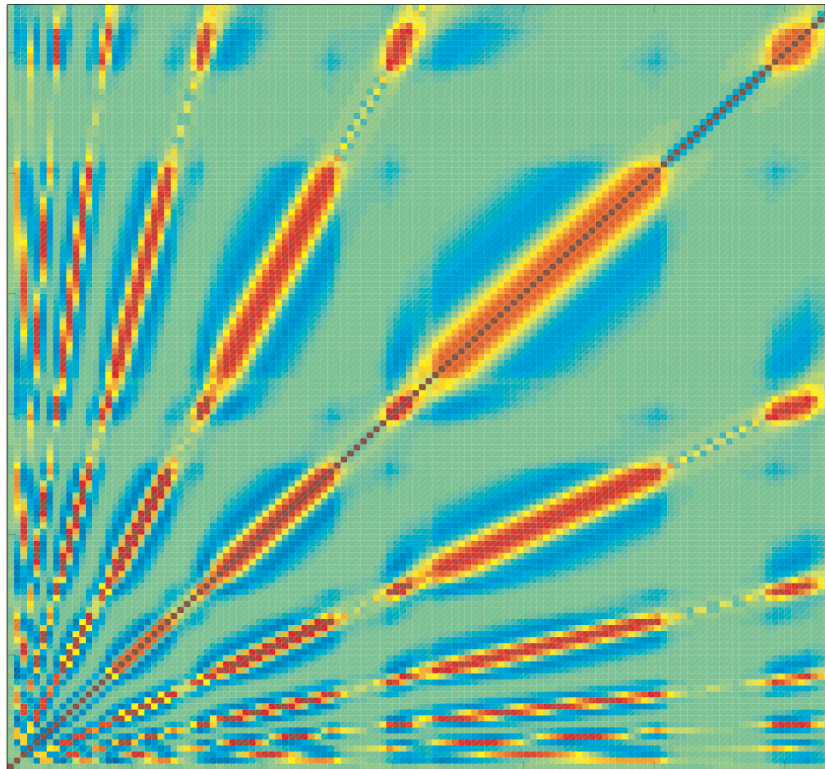


Linear Transforms and Averaging for Data Assimilation in Systems with a Range of Scales



Chris Snyder (NCAR)

Linear Transforms and Averaging for Data Assimilation in Systems with a Range of Scales

I'll show work by:

Altug Aksoy, David Dowell: Convective-scale EnKF with single radar

Glen Romine: EnKF with multiple radars, experiments in real-time mesoscale DA

Stan Trier: Analysis of convective initiation in high-resolution ensemble forecasts

Work on linear transforms and averaging is joint with:

Greg Hakim



Overview

1. An (incomplete) review of DA for convective scales
 - State of the art
 - Role of mesoscale motions
 - Issues posed for ensemble DA
2. Ensemble DA in presence of a range of scales
 - Localization
 - A simple example
 - Linear transforms

Convective-Scale DA

Useful to distinguish:

1. Analysis and prediction for individual convective cells
 - 0-3 h
2. Analysis and prediction for “watches” (i.e. areas where severe convective weather is likely)
 - 1-12 h
 - Beyond range of predictability for individual cells

Assimilation for Individual Cells

1. Require observations that resolve convection temporally and spatially
2. Update (very) frequently

Dense networks of Doppler radars provide enough observations

- Volume scans every 5 min with $O(1 \text{ km})$ spatial resolution
- Convective-scale error doubling time $\sim 10\text{-}30 \text{ min}$, so 2-6 scans/doubling time
- C.f. synoptic scale: error doubling time $\sim 2\text{-}3 \text{ d}$, so 4-6 radiosoundings/doubling time

Assimilation for Individual Cells (cont.)

Small domain, resolution $O(1 \text{ km})$

Single or a few Doppler radars

Simple mesoscale objective analysis or horizontally uniform environment

“Convection in a box”

[e.g. Sun 1997, Dowell et al. 2004, Marquis et al 2014, Snook et al. 2015, Wuersch P-22]

Tools

Weather Research and Forecasting model (WRF)

- Non-hydrostatic, time-split dynamics
- <http://www.mmm.ucar.edu/wrf/users/>

Data Assimilation Research Testbed (DART)

- A general-purpose software facility for ensemble filtering
- Development led by J. Anderson, NCAR/IMAGe
- <http://www.image.ucar.edu/DAReS/DART/>

Interfaces for WRF in DART

- Model grid to state vector, distances between elements of state vector
- Suite of observation operators, including Doppler radar

“WRF/DART” provides EnKF for WRF

Example of Assimilation for Individual Cells

WRF/DART EnKF

- 50 ensemble members
- Covariance localization within sphere of radius ~ 4 km

Three cases assimilating single Doppler radar

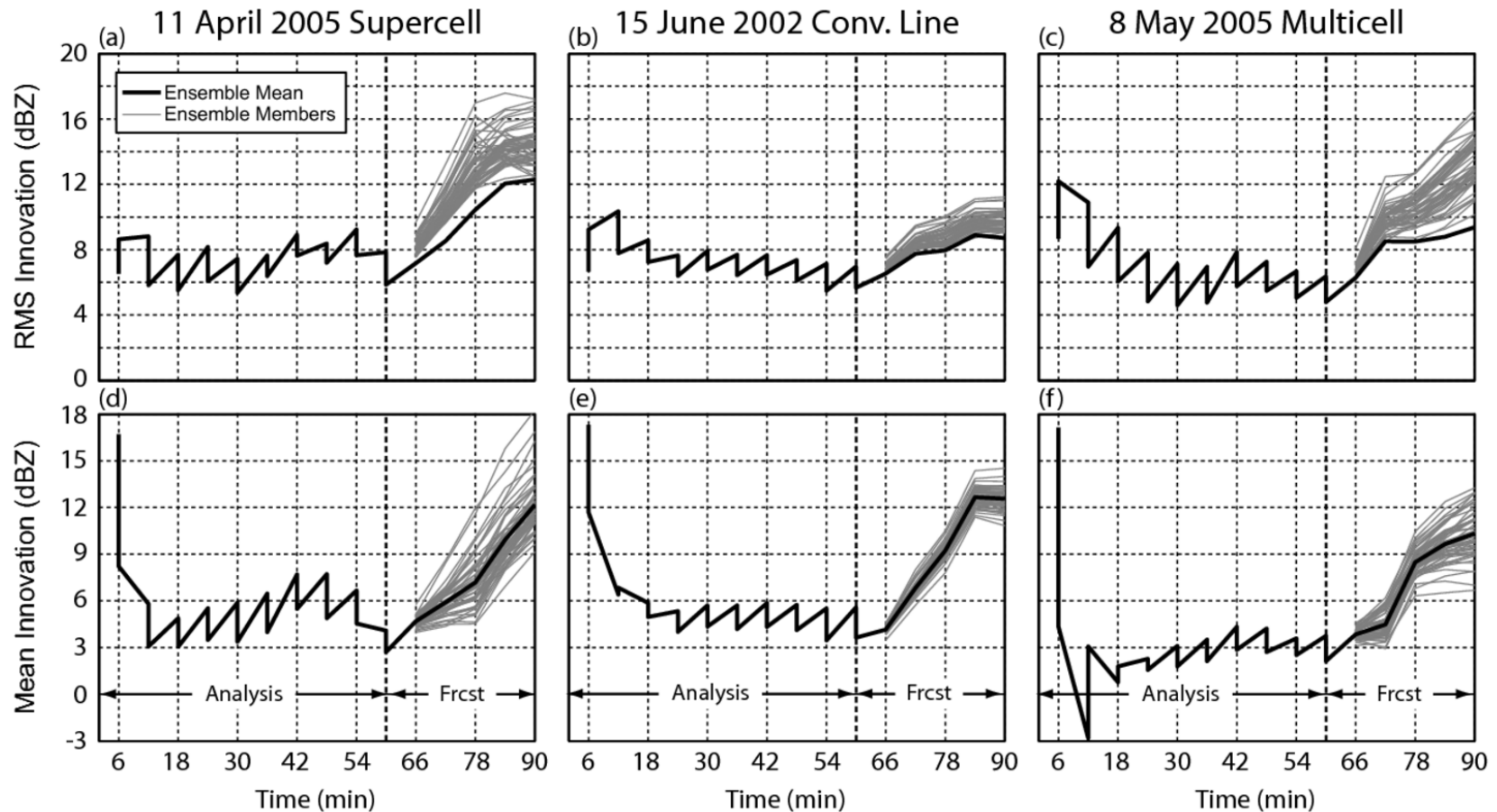
- Radial velocity
- “Clear air” reflectivity

2-min cycling: Assimilate scans at each elevation angle separately

[see Aksoy et al., 2010: *Mon. Wea. Rev.*]

