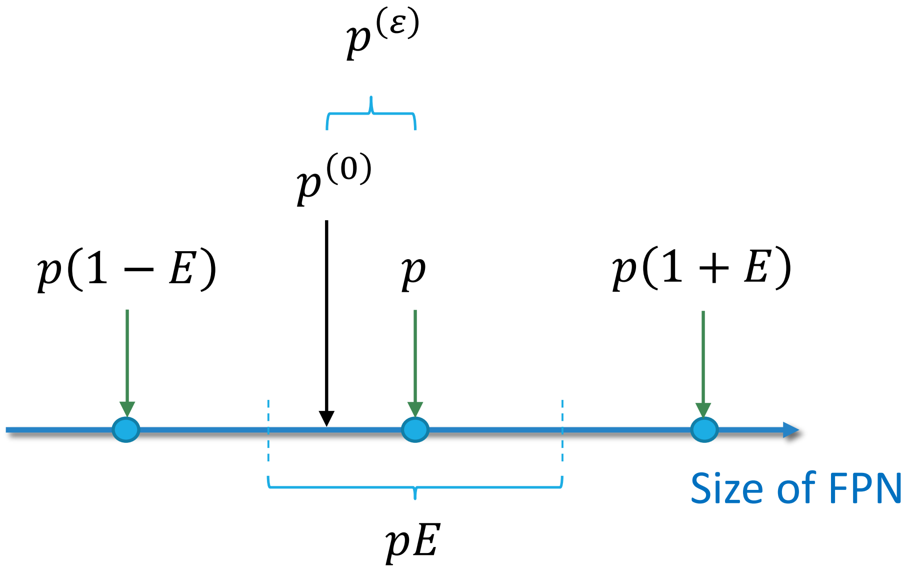
- Model tuning
- Use HPC system
- Use accelerator

Are double-precision (DP) floating point numbers (FPNs) really required in the numerical weather/climate models?

Expression of floating-point number



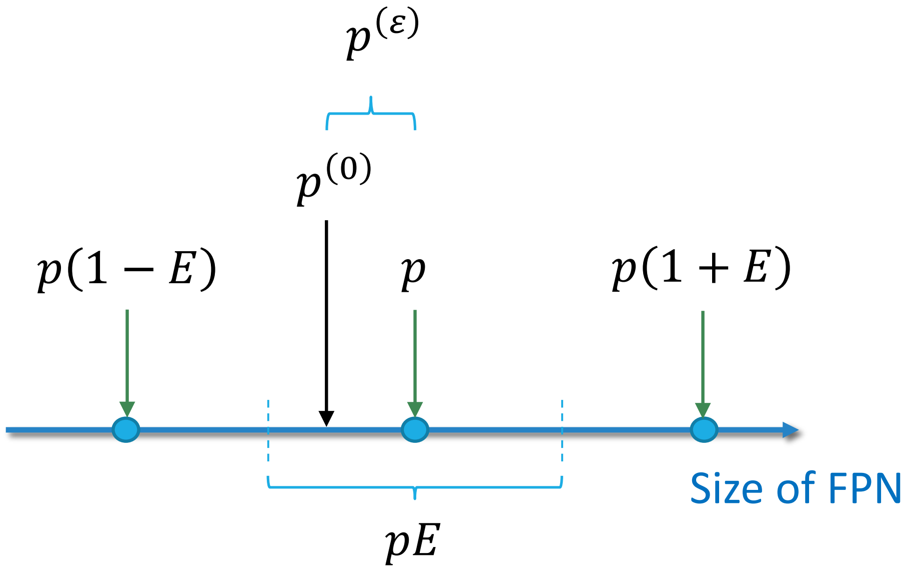
$$p = p^{(0)} + p^{(\varepsilon)}$$



The schematic illustration among floating-point number p, real number $p^{(0)}$, and the error of FPN $p^{(\varepsilon)}$. E is machine epsilon.

Difference in the different precision FPNs

- We consider the difference in high-precision FPN and low-precision one.
  - $p^{(L)} - p^{(H)} = p^{(\varepsilon_L)} - p^{(\varepsilon_H)} \equiv p^{(\delta)}$
  - High-precision FPN: $p^{(H)} = p^{(0)} + p^{(\varepsilon_H)}$
  - Low-precision FPN: $p^{(L)} = p^{(0)} + p^{(\varepsilon_L)}$
- To estimate the FPN errors from $p^{(\delta)}$, high-precision FPN error $p^{(\varepsilon_H)}$ has to be smaller than low-precision FPN error $p^{(\varepsilon_L)}$ significantly.
- Since the size of FPN errors satisfies $\left|p^{(\varepsilon)}\right| < \left|\frac{pE}{2}\right|$,
  - $\left|p^{(\varepsilon_H)}\right| \ll \left|p^{(\varepsilon_L)}\right| \Rightarrow \left|\frac{pE^{(H)}}{2}\right| \ll \left|\frac{pE^{(L)}}{2}\right| \Rightarrow E^{(H)} \ll E^{(L)}$
- $E = 2^{1-d}$ is machine epsilon, where d is mantissa bit width.
  - $2^{1-d_H} \ll 2^{1-d_L} \Rightarrow 2^{d_L - d_H} \ll 1$



The schematic illustration among floating-point number p, real number $p^{(0)}$, and the error of FPN $p^{(\varepsilon)}$. E is machine epsilon.

Difference equation of 2-D shallow-water model

- 2-dimensional shallow-water equations in this study
  - $\frac{\partial u}{\partial t} = -u\frac{\partial u}{\partial x} - v\frac{\partial u}{\partial y} - \frac{\partial \phi}{\partial x} + fv$
  - $\frac{\partial v}{\partial t} = -u\frac{\partial v}{\partial x} - v\frac{\partial v}{\partial y} - \frac{\partial \phi}{\partial y} - fu$
  - $\frac{\partial \phi}{\partial t} = -u\frac{\partial \phi}{\partial x} - v\frac{\partial \phi}{\partial y} - \phi\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)$

u: zonal velocity [m/s], v: meridional velocity [m/s]

ϕ: geopotential [m^2/s^2], f: Coriolis parameter [1/s]

- The differential equations are discretized as follows:
  - $\frac{\Delta_N u_{I,J,N}}{\Delta t} = -u_{I,J,N}\frac{\Delta_I u_{I,J,N}}{\Delta x} - v_{I,J,N}\frac{\Delta_J u_{I,J,N}}{\Delta y} - \frac{\Delta_I \phi_{I,J,N}}{\Delta x} + f v_{I,J,N} + O(\Delta x) + O(\Delta y) + O(\Delta t)$
  - $\frac{\Delta_N v_{I,J,N}}{\Delta t} = -u_{I,J,N}\frac{\Delta_I v_{I,J,N}}{\Delta x} - v_{I,J,N}\frac{\Delta_j v_{I,J,N}}{\Delta y} - \frac{\Delta_J \phi_{I,J,N}}{\Delta y} - f u_{I,J,N} + O(\Delta x) + O(\Delta y) + O(\Delta t)$
  - $\frac{\Delta_N \phi_{I,J,N}}{\Delta t} = -u_{I,J,N}\frac{\Delta_I \phi_{I,J,N}}{\Delta x} - v_{I,J,N}\frac{\Delta_j \phi_{I,J,N}}{\Delta y} - \phi_{I,J,N}\left(\frac{\Delta_I u_{I,J,N}}{\Delta x} + \frac{\Delta_J v_{I,J,N}}{\Delta y}\right) + O(\Delta x) + O(\Delta y) + O(\Delta t)$
  - Δ_N, Δ_I, and Δ_J are the forward difference for the t-, x-, and y-direction. For example, $\Delta_I p_{I,J,N} = p_{I+1,J,N} - p_{I,J,N}$.