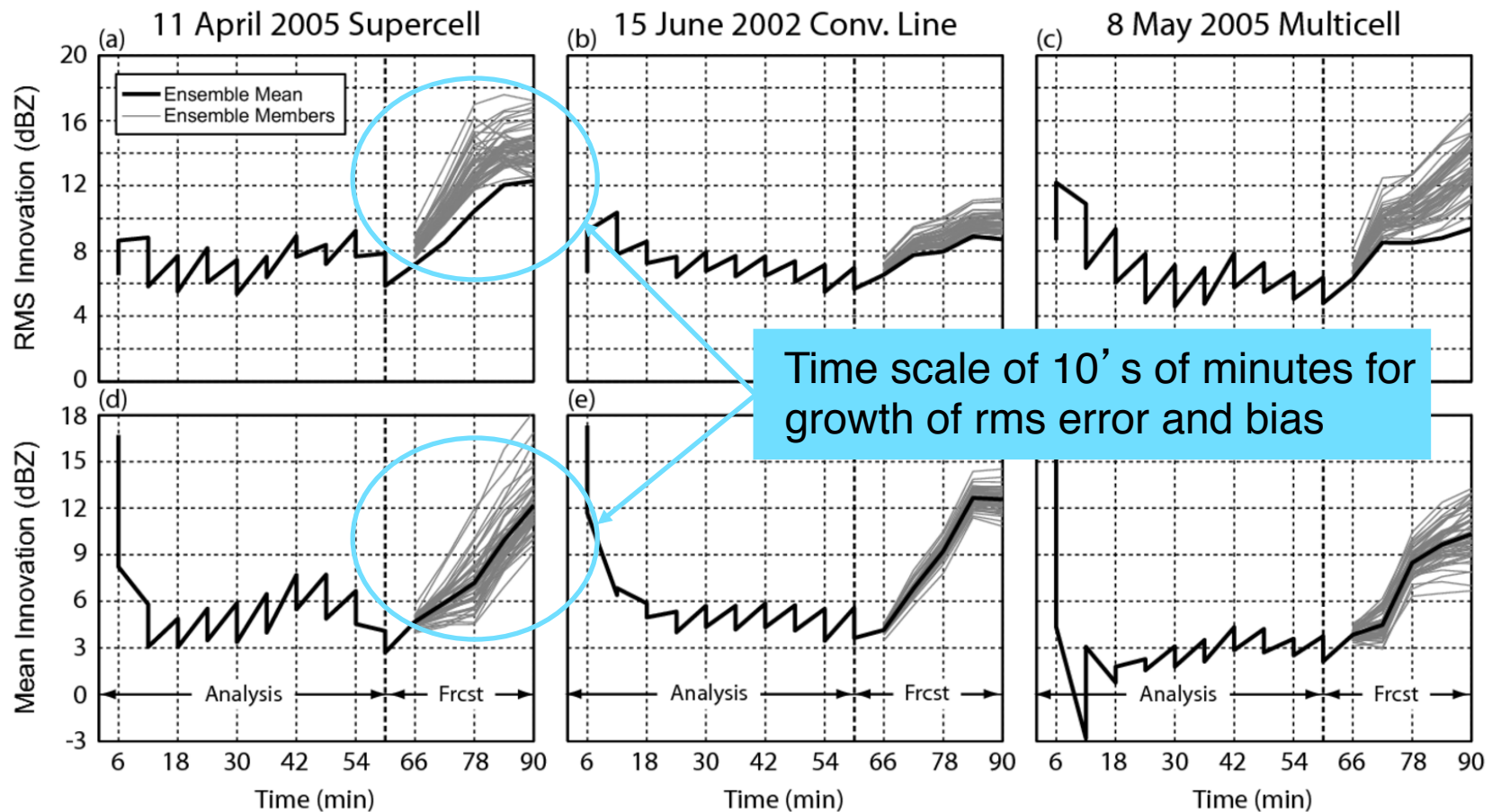
Obs-Minus-Forecast Statistics (reflectivity)

RMS: 6-10 dBZ for background forecast; area mean (bias): 3-8 dBZ



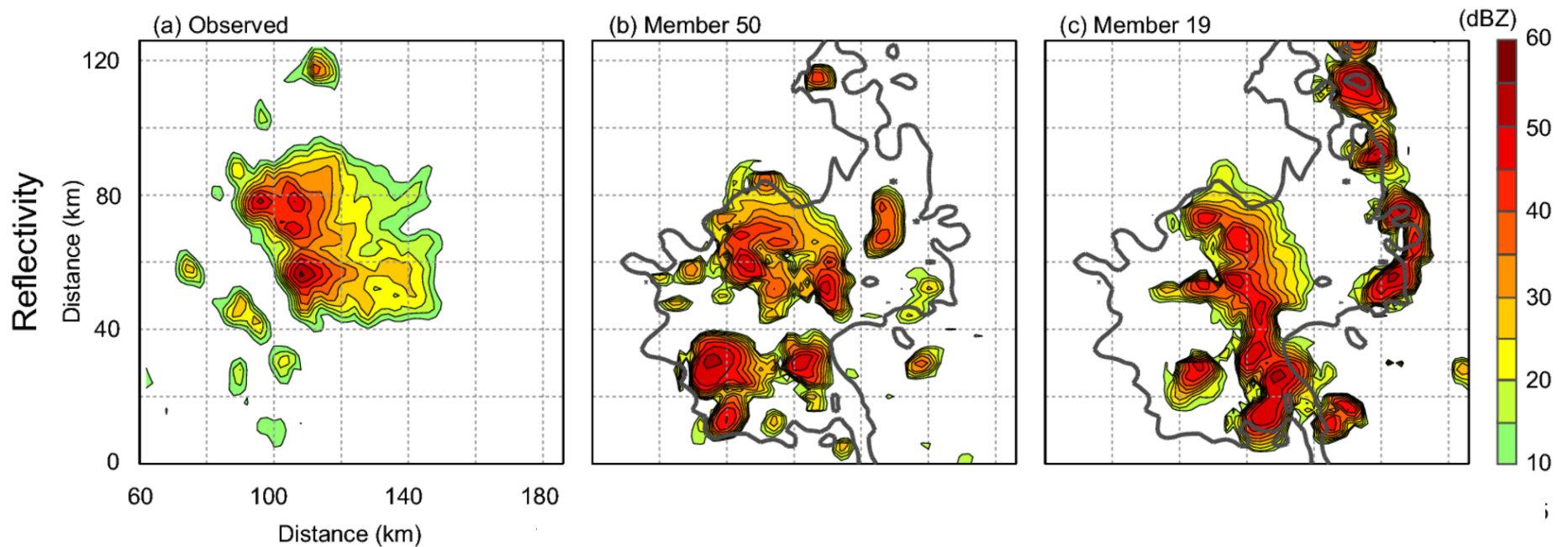
Obs-Minus-Forecast Statistics (reflectivity)

RMS: 6-10 dBZ for background forecast; area mean (bias): 3-8 dBZ



30-min Forecast: Multicell Case

All members produce too much convection



Aksoy et al., 2010: *Mon. Wea. Rev.*

Nature of Forecast Errors

Aksoy et al. (2010) find systematic errors in 30-min forecasts in all cases:

- Supercell: Cell decays in forecast but not in observations
- Convective line: Cells in forecast are systematically too shallow
- Multicell: Forecast produces too many cells, somewhat too intense

Potential causes

- Errors in environment/larger scales
- Errors in forecast model

Operational DA for Convective Scale

Broad similarities among many existing systems

- Convection “permitting,” 3-km resolution
- Cycling period between 1 h and 3 h
- 3DVar (except JMA 4DVar)
- Assimilate radial velocity (at coarsened resolution), plus nudging/retrievals from reflectivity or rainfall

Center	System	Resolution	Cycling	Reflectivity
Meteo France	AROME	3 km	3 h	RH retrieval
JMA	MA	5 km (4DVar)	3 h	
Met Office	UKV	1.5 km	3 h	RH retrieval
US NOAA	HRRR	3 km	1 h	DDFI
Germany	COSMO-DE	2.8 km	3 h	LH nudging

Operational DA for Convective Scale (cont.)

These systems do not analyze individual cells accurately

- Focus is on the mesoscale
- Cycling interval in particular is generally too infrequent to capture individual cells

Similar mesoscale EnKF systems exist for research

- E.g. Romine et al. 2012

Center	System	Resolution	Cycling	Reflectivity
Meteo France	AROME	3 km	3 h	RH retrieval
JMA	MA	5 km (4DVar)	3 h	
Met Office	UKV	1.5 km	3 h	RH retrieval
US NOAA	HRRR	3 km	1 h	DDFI
Germany	COSMO-DE	2.8 km	3 h	LH nudging

Research Systems for Convective Scale

Research frontier is to combine mesoscale and convective-scale analysis

4DVar

- Active efforts at JMA, Met Office and NCAR
- E.g., UK NDP: 1.5 km (3-km DA), 1-h window with 6 radar volumes

EnKF

- Work underway at NOAA labs, OU, Environment Canada, DWD and NCAR
- Resolutions between 1 km and 3 km, cycling interval of O(minutes)

“Serial” vs. simultaneous

- Easiest initial path is to perform mesoscale analysis then add radar in separate step

Influence of Larger Scales

WRF/DART mesoscale EnKF, 15-km North American domain

Spring real-time experiments as testbed for improving mesoscale

- Cycled DA for spring 2011-13 (2-3 months)
- Conventional obs + satellite winds + GPS; no radiances
- Covariance localization ~ 600 km

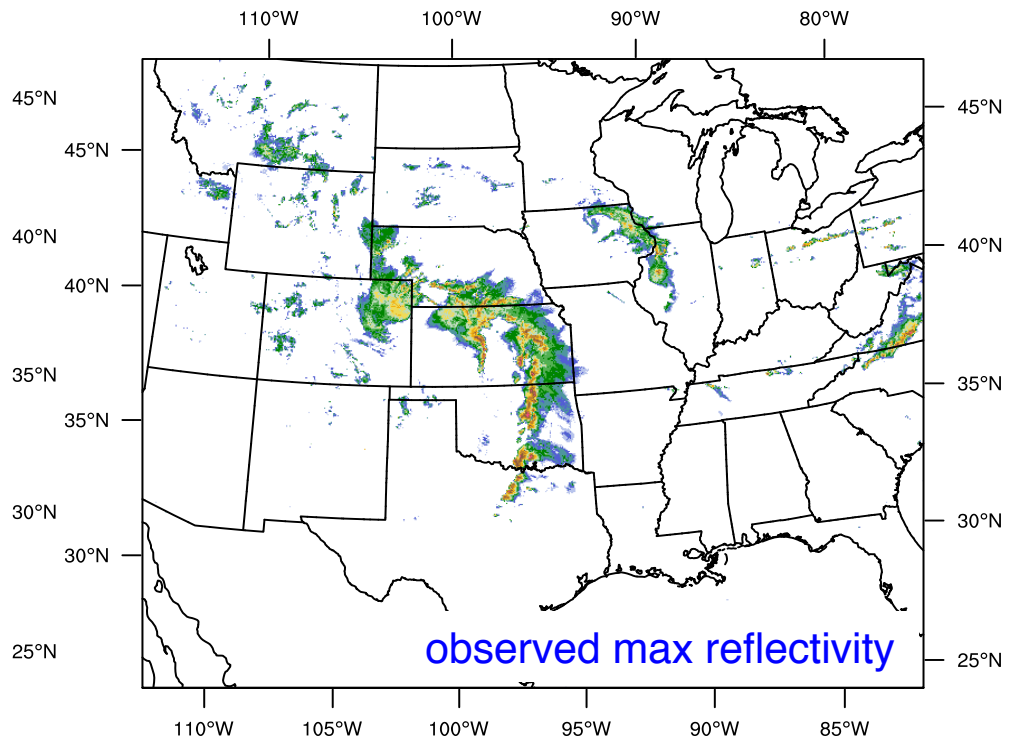
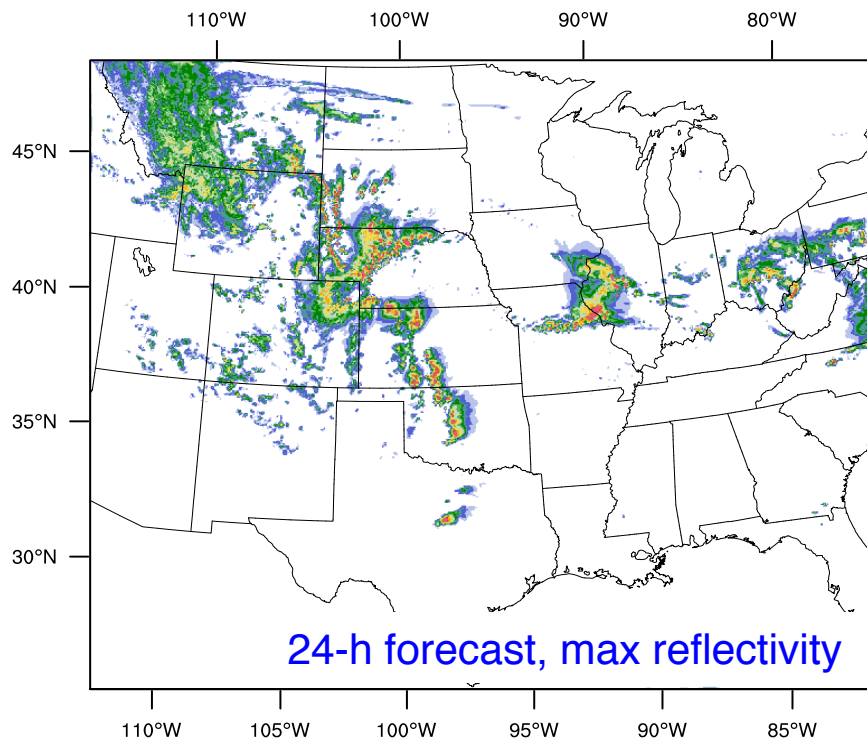
Evaluate quality based on high-resolution forecast

- Daily 3-km ‘convection permitting’ forecasts from 15-km analyses in 2011, 2012
- 30-member ensemble in 2013

2011 Results

Substantial room for improvement in 3-km forecasts

- Obvious mesoscale errors by 24 h
- Forecasts clearly worse than “cold starts” from NCEP global analysis

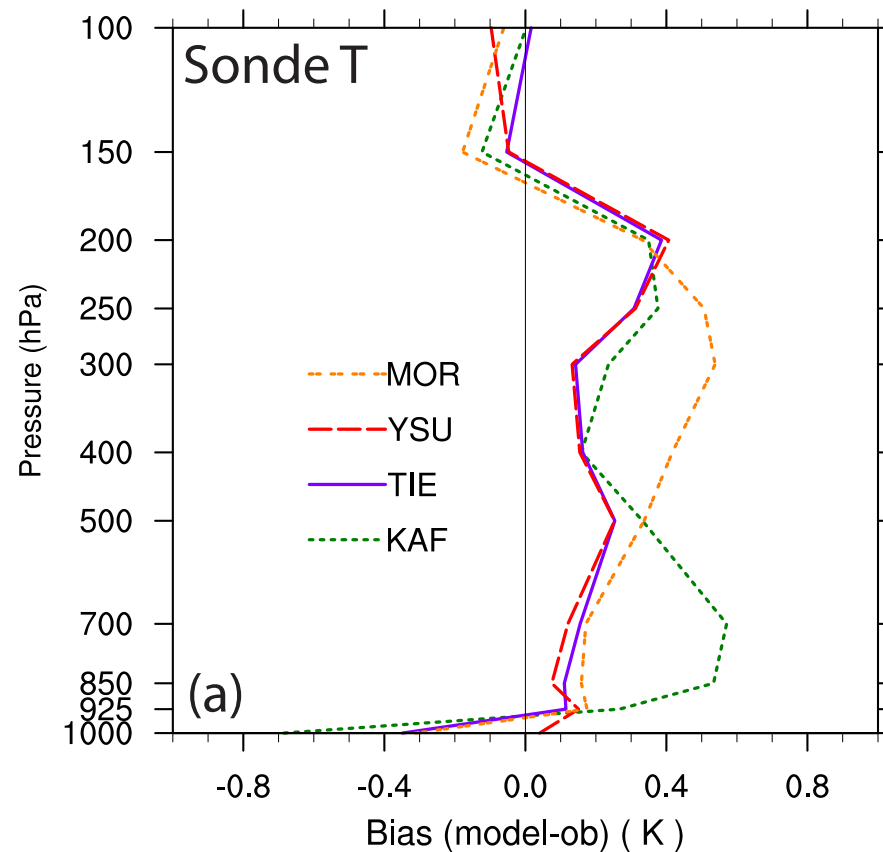


Romine et al., 2012, *Mon. Wea. Rev.*

2011 Results (cont.)

Real-time system employed Kain-Fritsch convective parameterization

- KF leads to substantial bias below 400 hPa, evident in 6-h fits to radiosondes

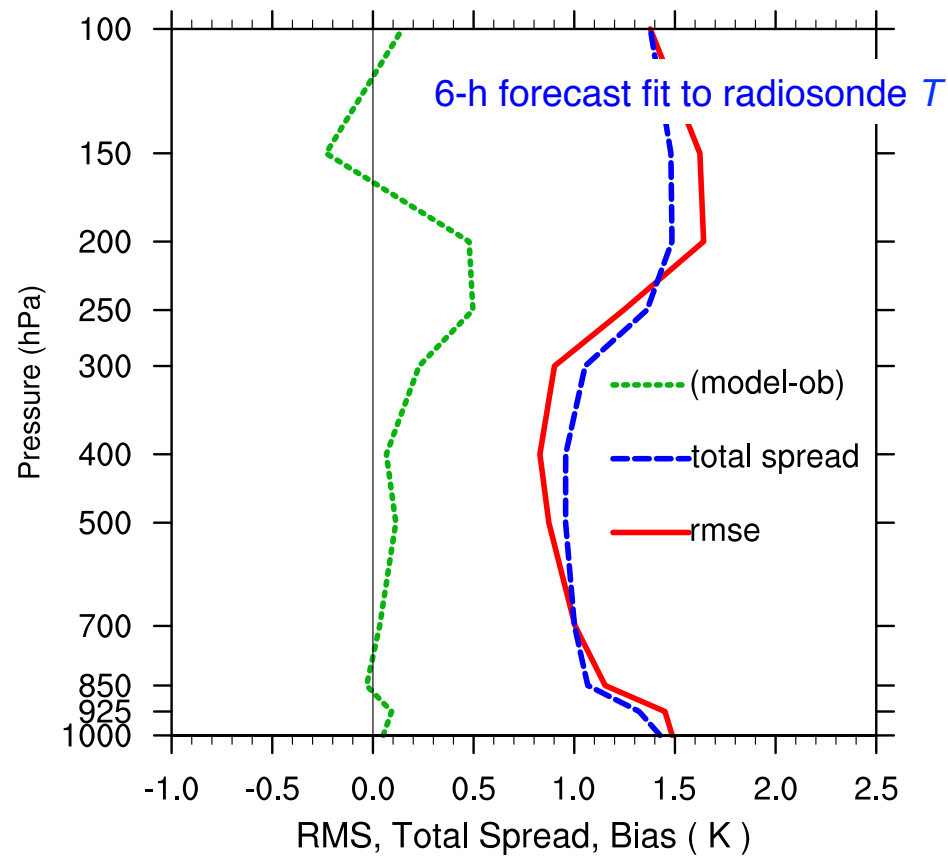


Romine et al., 2012, *Mon. Wea. Rev.*
(see also Torn and Davis, 2012, *Mon. Wea. Rev.*)