Difference of FPN errors

- Taylor expansion ($x \rightarrow x + \Delta x$) of $f(x)$ is expressed as follows:
  - $f(x+\Delta x) = f(x) + \Delta x \frac{\partial f(x)}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 f(x)}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 f(x)}{\partial x^3} + \cdots + \frac{\Delta x^n}{n!}\frac{\partial^n f(x)}{\partial x^n} + \cdots$
- Subtract $f(x)$ from both side and divided by Δx,
  - $\frac{f(x+\Delta x)-f(x)}{\Delta x} = \frac{\partial f(x)}{\partial x} + O(\Delta x)$
- We use p instead of $f(x)$,
  - $\frac{p_{I+1,J,N}-p_{I,J,N}}{\Delta x} = \frac{\Delta_I p_{I,J,N}}{\Delta x} = \frac{\partial p}{\partial x} + O(\Delta x)$
- The difference in the high-precision FPN $p^{(H)}$ and low-precision one $p^{(L)}$,
  - $\frac{p^{(L)}_{I+1,J,N}-p^{(L)}_{I,J,N}}{\Delta x} - \frac{p^{(H)}_{I+1,J,N}-p^{(H)}_{I,J,N}}{\Delta x} = \frac{p^{(\varepsilon_L)}_{I+1,J,N}-p^{(\varepsilon_L)}_{I,J,N}}{\Delta x} - \frac{p^{(\varepsilon_H)}_{I+1,J,N}-p^{(\varepsilon_H)}_{I,J,N}}{\Delta x} = \frac{\Delta_I p^{(\varepsilon_L)}_{I,J,N}}{\Delta x} - \frac{\Delta_I p^{(\varepsilon_H)}_{I,J,N}}{\Delta x} = \frac{\Delta_I p^{(\delta)}_{I,J,N}}{\Delta x}$

Difference equation of FPN errors

- Difference equation of FPN errors in the shallow-water model is
  - $$\frac{\Delta_N u_{I,J,N}^{(\delta)}}{\Delta t} = -\left(u_{I,J,N}^{(0)}\frac{\Delta_I u_{I,J,N}^{(\delta)}}{\Delta x} + u_{I,J,N}^{(\delta)}\frac{\Delta_I u_{I,J,N}^{(0)}}{\Delta x}\right) - \left(v_{I,J,N}^{(0)}\frac{\Delta_J u_{I,J,N}^{(\delta)}}{\Delta y} + v_{I,J,N}^{(\delta)}\frac{\Delta_J u_{I,J,N}^{(0)}}{\Delta y}\right) - \frac{\Delta_I \phi_{I,J,N}^{(\delta)}}{\Delta x} + f v_{I,J,N}^{(\delta)} + F_u$$
  - $$\frac{\Delta_N v_{I,J,N}^{(\delta)}}{\Delta t} = -\left(u_{I,J,N}^{(0)}\frac{\Delta_I v_{I,J,N}^{(\delta)}}{\Delta x} + u_{I,J,N}^{(\delta)}\frac{\Delta_I v_{I,J,N}^{(0)}}{\Delta x}\right) - \left(v_{I,J,N}^{(0)}\frac{\Delta_J v_{I,J,N}^{(\delta)}}{\Delta y} + v_{I,J,N}^{(\delta)}\frac{\Delta_J v_{I,J,N}^{(0)}}{\Delta y}\right) - \frac{\Delta_J \phi_{I,J,N}^{(\delta)}}{\Delta y} - f u_{I,J,N}^{(\delta)} + F_v$$
  - $$\frac{\Delta_N \phi_{I,J,N}^{(\delta)}}{\Delta t} = -\left(u_{I,J,N}^{(0)}\frac{\Delta_I \phi_{I,J,N}^{(\delta)}}{\Delta x} + u_{I,J,N}^{(\delta)}\frac{\Delta_I \phi_{I,J,N}^{(0)}}{\Delta x}\right) - \left(v_{I,J,N}^{(0)}\frac{\Delta_J \phi_{I,J,N}^{(\delta)}}{\Delta y} + v_{I,J,N}^{(\delta)}\frac{\Delta_J \phi_{I,J,N}^{(0)}}{\Delta y}\right) - \phi_{I,J,N}^{(0)}\left(\frac{\Delta_I u_{I,J,N}^{(\delta)}}{\Delta x} + \frac{\Delta_J v_{I,J,N}^{(\delta)}}{\Delta y}\right) - \phi_{I,J,N}^{(\delta)}\left(\frac{\Delta_I u_{I,J,N}^{(0)}}{\Delta x} + \frac{\Delta_J v_{I,J,N}^{(0)}}{\Delta y}\right) + F_\phi$$
- where
  - $$F_u = NL_u(\varepsilon_L, I, J, N) - NL_u(\varepsilon_H, I, J, N) + RN\left(u_{I,J,N}^{(L)}\right) - RN\left(u_{I,J,N}^{(H)}\right) + O(\Delta x) + O(\Delta y) + O(\Delta t)$$
  - $$F_v = NL_v(\varepsilon_L, I, J, N) - NL_v(\varepsilon_H, I, J, N) + RN\left(v_{I,J,N}^{(L)}\right) - RN\left(v_{I,J,N}^{(H)}\right) + O(\Delta x) + O(\Delta y) + O(\Delta t)$$
  - $$F_\phi = NL_\phi(\varepsilon_L, I, J, N) - NL_\phi(\varepsilon_H, I, J, N) + RN\left(\phi_{I,J,N}^{(L)}\right) - RN\left(\phi_{I,J,N}^{(H)}\right) + O(\Delta x) + O(\Delta y) + O(\Delta t)$$

Difference equation of FPN errors 2

- (continue)
  - $NL_u(\varepsilon, I, J, N) = -\left(u_{I,J,N}^{(\varepsilon)} \frac{\Delta_I u_{I,J,N}^{(\varepsilon)}}{\Delta x} + u_{I,J,N}^{(\varepsilon)} \frac{\Delta_I u_{I,J,N}^{(\varepsilon)}}{\Delta x}\right) - \left(v_{I,J,N}^{(\varepsilon)} \frac{\Delta_J u_{I,J,N}^{(\varepsilon)}}{\Delta y} + v_{I,J,N}^{(\varepsilon)} \frac{\Delta_J u_{I,J,N}^{(\varepsilon)}}{\Delta y}\right)$
  - $NL_v(\varepsilon, I, J, N) = -\left(u_{i,j,n}^{(\varepsilon)} \frac{\Delta_I v_{I,J,N}^{(\varepsilon)}}{\Delta x} + u_{I,J,N}^{(\varepsilon)} \frac{\Delta_I v_{I,J,N}^{(\varepsilon)}}{\Delta x}\right) - \left(v_{I,J,N}^{(\varepsilon)} \frac{\Delta_J v_{I,J,N}^{(\varepsilon)}}{\Delta y} + v_{I,J,N}^{(\varepsilon)} \frac{\Delta_J v_{I,J,N}^{(\varepsilon)}}{\Delta y}\right)$
  - $NL_\phi(\varepsilon, I, J, N) = -\left(u_{i,j,n}^{(\varepsilon)} \frac{\Delta_I \phi_{I,J,N}^{(\varepsilon)}}{\Delta x} + u_{I,J,N}^{(\varepsilon)} \frac{\Delta_I \phi_{I,J,N}^{(\varepsilon)}}{\Delta x}\right) - \left(v_{I,J,N}^{(\varepsilon)} \frac{\Delta_J \phi_{I,J,N}^{(\varepsilon)}}{\Delta y} + v_{I,J,N}^{(\varepsilon)} \frac{\Delta_J \phi_{I,J,N}^{(\varepsilon)}}{\Delta y}\right) - \phi_{I,J,N}^{(\varepsilon)} \left(\frac{\Delta_I u_{I,J,N}^{(\varepsilon)}}{\Delta x} + \frac{\Delta_J v_{I,J,N}^{(\varepsilon)}}{\Delta y}\right)$
- RN is newly generated errors by rounding operation during numerical calculation.

FPN errors in geostrophic wind balance

- Geostrophic wind balance does not change prognostic variables by time-step advance,
  - $u = -\frac{1}{f}\frac{d\phi}{dy}, \quad v = 0, \quad \phi = \phi(y)$
  - FPN errors in the x-direction is eliminated, because the value is the same along the x-direction.