2012 Results

Switch from KF to Tiedtke for convective parameterization

–Huge reduction in bias

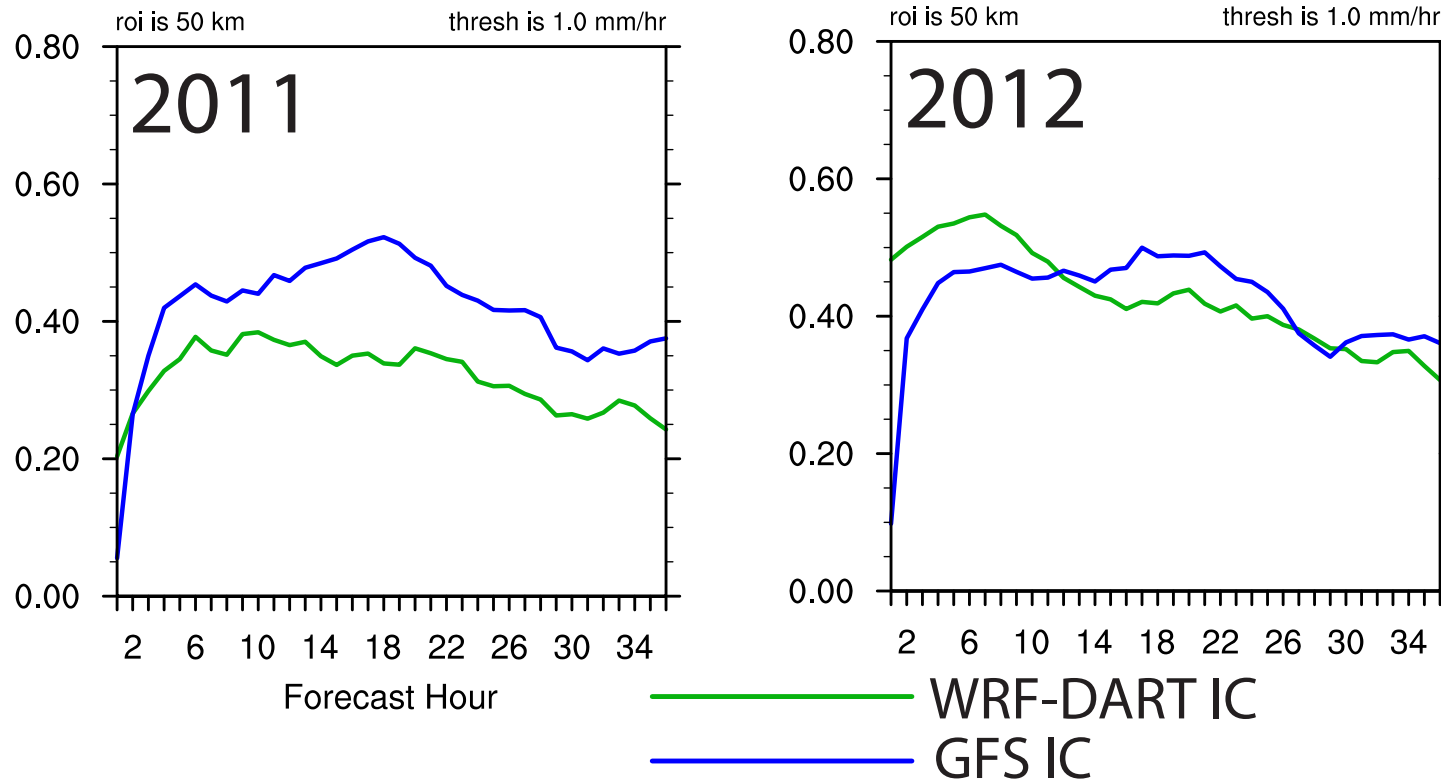


2012 Results (cont.)

Reduced bias leads to dramatic forecast improvement

- Note “multiplier” effect of better model in cycled DA/forecast system – little change in cold-start forecasts

Fractions skill score for precipitation vs. NCEP Stage IV analysis



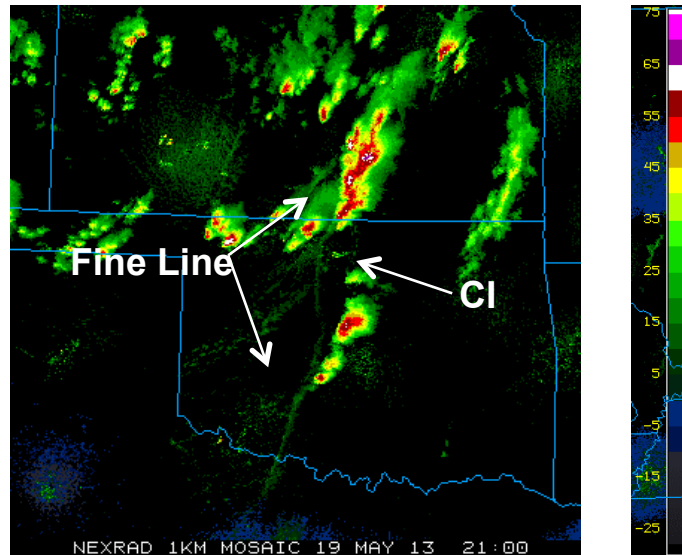
Influence of Mesoscale

3-km ensemble forecasts from spring 2013

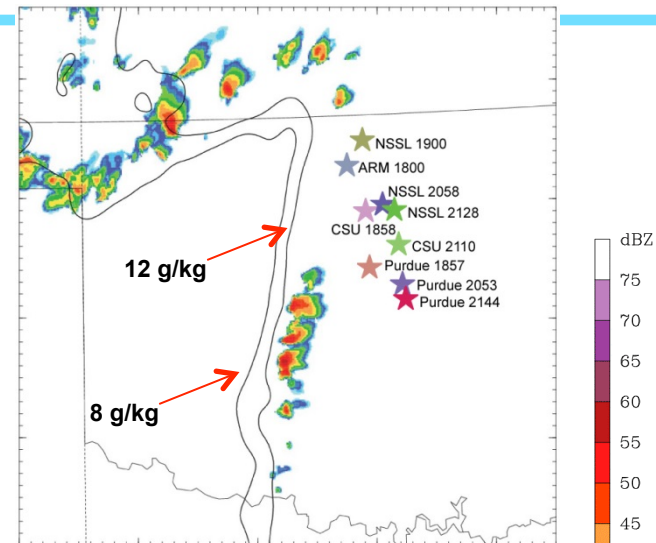
- 1-hourly cycling for selected cases
- Diagnose causes of convective-scale divergence
- Trier, Ahiyevich and Romine, 2015, *Mon. Wea. Rev.*, submitted

Observed/Simulated Reflectivity during Central Oklahoma Convection Initiation

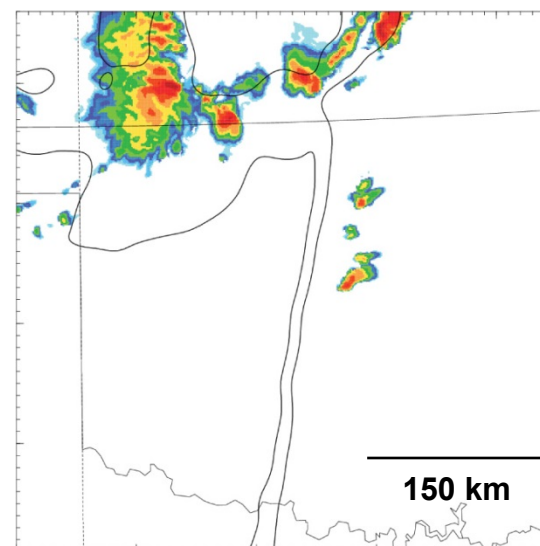
NEXRAD Radar Reflectivity at 2057 UTC 19 May 2013



WRF Ensemble Member 6 (CTRL) at 2048 UTC



WRF Ensemble Member 9 at 2148 UTC

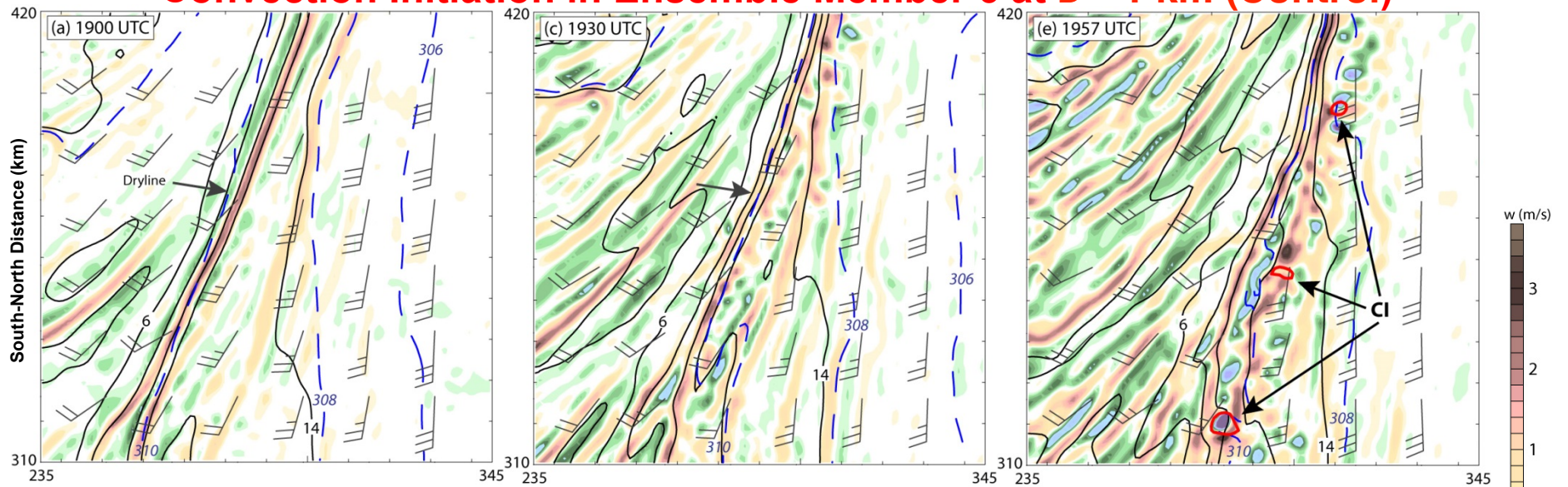


10-Member WRF Ensemble

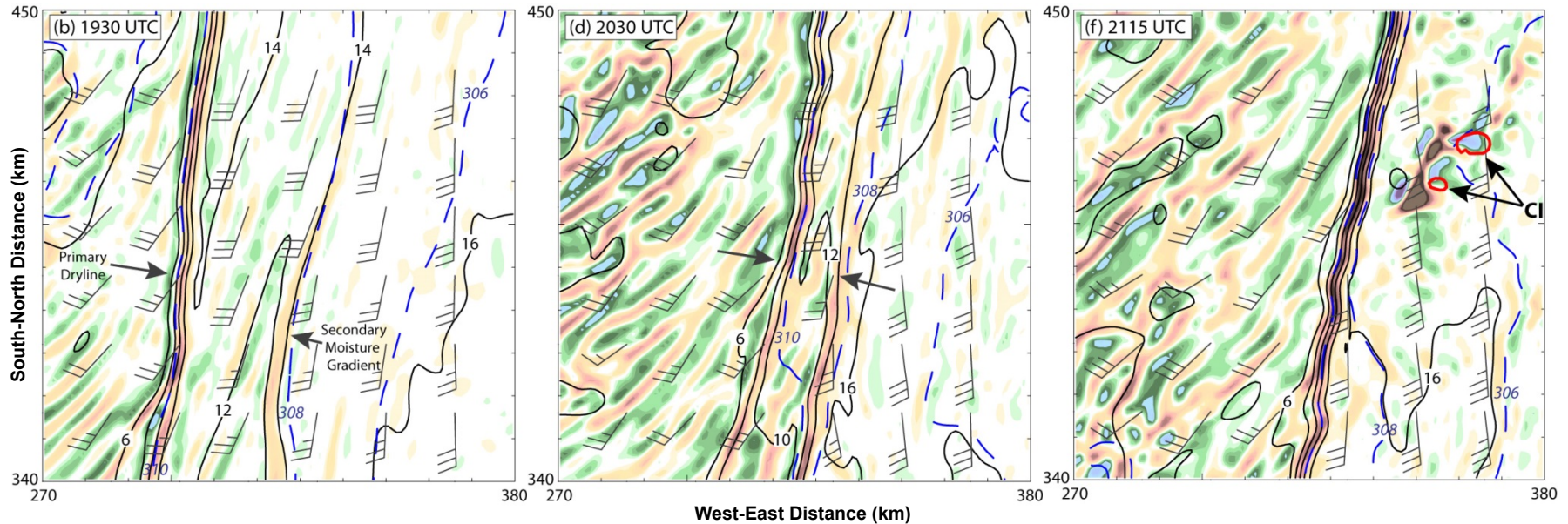
- WRF/DART ensemble analysis generated ICs
- 1500 UTC Initialization
- Domains with 9 and 3-km horizontal spacing
- One-Way Nest with 1-km spacing for selected members
- MYJ PBL Scheme
- Thompson Microphysics

Trier et al. 2015, *Mon. Wea. Rev.* submitted

Convection Initiation in Ensemble Member 6 at D = 1 km (Control)

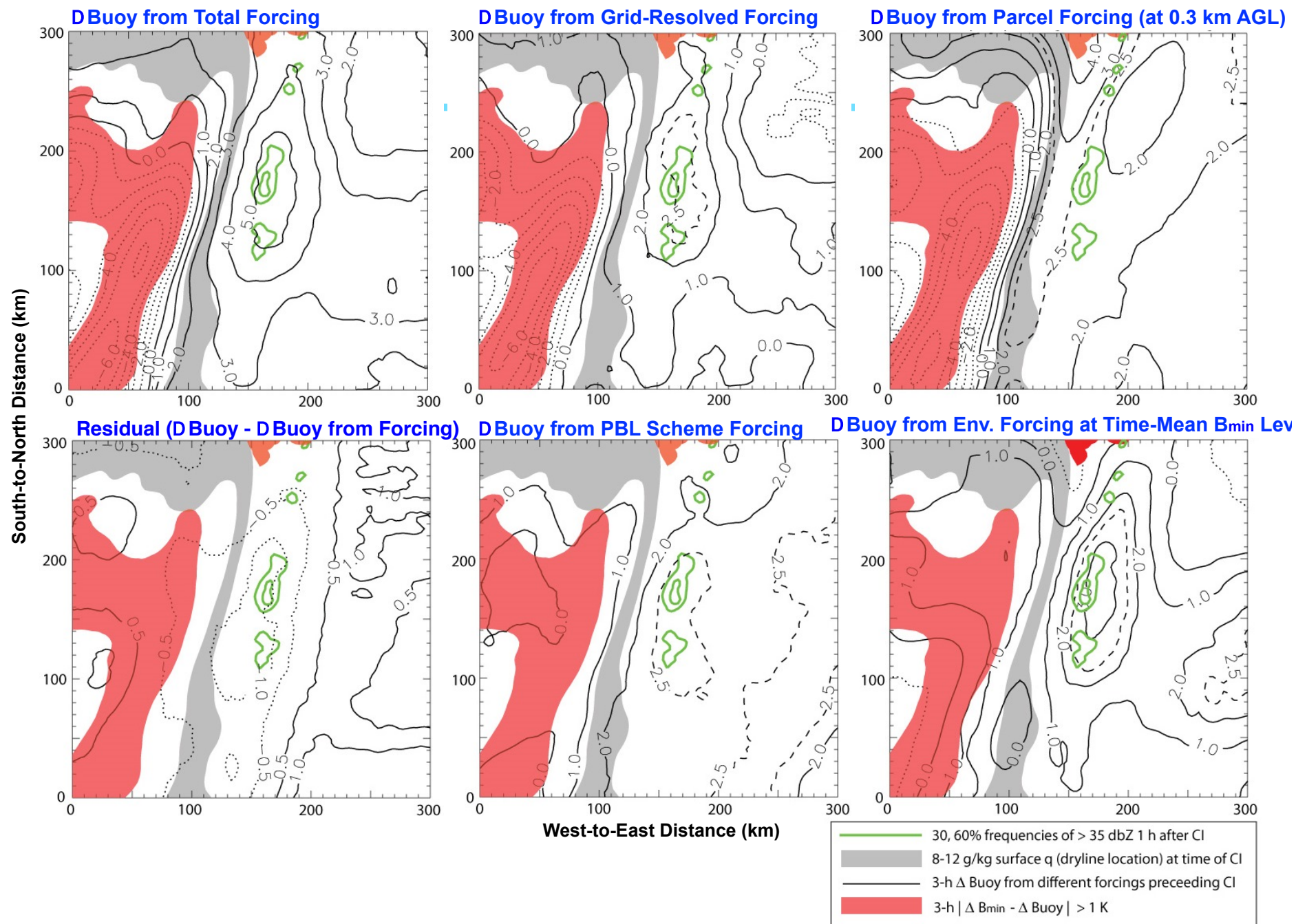


Convection Initiation in Ensemble Member 9 at D = 1 km



Color Shading: Model Level 7 (~0.9-km AGL) Vertical Velocity
 Black Barbs: Model Level 5 (~0.5-km AGL) Horizontal Winds
 Black Contours: Surface Water Vapor Mixing Ratio (2 g/kg intervals)
 Red Contours: Surface 35-dBZ Model-Derived Radar Reflectivity

Composite of 3-h Buoyancy Change from Different Forcings for Members 1, 3, 6, 8 and 10



Sensitivity of Moist Convection Forced by Boundary Layer Processes to Low-Level Thermodynamic Fields

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(Manuscript received 21 August 1995, in final form 18 January 1996)

ABSTRACT

The sensitivity of moist convection to a number of low-level thermodynamic parameters is examined with a high-resolution, nonhydrostatic numerical model. The parameters examined are the surface temperature dropoff (defined as the difference between the potential temperature measured at the surface and that in the well-mixed boundary layer), the surface moisture dropoff (defined similarly for moisture), the boundary layer moisture dropoff (defined as the vertical decrease in moisture within the boundary layer), and the depth of the moisture. The typical variability in these parameters is estimated from two field experiments in northeastern Colorado. Sensitivity is then defined relative to this typical observational variability.

Two convection initiation cases from northeastern Colorado are examined. In both cases, convection initiation is found to be most sensitive to the surface temperature dropoff and the surface moisture dropoff. It is found that variations in boundary layer temperature and moisture that are within typical observational variability (1°C and 1 g kg^{-1} , respectively) can make the difference between no initiation and intense convection. For cases in which convection is well developed, the storm's strength is more sensitive to the typical observational variability in moisture than in temperature. However, at the convection/no convection boundary, the storm's strength is more sensitive to the surface temperature dropoff than to the surface moisture dropoff (both in terms of equivalent moist static energy and, for the cases studied, in terms of typical observational variability). It is shown that this is due to the greater sensitivity of the negative area (or convective inhibition) to temperature variations than to moisture variations. The implications of these results for the predictability of convection initiation are then briefly discussed.

1. Introduction

Accurately predicting the development of severe thunderstorms is one of the most challenging problems facing weather forecasters. Currently, forecasters rely on a number of different data sources to aid them in these predictions. These sources include the National Weather Service soundings, surface stations, satellites, radars and lightning detection networks. In recent years there has been a considerable effort to combine these separate data sources into gridded analysis and short-term prediction systems. These systems include the Mesoscale Analysis and Prediction System (MAPS, Benjamin 1989) the Local Analysis and Prediction System (LAPS, McGinley 1989), as well as a number of variants on LAPS such as Oklahoma-LAPS (O-LAPS, Brewster et al. 1994) and Terminal-LAPS (T-LAPS, Cole et al. 1993).

The analysis cycles of these systems should provide guidance to the human forecaster in short-term thunderstorm prediction. However, it is still an open question as to whether the predictive cycle of these systems will be able to produce accurate, *explicit*, thunderstorm forecasts. The purpose of this study is to explore the prospects for explicit thunderstorm prediction by examining the sensitivity of convection initiation to a number of boundary layer thermodynamic parameters. The approach taken is sometimes called the "forward" method, where a forward simulation is performed from an initial state and then compared with simulations from slightly perturbed initial states. Examples of studies utilizing this approach are Weisman and Klemp (1982), Droegemeier and Wilhelmson (1985a) and (1985b), and Lee et al. (1991). Clearly, this is a somewhat limited approach since to calculate the sensitivity to each perturbation around each initial state requires a separate, forward, integration.

Recently, the adjoint method has proven to be a powerful tool for examining the sensitivity of atmospheric flows (see, e.g., Errico and Vukicevic 1992). With this method, it is possible to examine the sensitivity of some measure (e.g., the strength of an updraft) to the initial fields with a single integration of the forward model and one of the adjoint. However, adjoint models are

* The National Center for Atmospheric Research is sponsored by the National Science Foundation.

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Issues for Ensemble DA

Ensemble DA relies on covariance localization

- Radius of localization $\sim$ length scale of flow
- E.g.:
 - $R \sim 10$ km for convective applications
 - $R \sim 100$ km for mesoscale
 - $R \sim 1000$ km for synoptic-scale and global

Would like to analyze convective scale simultaneously with larger scales.

How to choose localization?

Brief Review: Sampling Error in EnKF _____

Recall that, in its raw form,

- ▷ EnKF implements KF update, but with $\mathbf{P}^f$ replaced by $\hat{\mathbf{P}}^f = \mathbf{X}\mathbf{X}^T$
- ▷ Sampling errors in elements of $\hat{\mathbf{P}}^f$ are $O(N_e^{-1/2})$, but correlated across elements
- ▷ global properties of $\mathbf{P}^f$ poorly estimated; most eigenvalues are zero
- ▷ Deviations from mean for analysis ensemble are linear combinations of forecast deviations

$$\mathbf{X}^a = \mathbf{X}\mathbf{F}$$

How does sampling error affect estimated $\hat{\mathbf{x}}^a$, $\hat{\mathbf{P}}^a$?