- We rearrange the governing equations of FPN errors using initial condition,
  - $\frac{\Delta_N u_{I,J,N}^{(\delta)}}{\Delta t} = -v_{I,J,N}^{(\delta)} \frac{\Delta_J u_{I,J,N}^{(0)}}{\Delta y} + f v_{I,J,N}^{(\delta)} + F_u$
  - $\frac{\Delta_N v_{I,J,N}^{(\delta)}}{\Delta t} = -\frac{\Delta_J \phi_{I,J,N}^{(\delta)}}{\Delta y} - f u_{I,J,N}^{(\delta)} + F_v$
  - $\frac{\Delta_N \phi_{I,J,N}^{(\delta)}}{\Delta t} = -v_{I,J,N}^{(\delta)} \frac{\Delta_J \phi_{I,J,N}^{(0)}}{\Delta y} - \phi_{I,J,N}^{(0)} \frac{\Delta_J v_{I,J,N}^{(\delta)}}{\Delta y} + F_\phi$

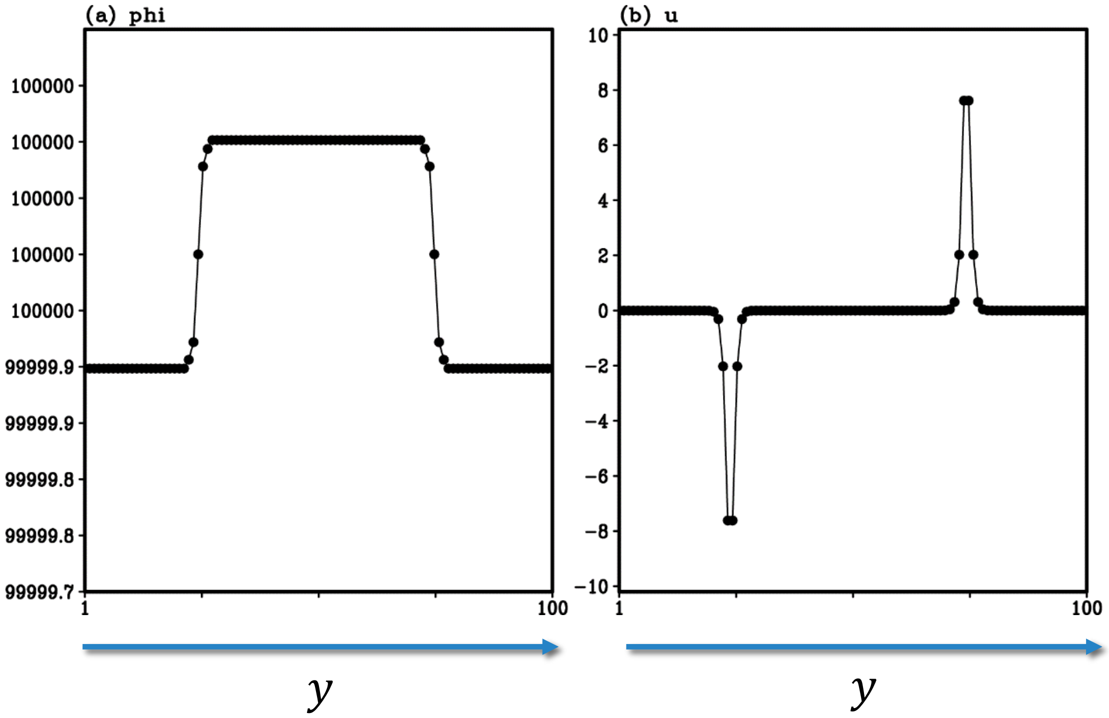
- The equation is led by geostrophic wind balance,
  - $u_{I,J,N}^{(0)} = -\frac{1}{f}\frac{\Delta_J \phi_{I,J,N}^{(0)}}{\Delta y},$

FPN errors in geostrophic wind balance 2

- We rearrange the governing equations more and divide Δt,
  - $$\frac{v^{(\delta)}_{I,J,N+2}-2v^{(\delta)}_{I,J,N+1}+v^{(\delta)}_{I,J,N}}{\Delta t^2} = -\frac{1}{\Delta y}\left(\frac{\Delta_N \phi^{(\delta)}_{I,J+1,N}}{\Delta t} - \frac{\Delta_N \phi^{(\delta)}_{I,J,N}}{\Delta t}\right) - f\,\frac{\Delta_N u^{(\delta)}_{I,J,N}}{\Delta t} + \frac{\Delta_J F_v}{\Delta t}$$
  - where $p^{(\delta)}_{I,J,N+2} - 2p^{(\delta)}_{I,J,N+1} + p^{(\delta)}_{I,J,N} = \Delta_N p^{(\delta)}_{I,J,N+1} - \Delta_N p^{(\delta)}_{I,J,N} = \Delta_N \left\{\Delta_N p^{(\delta)}_{I,J,N}\right\} = \Delta_N^2 p^{(\delta)}_{I,J,N}$,
- Finally, we obtain the following equation:
  - $$\frac{\Delta_N^2 v^{(\delta)}_{I,J,N}}{\Delta t^2} = \phi^{(0)}_{I,J+1,N}\frac{\Delta_J^2 v^{(\delta)}_{I,J,N}}{\Delta y^2} - f\left(u^{(0)}_{I,J+1,N} + u^{(0)}_{I,J,N}\right)\frac{\Delta_J v^{(\delta)}_{I,J,N}}{\Delta y} - f^2 v^{(\delta)}_{I,J,N} - fF_u + \frac{\Delta_N F_v}{\Delta t} - \frac{\Delta_J F_\phi}{\Delta y}$$
- This is similar to the difference equation of linear inhomogeneous wave equation.

Geostrophic wind balance experiment

Bickley Jet

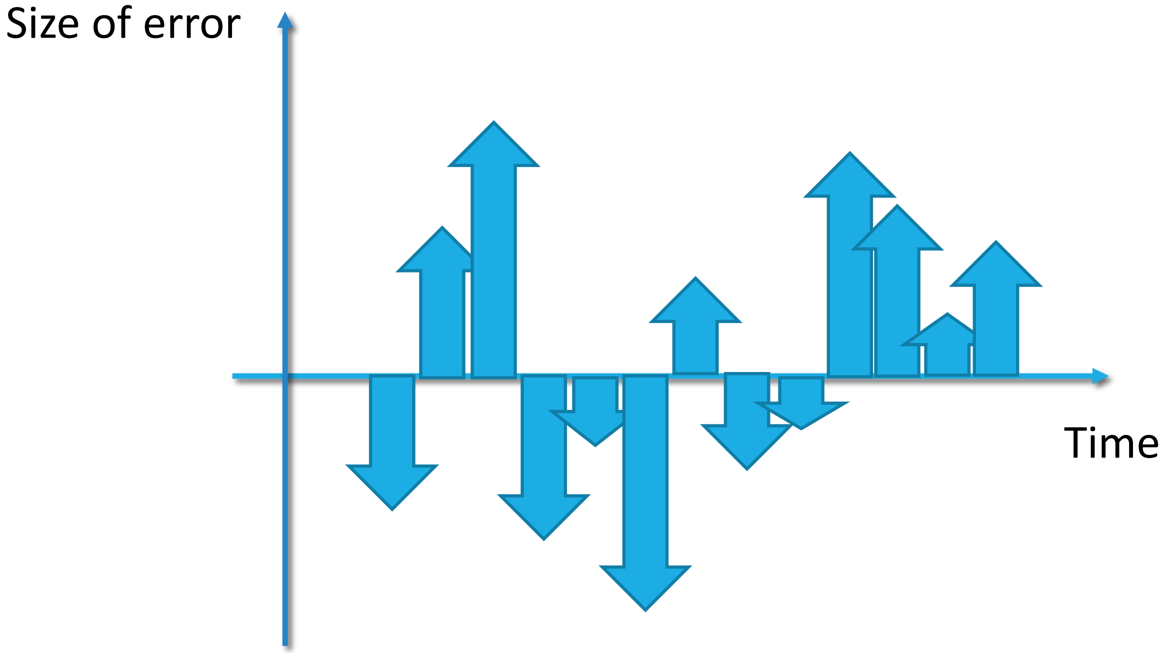


- Initiation
  - $u_{I,J,N} = -\frac{1}{f}\frac{\Delta_J \phi_{I,J,N}}{\Delta y}$
  - $v_{I,J,N} = 0$
  - $\phi_{I,J,N} = \Phi - fU\Delta y \tanh\left(\left|J - \frac{Jmax}{2}\right| - \frac{Jmax}{4}\right)$
- $f = 10^{-4}, U = 10^{1}, \Phi = 10^{5}$
  - Typical parameters in the mid-latitude
- $\Delta y = 10^{4}, \Delta t = 10^{1}, J_{\max} = 100$
  - We should set Δt smaller than the phase speed of surface gravity wave $\sqrt{\phi^{(0)}}$.
- Cyclic lateral boundary condition

FPN errors in geostrophic wind balance 3

- We rearrange the governing equation using scale analysis,
  - $$\frac{\Delta_N^2 v_{I,J,N}^{(\delta)}}{\Delta t^2} = \phi_{I,J+1,N}^{(0)} \frac{\Delta_J^2 v_{I,J,N}^{(\delta)}}{\Delta y^2} - f\left(u_{I,J+1,N}^{(0)} + u_{I,J,N}^{(0)}\right)\frac{\Delta_J v_{I,J,N}^{(\delta)}}{\Delta y} - f^2 v_{I,J,N}^{(\delta)} - fF_u + \frac{\Delta_N F_v}{\Delta t} - \frac{\Delta_J F_\phi}{\Delta y}$$
- The first term of right-hand side is left, $O\left(\phi_{I,J+1,N}^{(0)} \frac{\Delta_J^2 v_{I,J,N}^{(\delta)}}{\Delta y^2}\right) \gg O\left(f u_{I,J,N}^{(0)} \frac{\Delta_J v_{I,J,N}^{(\delta)}}{\Delta y}\right) \gg O\left(f^2 v_{I,J,N}^{(\delta)}\right)$
- Rounding error is dominant in the forcing term,
  - $O(F_p) \cong O\left(RN(F_p)\right) \cong O\left(\frac{pE}{\Delta t}\right)$ E: machine epsilon
- The sixth term of right-hand side is left, $O\left(\frac{\Delta_J F_\phi}{\Delta y}\right) \gg O(fF_u) \gg O\left(\frac{\Delta_N F_v}{\Delta t}\right)$
  - $$\frac{\Delta_N^2 v_{I,J,N}^{(\delta)}}{\Delta t^2} = \phi_{I,J+1,N}^{(0)} \frac{\Delta_J^2 v_{I,J,N}^{(\delta)}}{\Delta y^2} - \frac{\Delta_J F_\phi}{\Delta y}$$

Integration of numerical errors



It's difficult to predict the size of rounding up and down …
⇒ we assume the change is stochastic.

- The governing equations of geostrophic wind balance can be written as follows:
  - $$\frac{\Delta_N^2 v_{I,J,N}^{(\delta)}}{\Delta t^2} = \phi_{I,J+1,N}^{(0)} \frac{\Delta_J^2 v_{I,J,N}^{(\delta)}}{\Delta y^2} - \frac{\Delta_J F_\phi}{\Delta y}$$
- This equation is similar to the linear inhomogeneous wave equation.
  - FPN errors will show wave-like oscillation in the geostrophic wind balance, and the amplitude is increased gradually.
  - Note that the forcing term is stochastic variable.

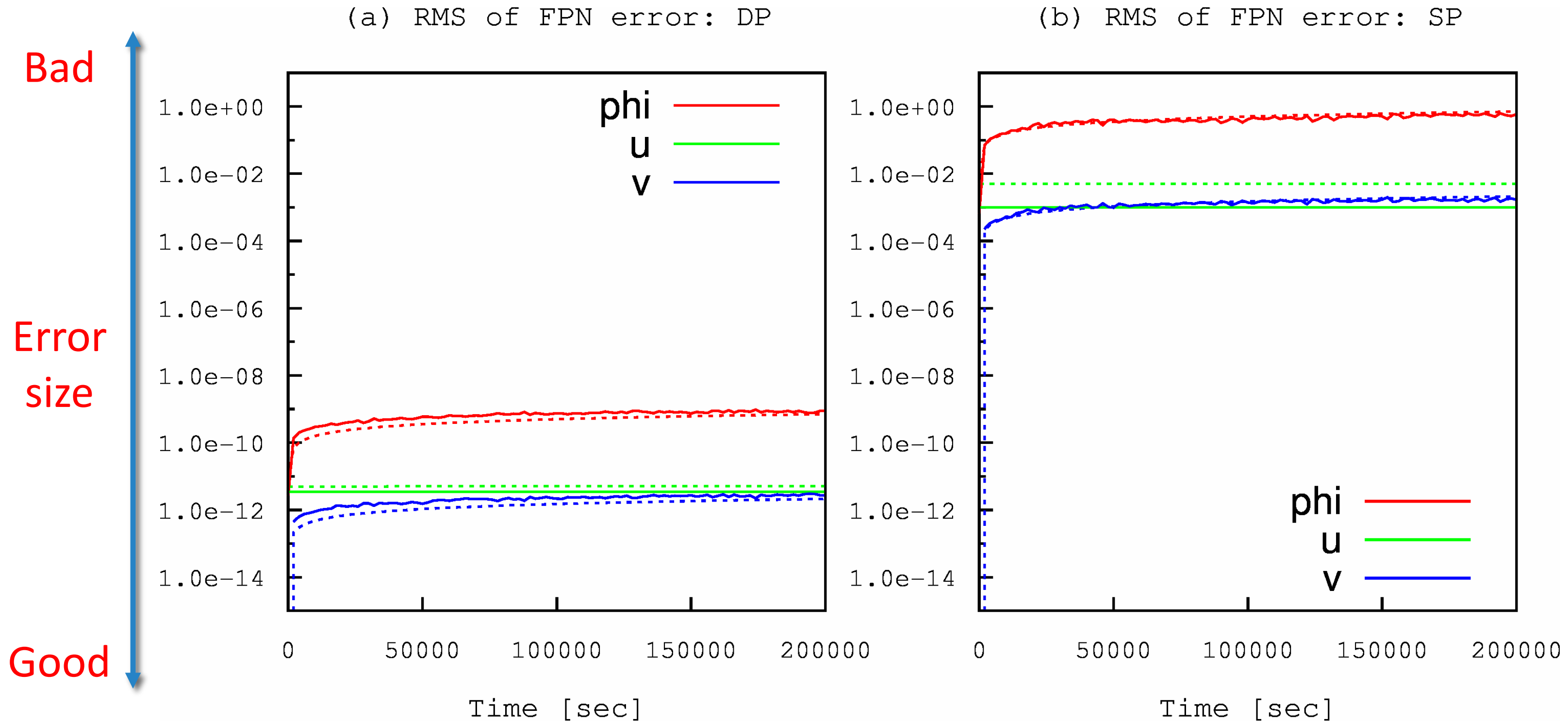
How large is the numerical error?

- We define root-mean-square (RMS) of FPN error:
  - $\{RMS(p^{(\delta)})\}_N = \sqrt{\frac{1}{I_{max}} \sum_{I=1}^{I_{\max}} \frac{1}{J_{\max}} \sum_{J=1}^{J_{\max}} \left(p_{I,J,N}^{(\delta)}\right)^2}$
- The RMS of FPN errors changes by time-step advance.
- Forcing term is worked stochastically. The chances of rounding up/down operation are 50%.
  - We assume the random forcing has the almost same scale, but the sign is different (pos/neg).
  - If the random forcing effects are integrated, the mean is zero and the variance increase with respect to time-step N. Therefore, the RMS increases with respect to $\sqrt{N}$.