Example I

Consider update using single ob that is unrelated to state, $\mathbf{c} = \text{cov}(\mathbf{x}, y) = 0$.

$$\hat{\mathbf{x}}^a = \hat{\mathbf{x}}^f + (\hat{\mathbf{c}}/\hat{d}) (y^o - \mathbf{H}\hat{\mathbf{x}}^f)$$

The sample covariance $\hat{\mathbf{c}}$ is nonzero because of sampling error. Thus, $\hat{\mathbf{x}}^a$ is updated with noise and degraded.

EnKF covariance update gives

$$\hat{\mathbf{P}}^a = \hat{\mathbf{P}}^f - \hat{\mathbf{c}}\hat{\mathbf{c}}^T/\hat{d}.$$

The updated covariances decrease, despite the uninformative observation.

Example II

Suppose that $\mathbf{x} \sim N(0, \mathbf{I})$, $\epsilon \sim N(0, \mathbf{I})$ and $\mathbf{H} = \mathbf{I}$, i.e.,

$$\mathbf{y} = \mathbf{x} + \epsilon$$

Suppose also that we have an ensemble of 2 members.

Let $\delta\mathbf{x}$ be deviation of member 1 from mean. (Member 2 deviation is $-\delta\mathbf{x}$.)

Normalize so that

$$\text{tr}(\hat{\mathbf{P}}^f) = \text{tr}(\delta\mathbf{x}\delta\mathbf{x}^T) = \delta\mathbf{x}^T\delta\mathbf{x} = \text{tr}(\mathbf{P}^f) = N_x$$

Example II (cont.)

$\mathbf{x} \sim N(0, \mathbf{I})$, $\epsilon \sim N(0, \mathbf{I})$ and $\mathbf{H} = \mathbf{I}$

For the KF,

$$\begin{aligned}\mathbf{P}^a &= \mathbf{P}^f - \mathbf{P}^f \mathbf{H}^T (\mathbf{H} \mathbf{P}^f \mathbf{H}^T + \mathbf{R})^{-1} \mathbf{H} \mathbf{P}^f \\ &= \mathbf{I} - (\mathbf{I} + \mathbf{I})^{-1} \\ &= \frac{1}{2} \mathbf{I}\end{aligned}$$

$$\text{tr}(\mathbf{P}^a) = N_x/2$$

Example II (cont.)

Ensemble of 2, with deviation $\pm\delta\mathbf{x}$ from mean and $\delta\mathbf{x}^T\delta\mathbf{x} = N_x$.

Also $\epsilon \sim N(0, \mathbf{I})$ and $\mathbf{H} = \mathbf{I}$

For the EnKF,

$$\hat{\mathbf{P}}^a = \mathbf{X}^a \mathbf{X}^{aT}, \quad \mathbf{X}^a = \mathbf{X} \mathbf{F}, \quad \mathbf{F} \mathbf{F}^T = (\mathbf{I} - \mathbf{X}^T \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} \mathbf{X})^{-1}$$

(And note: $\mathbf{X} = \delta\mathbf{x}$, $\mathbf{F} = f$, a scalar)

Example II (cont.)

Ensemble of 2, with deviation $\pm\delta\mathbf{x}$ from mean; $\epsilon \sim N(0, \mathbf{I})$ and $\mathbf{H} = \mathbf{I}$
Also $\epsilon \sim N(0, \mathbf{I})$ and $\mathbf{H} = \mathbf{I}$

For the EnKF,

$$\hat{\mathbf{P}}^a = \mathbf{X}^a \mathbf{X}^{aT}, \quad \mathbf{X}^a = \mathbf{X} \mathbf{F}, \quad \mathbf{F} \mathbf{F}^T = (\mathbf{I} - \mathbf{X}^T \mathbf{H}^T \mathbf{R}^{-1} \mathbf{H} \mathbf{X})^{-1}$$

$$f^2 = (1 + \delta\mathbf{x}^T \delta\mathbf{x})^{-1} = (N_x + 1)^{-1}$$

$$\hat{\mathbf{P}}^a = \mathbf{X} \mathbf{F} \mathbf{F}^T \mathbf{X}^T = (N_x + 1)^{-1} \hat{\mathbf{P}}^f$$

$$\text{tr}(\hat{\mathbf{P}}^a) = N_x / (N_x + 1)$$

Example II (cont.)

Correct update:

$$\text{tr}(\mathbf{P}^a) = N_x/2$$

For EnKF:

$$\text{tr}(\hat{\mathbf{P}}^a) = N_x/(N_x + 1)$$

2-member update underestimates variance by factor $\approx 2/N_x$

With small ensemble, EnKF sees all prior variance confined to small subspace. Typically (unless rows of $\mathbf{H}$ are nearly orthogonal to the ensemble subspace), obs-error variance will be small compared to (apparent) prior variance. Update reduces variance in subspace to $\sim$ obs-error variance.

Example II (cont.)

Also easy to show that $\hat{\mathbf{x}}^a$ is degraded by noise in subspace orthogonal to $\mathbf{y}^o - \mathbf{x}^f$

Covariance Localization

Use a priori info that flow has finite decorrelation length

- ▷ variables separated by sufficient distance should have small covariance
- ▷ implement by $\hat{\mathbf{P}}_{loc}^f = \mathbf{C} \circ \hat{\mathbf{P}}^f$, w/ $\mathbf{C}$ a correlation matrix
- ▷ explicitly removes covariances at large separations; $\hat{\mathbf{P}}_{loc}^f$ has increased rank and analysis increment not confined to ensemble subspace

An **essential** ingredient for NWP applications

Covariance Localization (cont.) ---

More generally, “localize” by enforcing sparsity—set small covariances to zero

- ▷ parameter-state covariances
- ▷ cross-variable covariances

Data Assimilation Using an Ensemble Kalman Filter Technique

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(Manuscript received 18 December 1996, in final form 10 June 1997)

ABSTRACT

The possibility of performing data assimilation using the flow-dependent statistics calculated from an ensemble of short-range forecasts (a technique referred to as ensemble Kalman filtering) is examined in an idealized environment. Using a three-level, quasigeostrophic, T21 model and simulated observations, experiments are performed in a perfect-model context. By using forward interpolation operators from the model state to the observations, the ensemble Kalman filter is able to utilize nonconventional observations.

In order to maintain a representative spread between the ensemble members and avoid a problem of inbreeding, a pair of ensemble Kalman filters is configured so that the assimilation of data using one ensemble of short-range forecasts as background fields employs the weights calculated from the other ensemble of short-range forecasts. This configuration is found to work well: the spread between the ensemble members resembles the difference between the ensemble mean and the true state, except in the case of the smallest ensembles.

A series of 30-day data assimilation cycles is performed using ensembles of different sizes. The results indicate that (i) as the size of the ensembles increases, correlations are estimated more accurately and the root-mean-square analysis error decreases, as expected, and (ii) ensembles having on the order of 100 members are sufficient to accurately describe local anisotropic, baroclinic correlation structures. Due to the difficulty of accurately estimating the small correlations associated with remote observations, a cutoff radius beyond which observations are not used, is implemented. It is found that (a) for a given ensemble size there is an optimal value of this cutoff radius, and (b) the optimal cutoff radius increases as the ensemble size increases.

1. Introduction

For two decades, statistical interpolation (e.g., Lorenc 1981), and more recently the closely related three-dimensional variational (3DVAR) algorithm (e.g., Parrish and Derber 1992), have been the foremost data assimilation methods for operational numerical weather prediction (NWP). Applied multivariately, with different observational errors for different types of observations and using a short-range forecast (typically 6 h) as background, these methods have proved capable of combining information from model forecasts and the heterogeneous set of available observations. They have supported a remarkable increase in forecast accuracy over the period, as shown by, for example, Kalnay et al. (1990, Figs. 4 and 5), Hollingsworth and Lönnerberg (1990), and Mitchell et al. (1993, Fig. 14).

Of the various aspects of statistical interpolation, the one that has received the most attention in the literature has been the determination and specification of the forecast and observation error statistics (see, e.g., Hollingsworth and Lönnerberg 1986; Lönnerberg and Hollingsworth 1986; Bartello and Mitchell 1992; Polavarapu 1995; and

references therein). Consideration of statistical interpolation in the context of Kalman filtering validates this preoccupation, since, as discussed, for example, by Cohn and Parrish (1991), the analysis of data is the same in the two methods. Rather, it is in the specification of the forecast-error statistics, or more precisely covariances, that the two methods differ. On the one hand, the Kalman filter gives a systematic way to calculate the time evolution of the forecast-error statistics according to the dynamics of the forecast model. In contrast, the statistics used in statistical interpolation and the 3DVAR algorithm are generally taken to be isotropic and largely homogeneous with little variation in time (see e.g., Rabier and McNally 1993 and the references cited above). Despite the demonstrated deficiencies of these restrictions [due to, e.g., data discontinuities (Cohn and Parrish 1991; Daley 1992b), baroclinic zones (Jørgensen 1987), and fronts (Desroziers and Lafore 1993)], it has not yet been possible to eliminate them in operational data assimilation systems.