FPN errors in geostrophic wind balance 4

- We guess the scale of initial RMS,
  - $O\left(\{RMS\left(\phi^{(\delta)}\right)\}_0\right) = O\left(\phi^{(0)}E\right) = O(10^5 E)$
  - $O\left(\{RMS\left(u^{(\delta)}\right)\}_0\right) = O\left(\frac{1}{f}\frac{\Delta_J \phi^{(0)}}{\Delta y}\right) = O(10^5 E)$ $\left(O\left(\{RMS\left(v^{(\delta)}\right)\}_0\right) = 0\right)$
- and random forcing coefficients,
  - $O\left(\gamma_\phi\right) = O\left(F_\phi \Delta t\right) = O(\phi E) = O(10^5 E)$
  - $O(\gamma_v) = O\left(F_\phi \frac{\Delta t^2}{\Delta y}\right) = O\left(\phi E \frac{\Delta t^2}{\Delta y}\right) = O(10^2 E)$
  - $O(\gamma_u) = O\left(v_{I,J,N}^{(\delta)} \frac{\Delta_J u_{I,J,N}^{(0)}}{\Delta y} \Delta t\right) + O(F_u \Delta t) = O\left(10^{-3} v_{I,J,N}^{(\delta)}\right) + O(10^1 E)$
- $u^{(\delta)}$ is adding with respect to $v^{(\delta)}$, but this is hardly affected when $N < 10^6$.

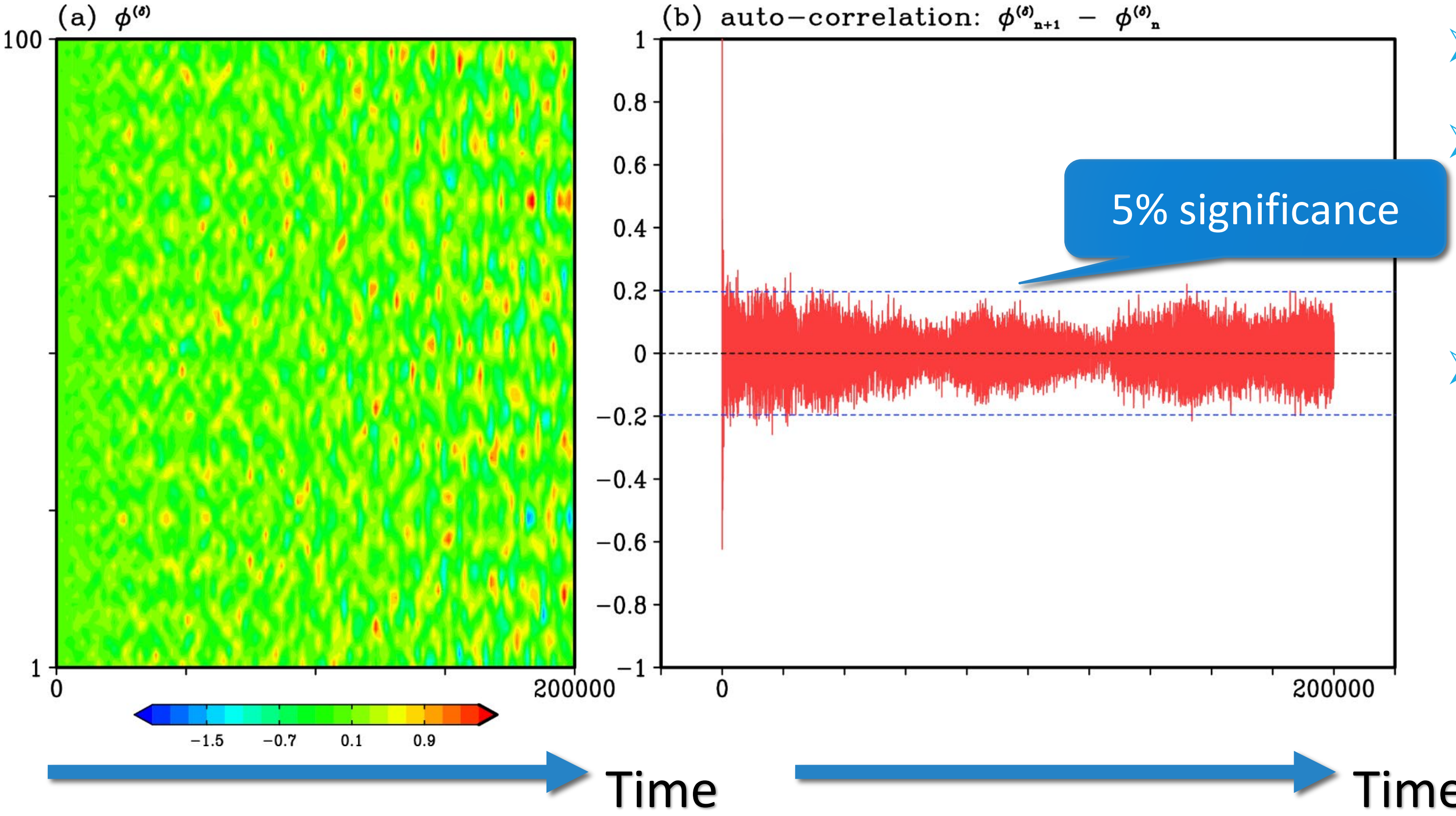
Time evolution of FPN errors in the geostrophic wind balance experiment



DP FPN $(E = 10^{-16})$ SP FPN $(E = 10^{-7})$

- RMS of each prognostic variable
  - $RMS\left(\phi^{(\delta)}\right) = O(10^5 E) \cdot \alpha_\phi \sqrt{1+N}$
  - $RMS\left(u^{(\delta)}\right) = O(10^5 E) \cdot \alpha_u$
  - $RMS\left(v^{(\delta)}\right) = O(10^2 E) \cdot \alpha_v \sqrt{N}$
- What's this figure?
  - FPN errors increase with the root of time-step, which is almost same as the ideal case.
  - The random forcing is dominated by F_ϕ that is the largest term in the equation.

FPN error distribution and auto-correlation



- Left: spatial distribution of $\phi^{(\delta)}$.
- Right: Time evolution of changing $\phi^{(\delta)}$, which corresponds to the auto-correlation of $\phi^{(\delta)}$.
- What’s this figure?
  - (left) The amplitude of $\phi^{(\delta)}$ becomes gradually large.
  - (right) The auto correlation has no significant correlation except for the initial stage.

FPN errors in the barotropic instability

- For the barotropic instability, we add small disturbances into the geostrophic wind balance state.
  - $u = -\frac{1}{f}\frac{d\phi}{dy}, \quad v = v(x), \quad \phi = \phi(y) \qquad\qquad (u \gg v)$
- We assume that non-convergence condition (rigid lid condition),
  - $\frac{\Delta_I u_{I,J,N}^{(0)}}{\Delta x} = \frac{\Delta_J v_{I,J,N}^{(0)}}{\Delta y} = \frac{\Delta_I u_{I,J,N}^{(\delta)}}{\Delta x} = \frac{\Delta_J v_{I,J,N}^{(\delta)}}{\Delta y} = 0$
- That is, the change of $\phi_{I,J,N}^{(\delta)}$ is nothing. We rearrange the governing equations,
  - $\frac{\Delta_N u_{I,J,N}^{(\delta)}}{\Delta t} = -\left(v_{I,J,N}^{(0)} \frac{\Delta_J u_{I,J,N}^{(\delta)}}{\Delta y} + v_{I,J,N}^{(\delta)} \frac{\Delta_J u_{I,J,N}^{(0)}}{\Delta y}\right) + f v_{I,J,N}^{(\delta)} + F_u$
  - $\frac{\Delta_N v_{I,J,N}^{(\delta)}}{\Delta t} = -\left(u_{I,J,N}^{(0)} \frac{\Delta_I v_{I,J,N}^{(\delta)}}{\Delta x} + u_{I,J,N}^{(\delta)} \frac{\Delta_I v_{I,J,N}^{(0)}}{\Delta x}\right) - f u_{I,J,N}^{(\delta)} + F_v$

FPN errors in the barotropic instability 2

- Then, we make the vorticity equation of FPN errors,
  - $$\frac{\Delta_N}{\Delta t}\left(\frac{\Delta_I v^{(\delta)}_{I,J,N}}{\Delta x}-\frac{\Delta_J u^{(\delta)}_{I,J,N}}{\Delta y}\right)=-u^{(0)}_{I,J,N}\frac{\Delta_I^2 v^{(\delta)}_{I,J,N}}{\Delta x^2}-u^{(\delta)}_{I,J,N}\frac{\Delta_I^2 v^{(0)}_{I,J,N}}{\Delta x^2}+v^{(0)}_{I,J,N}\frac{\Delta_J^2 u^{(\delta)}_{I,J,N}}{\Delta y^2}+v^{(\delta)}_{I,J,N}\frac{\Delta_J^2 u^{(0)}_{I,J,N}}{\Delta y^2}+\frac{\Delta_I F_v}{\Delta x}-\frac{\Delta_J F_u}{\Delta y}$$
- Furthermore, the second and third terms in the right-hand side can be ignored because those terms are smaller than the first and fourth terms ($u^{(0)} \gg v^{(0)}$). Then, we use the stream function of FPN errors,
  - $$\frac{\Delta_N}{\Delta t}\left(\frac{\Delta_I^2 \psi^{(\delta)}_{I,J,N}}{\Delta x^2}+\frac{\Delta_J^2 \psi^{(\delta)}_{I,J,N}}{\Delta y^2}\right)=-u^{(0)}_{I,J,N}\frac{\Delta_I}{\Delta x}\left(\frac{\Delta_I^2 \psi^{(\delta)}_{I,J,N}}{\Delta x^2}+\frac{\Delta_J^2 \psi^{(\delta)}_{I,J,N}}{\Delta y^2}\right)+\frac{\Delta_I \psi^{(\delta)}_{I,J,N}}{\Delta x}\frac{\Delta_J^2 u^{(0)}_{I,J,N}}{\Delta y^2}+\frac{\Delta_I F_v}{\Delta x}-\frac{\Delta_J F_u}{\Delta y}$$
  - where $u^{(\delta)}_{I,J,N}=-\frac{\Delta_J \psi^{(\delta)}_{I,J,N}}{\Delta y},\quad v^{(\delta)}_{I,J,N}=\frac{\Delta_I \psi^{(\delta)}_{I,J,N}}{\Delta x},\quad \frac{\Delta_I}{\Delta x}\left(\frac{\Delta_J^2 \psi^{(\delta)}_{I,J,N}}{\Delta y^2}\right)=\frac{\Delta_I}{\Delta x}\left(-\frac{\Delta_J u^{(\delta)}_{I,J,N}}{\Delta y}\right)=-\frac{\Delta_J}{\Delta y}\left(\frac{\Delta_I u^{(\delta)}_{I,J,N}}{\Delta x}\right)=0$

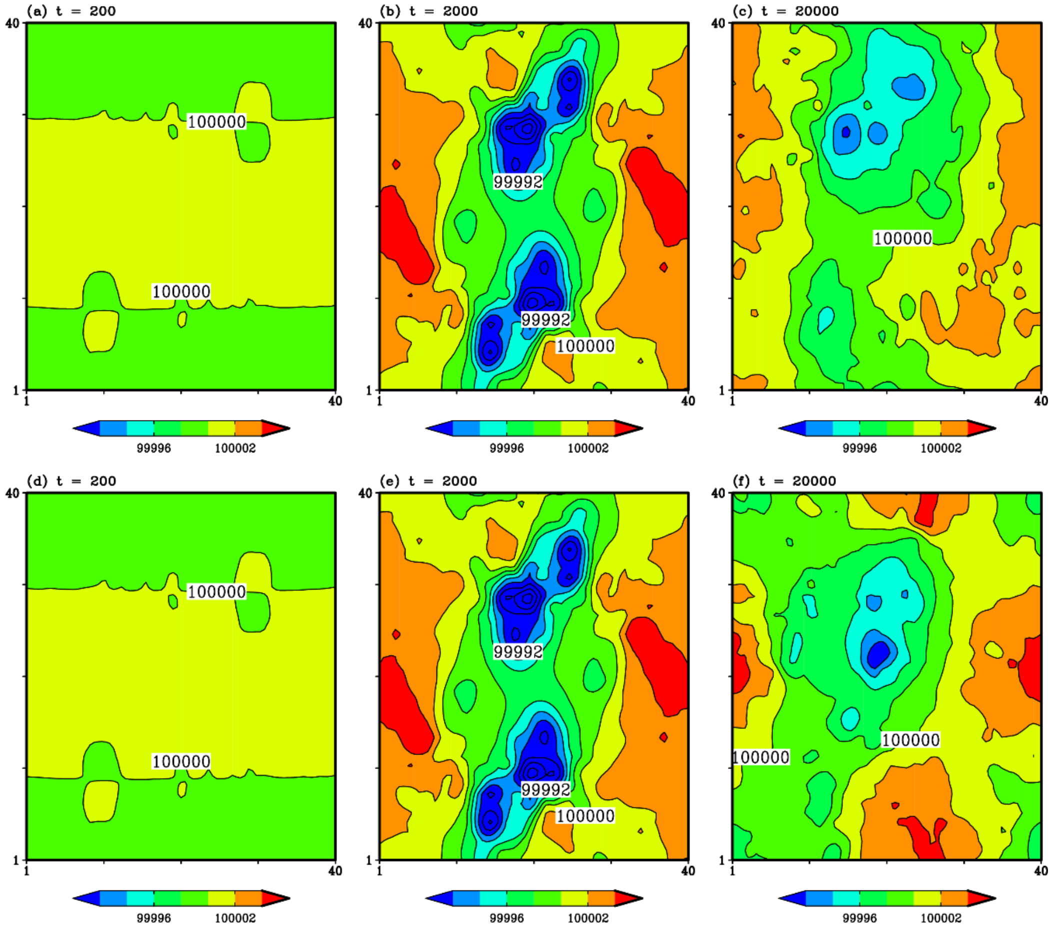
FPN errors in the barotropic instability 3

- We assume the normal mode solution $\psi^{(\delta)} = \Psi^{(\delta)}(y)e^{ik(x-ct)}$,
  - $$\frac{\Delta_J^2 \Psi_{I,J,N}^{(\delta)}}{\Delta y^2} - \Psi_{I,J,N}^{(\delta)} \left(k^2 \left(\frac{e^{ik\Delta x}-1}{k\Delta x} \right)^2 - \frac{1}{u_{I,J,N}^{(0)} + c\left(\frac{e^{-ick\Delta t}-1}{ck\Delta t} / \frac{e^{ik\Delta x}-1}{k\Delta x} \right)} \frac{\Delta_J^2 u_{I,J,N}^{(0)}}{\Delta y^2} \right) = D$$
  - where $D = \frac{1}{e^{ik(x-ct)}\left\{ u_{I,J,N}^{(0)} + c\left(\frac{e^{-ick\Delta t}-1}{ck\Delta t} / \frac{e^{ik\Delta x}-1}{k\Delta x} \right) \right\}} \left(\frac{\Delta_I F_v}{\Delta x} - \frac{\Delta_J F_u}{\Delta y} \right)$
- We solve the eigenvalue problem and can obtain the growth rate of FPN errors in the barotropic instability since $\lim_{\Delta x \to 0} \frac{e^{ik\Delta x}-1}{k\Delta x} = i,\ \lim_{\Delta t \to 0} \frac{e^{-ick\Delta t}-1}{ck\Delta t} = -i.$
- If $D = 0$, the eigenvalue problem corresponds to the initial problem of small disturbance in the barotropic instability. Because Duhamel's principle, FPN errors in the barotropic instability grow like a barotropic instability wave.