Following the approach of stochastic-dynamic prediction, proposed by Epstein (1969), Evensen (1994) recently considered the use of an ensemble to estimate the forecast-error statistics. Using Monte Carlo methods to generate the ensemble and assuming four available measurements, he considered data assimilation in the context of a two-layer quasigeostrophic ocean model on

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Multiple Scales in Other Contexts

Vertical

- Large-scale (balanced) flows have $H/L \sim f/N$
- Consistent with this, vertical localization typically narrow relative to horizontal
- Convective-scale flows have $H/L = O(1)$ and corresponding ratio of vertical, horizontal localizations

Temporal

- Initializing slow motions in presence of fast motions

Coupled atmosphere–ocean data assimilation experiments with a low-order model and CMIP5 model data

Robert Tardif · Gregory J. Hakim · Chris Snyder

Received: 30 June 2014 / Accepted: 20 October 2014
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Abstract Coupled atmosphere–ocean data assimilation (DA) experiments are performed for estimating the Atlantic meridional overturning circulation (AMOC). Recovery of the AMOC with an ensemble Kalman filter is assessed for a range of experiments over observation availability (atmosphere, upper and deep ocean) and for assimilating high-frequency observations compared to time averages. For an idealised low-order coupled climate model, the traditional DA approach using an ensemble of model trajectories to estimate covariances is compared to a simplified “no-cycling” approach involving climatological covariances derived from a single long model integration. Robustness of the no-cycling method is also tested on data from a millennial-scale simulation of a comprehensive coupled atmosphere–ocean climate model. Results show that the no-cycling approach provides a good approximation to the traditional approach, and that assimilation of time-averaged observations improves AMOC recovery using drastically smaller ensembles than would be required for the case of instantaneous observations. Even in the limit of no ocean observations, the no-cycling approach is capable of recovering the low-frequency AMOC with time-averaged observations; assimilation of noisy instantaneous atmospheric observations fails to recover decadal-scale AMOC variability.

Keywords Cost-effective · Coupled atmosphere–ocean data assimilation · Atlantic meridional overturning circulation · Estimation

1 Introduction

Considerable research is currently devoted to improving near-term (i.e. interannual to interdecadal) climate predictions using coupled atmosphere–ocean climate models (e.g., Meehl et al. 2014). One essential aspect is the initialisation of such predictions using available observations to provide more accurate representations of phases in atmospheric and oceanic states associated with internal variability of the coupled system. Using initialised states generally results in more skillful predictions, e.g. see Troccoli and Palmer (2007), Keenlyside et al. (2008), Pohlmann et al. (2009), Mochizuki et al. (2010), Matei et al. (2012), Meehl and Teng (2012), Robson et al. (2012), Doblas-Reyes et al. (2013), Smith et al. (2013) among others. However, a lack of consensus on the most effective initialization strategies is apparent in the wide variety of approaches, particularly with respect to initialisation of low-frequency components of the climate system. Here we address this problem by considering coupled atmosphere–ocean ensemble data assimilation strategies and, in particular, conditions for which the Atlantic meridional overturning circulation (AMOC) may be recovered from a limited set of observations, including the case where only the atmosphere is observed.

Extant initialisation methods include forcing an ocean model with analysed surface fields (e.g. atmospheric and/or sea surface temperature) (e.g., Keenlyside et al. 2008; Meehl and Teng 2012; Yeager et al. 2012; Pohlmann et al. 2013); performing data assimilation (DA) in the ocean

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Multiple Scales in 4DVar

Related issues arise for 4DVar

- Typically, length of assimilation window $\sim$ time scale of motion

How to choose length of assimilation window for multiple scales?

- Short window captures fast/small-scale motions, but fails to propagate covariances for slower/larger scales
- Long window makes minimization extremely difficult for fast/small scales

Different Looks at the Same Problem

Buehner and Charron 2007, Buehner 2012

- Localization after transforming to spectral or wavelet space

Miyoshi and Kondo 2013

- Combine low-res, large-localization analysis and high-resolution, short-localization analysis

The Simplest 2-Scale Idealization ---

Consider 1D, periodic spatial domain, discretized as N_x grid points

State is sum of two independent processes,

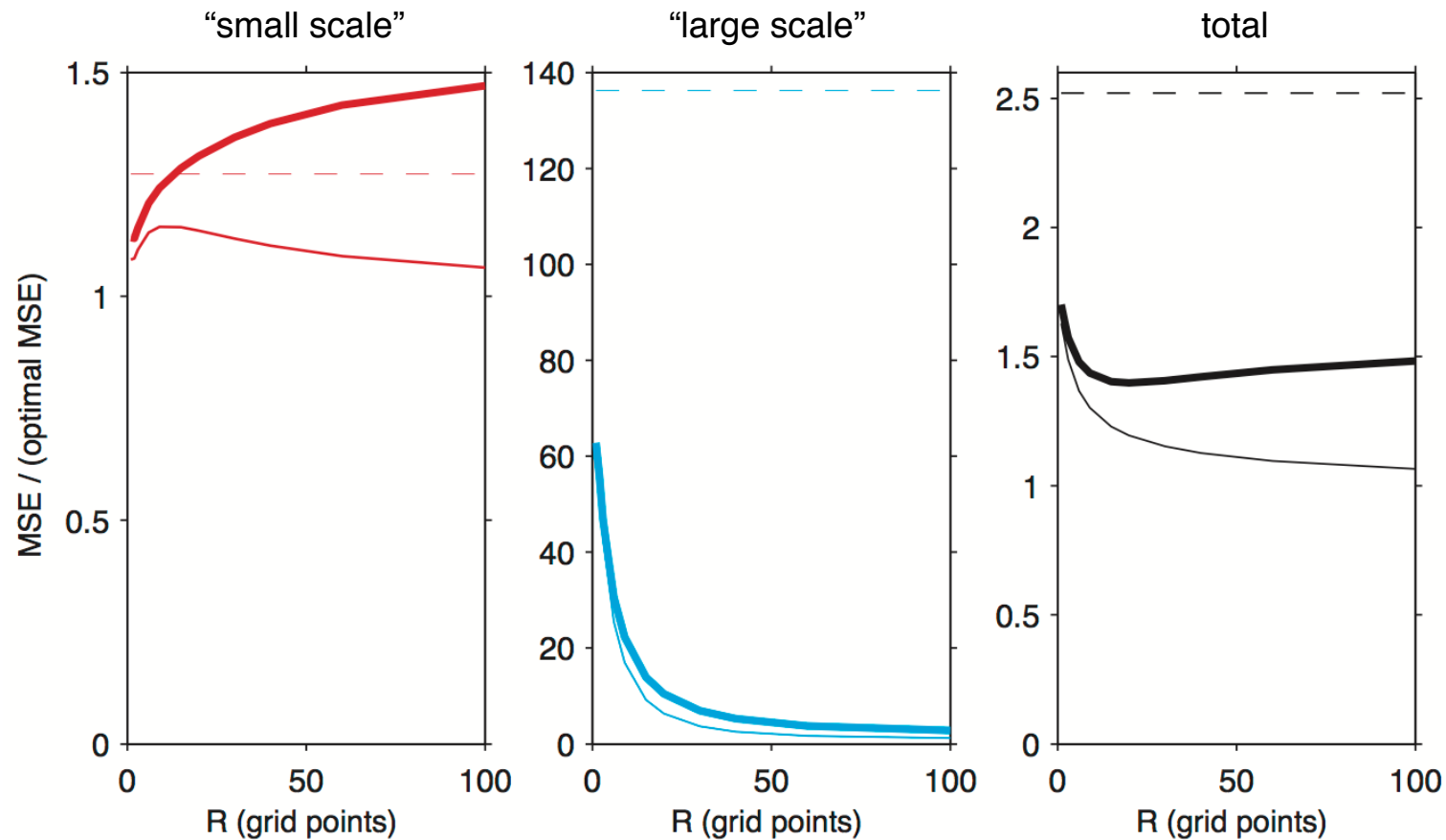
$$\mathbf{x} = \mathbf{x}_s + \mathbf{x}_l \quad \text{with} \quad \mathbf{x}_s \sim N(0, \sigma_s^2 \mathbf{I}), \quad \mathbf{x}_l \sim N(0, \sigma_l^2 \mathbf{1})$$

$\mathbf{x}_s$ is spatially white; $\mathbf{x}_l$ is identically correlated

Observations

$$\mathbf{y} = \mathbf{x} + \epsilon, \quad \epsilon \sim N(0, I)$$

A Simple Example (cont.)



A Simple Example (cont.)

Basic results as expected

- $\mathbf{x}_s$ analysis best with narrow localization, $\mathbf{x}_l$ analysis best with broad
- "optimal" localization (minimizing total MSE) is a compromise

Presence of another scale can degrade analysis at a given scale

What Defines a Multi-Scale Problem?

Two-scale example has homogeneous covariances

- diagonal in spectral space and easily handled by standard variational schemes
- (Is the classic DA problem with homogeneous, isotropic covariances a multiscale problem?)