Barotropic instability experiment

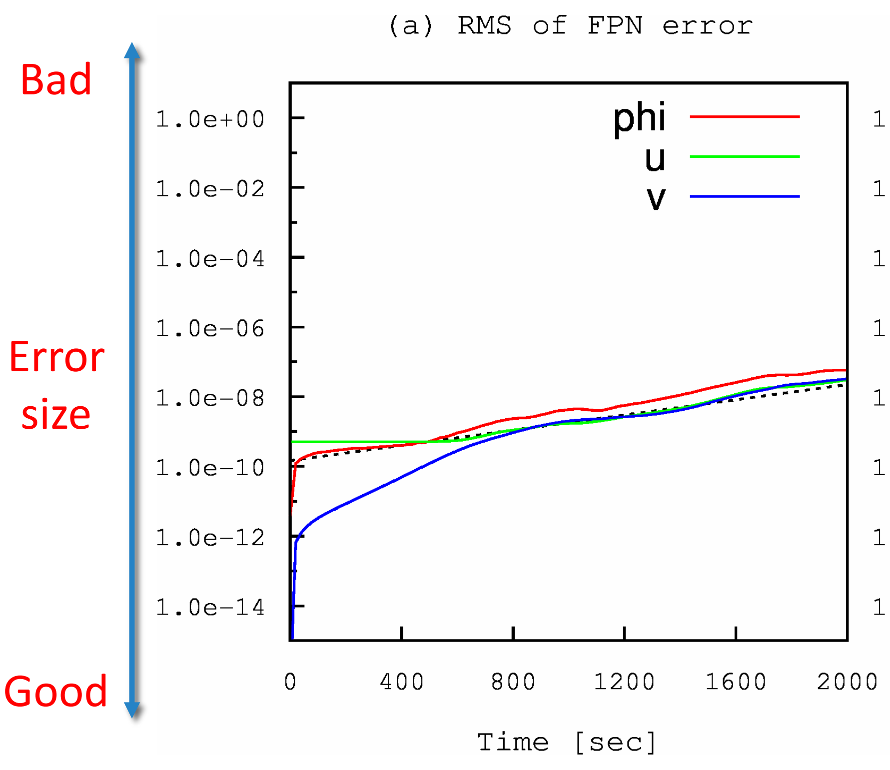
DP FPN

QP FPN



- Initiation
  - $u_{I,J,N} = -\frac{1}{f}\frac{\Delta_J \phi_{I,J,N}}{\Delta y}$
  - $v_{I,J,N} = V \sin\left(\frac{2\pi k}{\Delta x I_{\max}}\left(I - \frac{I_{max}}{2}\right)\right)$
  - $\phi_{I,J,N} = \Phi - fU\Delta y \tanh\left(\left|J - \frac{J_{max}}{2}\right| - \frac{J_{max}}{4}\right)$
- $f = 10^{-4}, U = 10^{1}, U = 10^{5}$
  - Typical parameters in the mid-latitude
- $\Delta y = 10^2, \Delta t = 10^{-1}, I_{\max} = J_{\max} = 40$
  - We should set Δt smaller than the phase speed of surface gravity wave $\sqrt{\phi^{(0)}}$.
- Cyclic lateral boundary condition

Time evolution of FPN errors in the barotropic instability experiment



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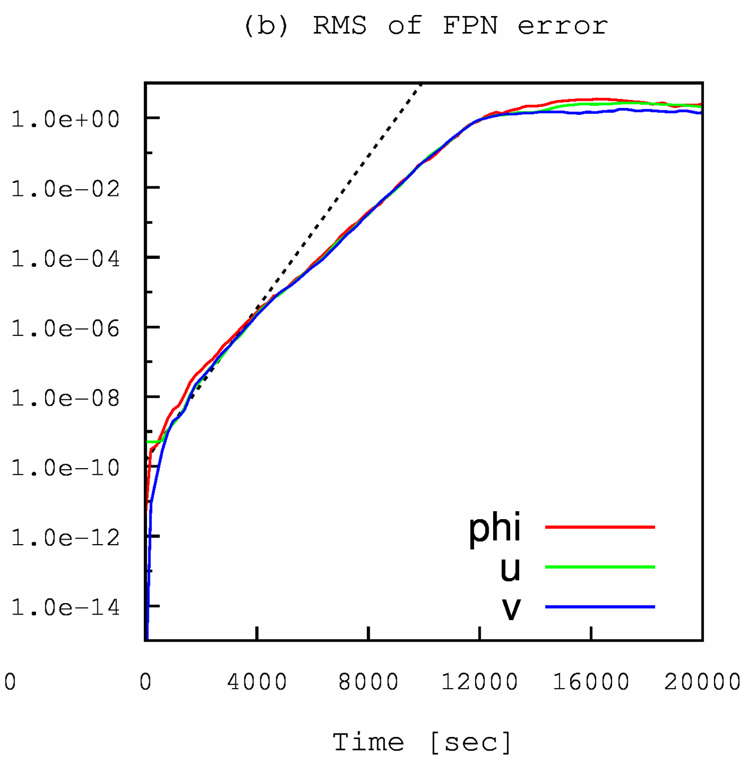

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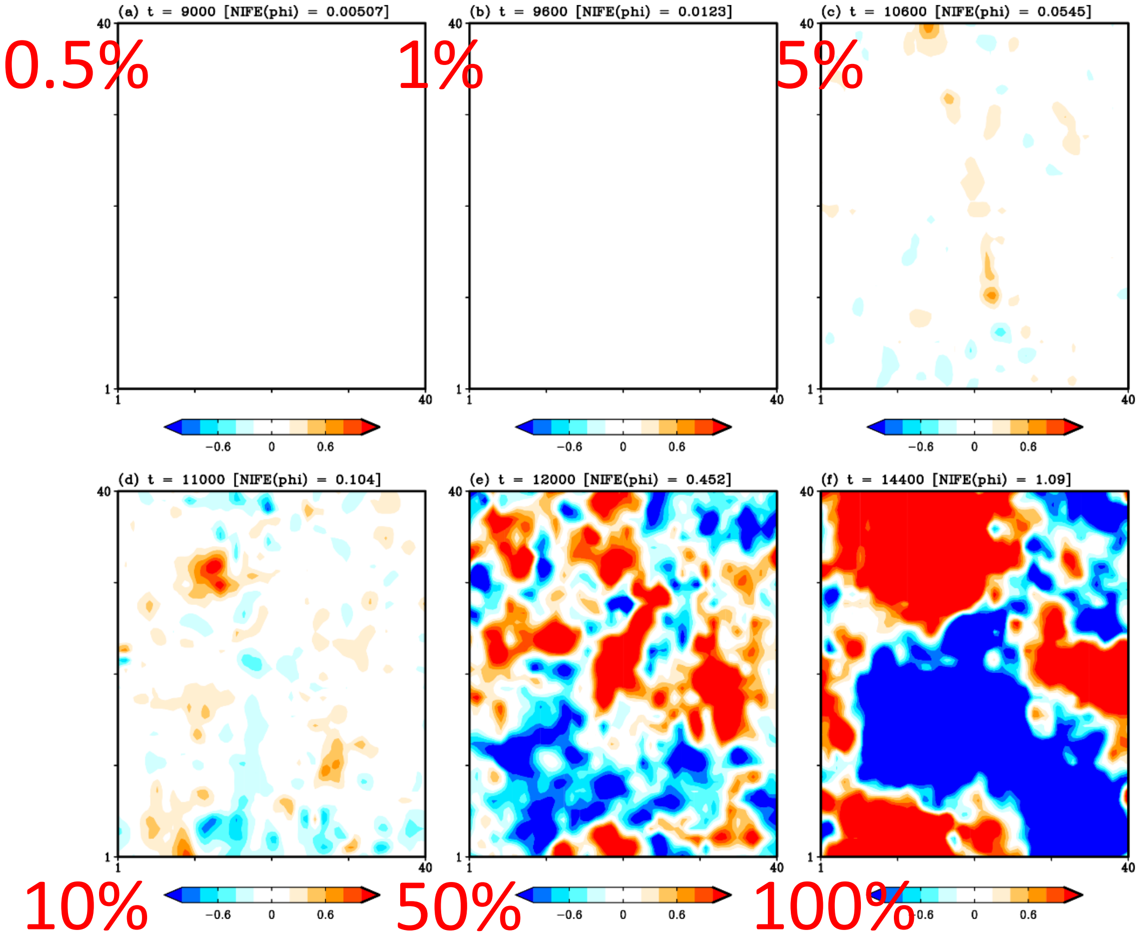
- Time evolution of FPN errors in BI
  - $RMS_n = RMS_0 \exp\left(\frac{U\delta_{\max}}{L_x} t\right)$
- The maximum growth rate in the *f*-plane is 0.16 with wave number 1.
  - $\delta_{\max} \approx 0.16 \times 2\pi$ (Kuo 1973)
- Which is dominant for the growth of the FPN errors, the root of time-step (numeric) or exponential term (physics)?

Normalized Index of FPN errors

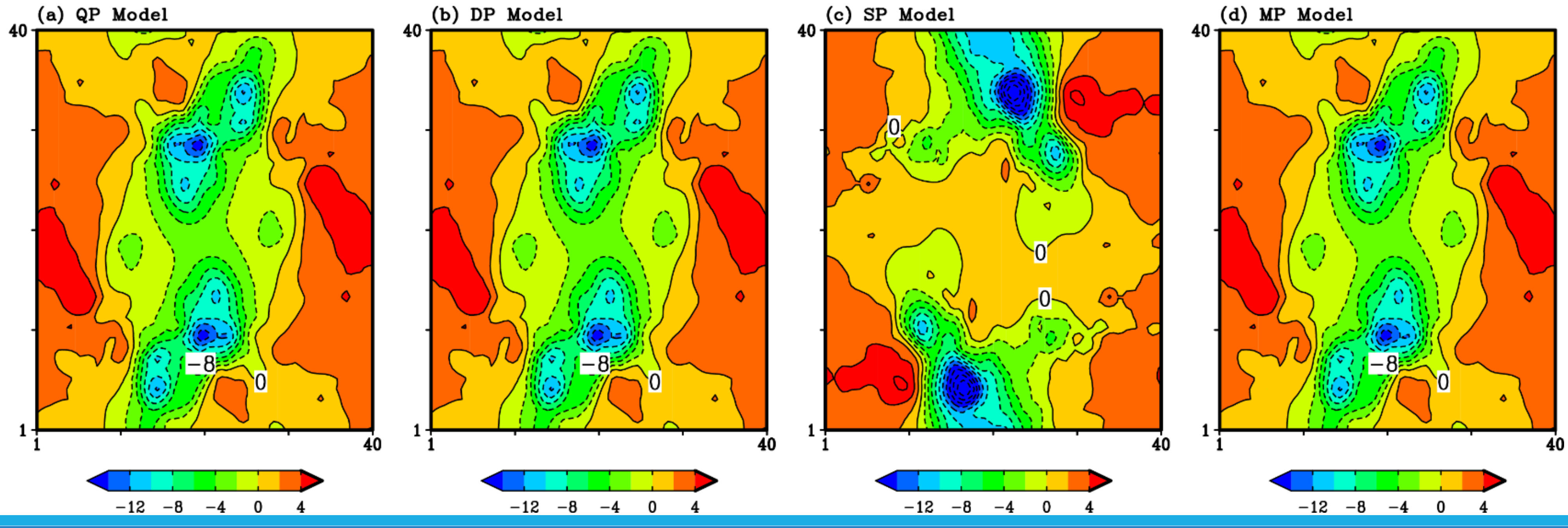


- Normalize the size of FPN errors
  - $$NIFE(p_N) = \frac{RMS\left(p_N^{(\varepsilon_L)}\right)}{STD(p_N)}$$
- We can evaluate the relative contribution of FPN errors to numerical simulation easily, and compare the relative contribution between prognostic variables.
- Shading can be seen over 2% in this figure
  - Can we ignore the FPN errors in the top-left and top-center figures?

Improving performance with mixed-precision FPN

Module	QP model	DP model	SP model	MP model
Program	318.873 ± 0.376	2.745 ± 0.008	1.349 ± 0.002	1.462 ± 0.004
Dynamics	318.207 ± 0.017	2.687 ± 0.008	1.299 ± 0.002	1.408 ± 0.004

[sec]



Heterogeneous Computing with FPGA

Tsukuba University (https://www.ccs.tsukuba.ac.jp)

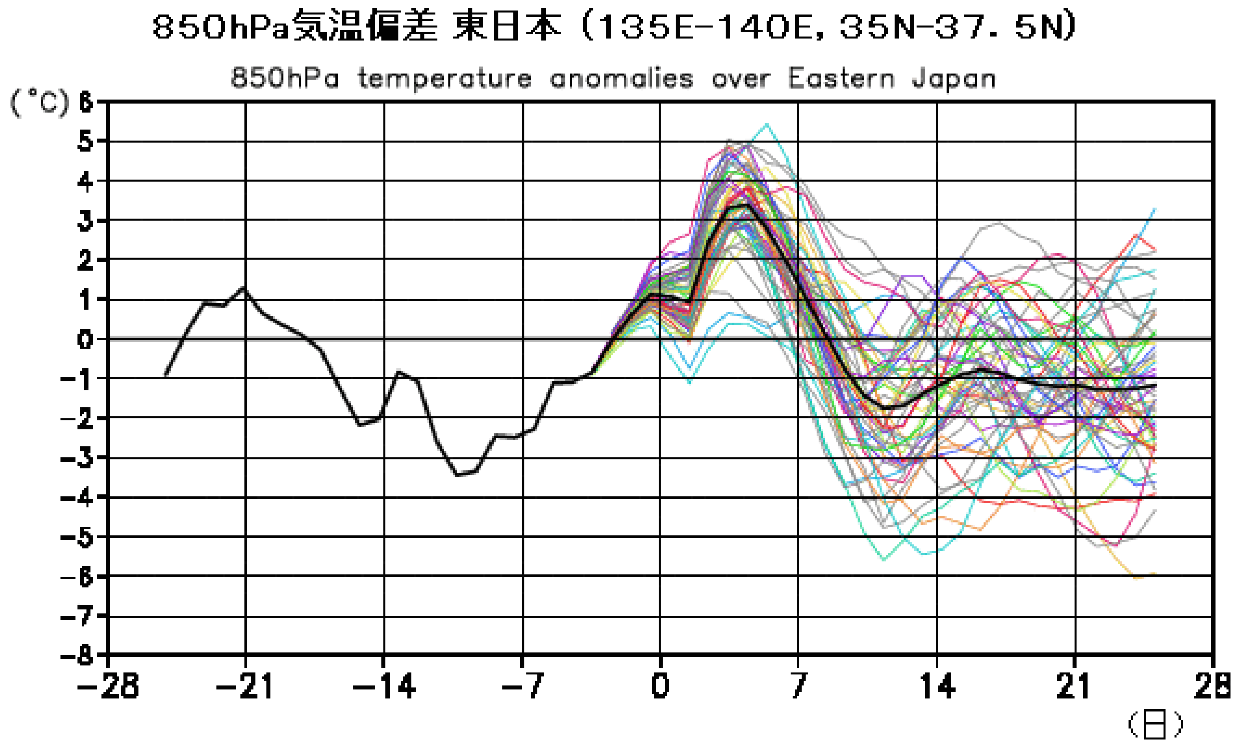
Field-Programmable Gate Array (FPGA)

Summary

- Introduction of SCALE library and LETKF system
  - We will implement data assimilation driver and then improve the SCALE-LETKF system!
- Reduced-precision FPN
  - We obtained the governing equation of floating-point number (FPN) errors in the shallow-water model.
  - We obtained the theoretical solution of FPN errors in the geostrophic wind balance and barotropic instability state, and compared with experimental result.
  - We tested the computational performance using mixed-precision FPN.
- FPGA can provide the simulation with mixed-precision FPN on the hardware architecture.
  - We need the best length of mantissa bit. Parameter estimation can provide it.

Yamaura, T., S. Nishizawa, and H. Tomita, 2019: Theoretical Time Evolution of Numerical Errors When Using Floating Point Numbers in Shallow-Water Models. *J. Adv. Model. Earth. Sy.*, doi:10.1029/2019MS001615.

Ensemble simulation using reduced-precision FPN



A example of ensemble simulation in weather forecasting systems (Japan Meteorological Agency)

FPN errors give random forcing
⇒ it is useful for ensemble simulation!

Challenging points

- New method for preparing ensemble members with very low costs
- New technique with stochastic forcing by FPN errors during simulation

RIKEN Incentive Research Projects 2019, “Establishing the new method for ensemble weather/climate forecasting simulations due to reduced-precision floating point numbers”.