Idealization with Inhomogeneity

Introducing Inhomogeneity

Convection is inhomogeneous owing to variation of larger-scale environment

Idealize, again on 1D, periodic spatial domain

State is sum of two processes,

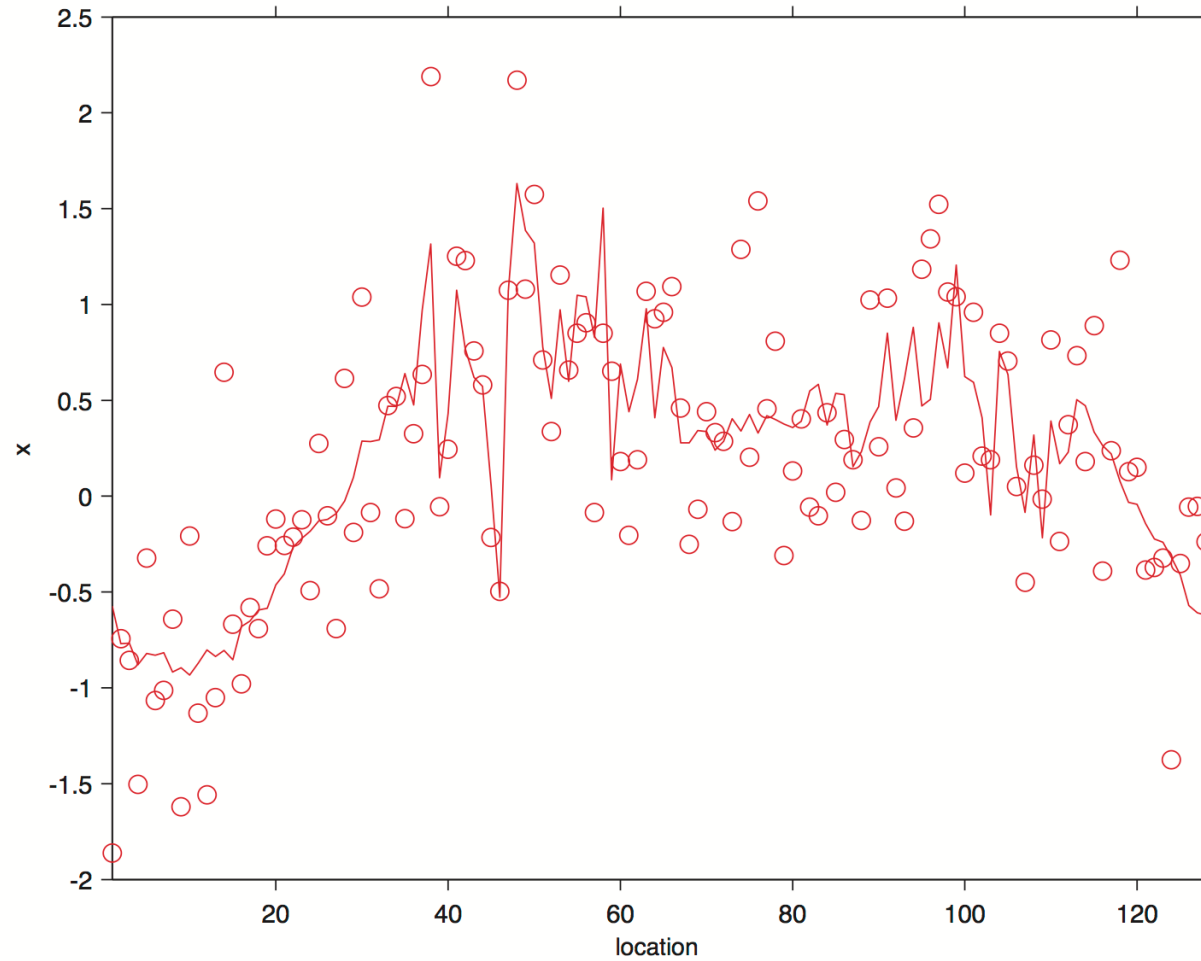
$$\mathbf{x} = \mathbf{x}_s + \mathbf{x}_l \quad \text{with} \quad \mathbf{x}_l \sim N(0, \mathbf{P}_l), \quad \mathbf{x}_s \sim N(0, \mathbf{P}_s(\mathbf{x}_l))$$

where $\mathbf{P}_s$ has fixed, narrow correlation structure and variance depending on $\mathbf{x}_l$ at each grid point. For example, at i th point, $\text{var}(x_{s,i}) \propto \max(x_{l,i}, 0)$.

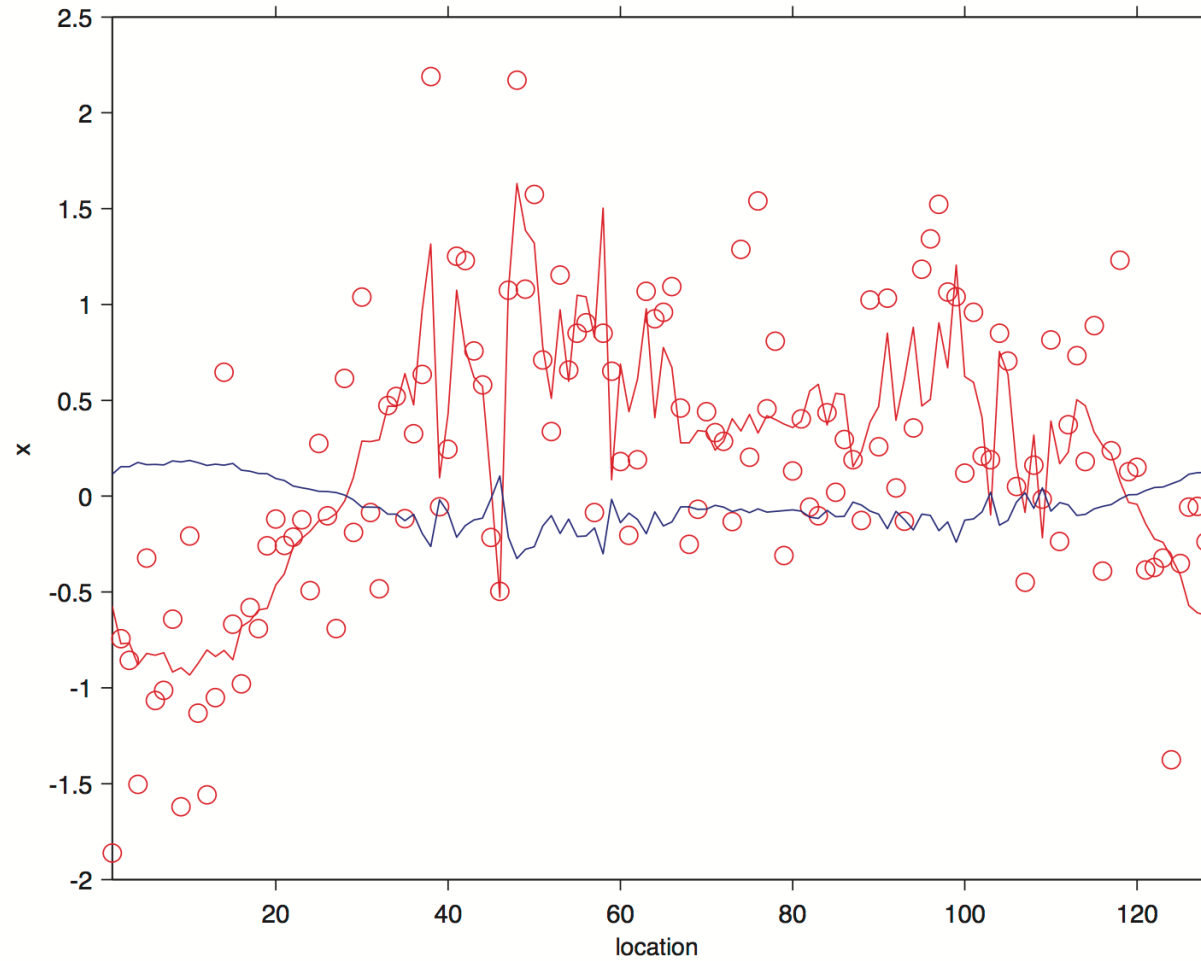
Observations

$$\mathbf{y} = \mathbf{x} + \epsilon, \quad \epsilon \sim N(0, I)$$

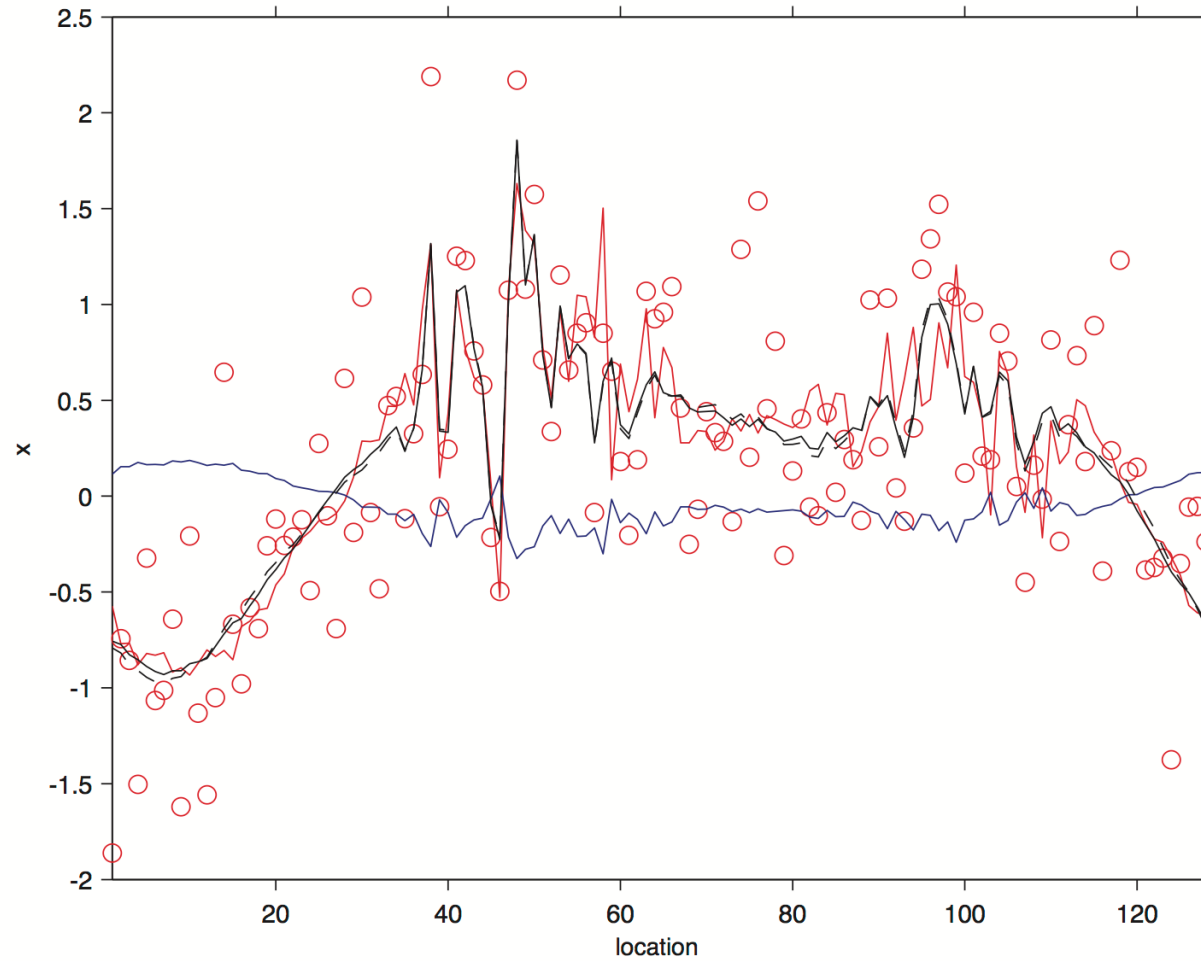
Idealization with Inhomogeneity (cont.)



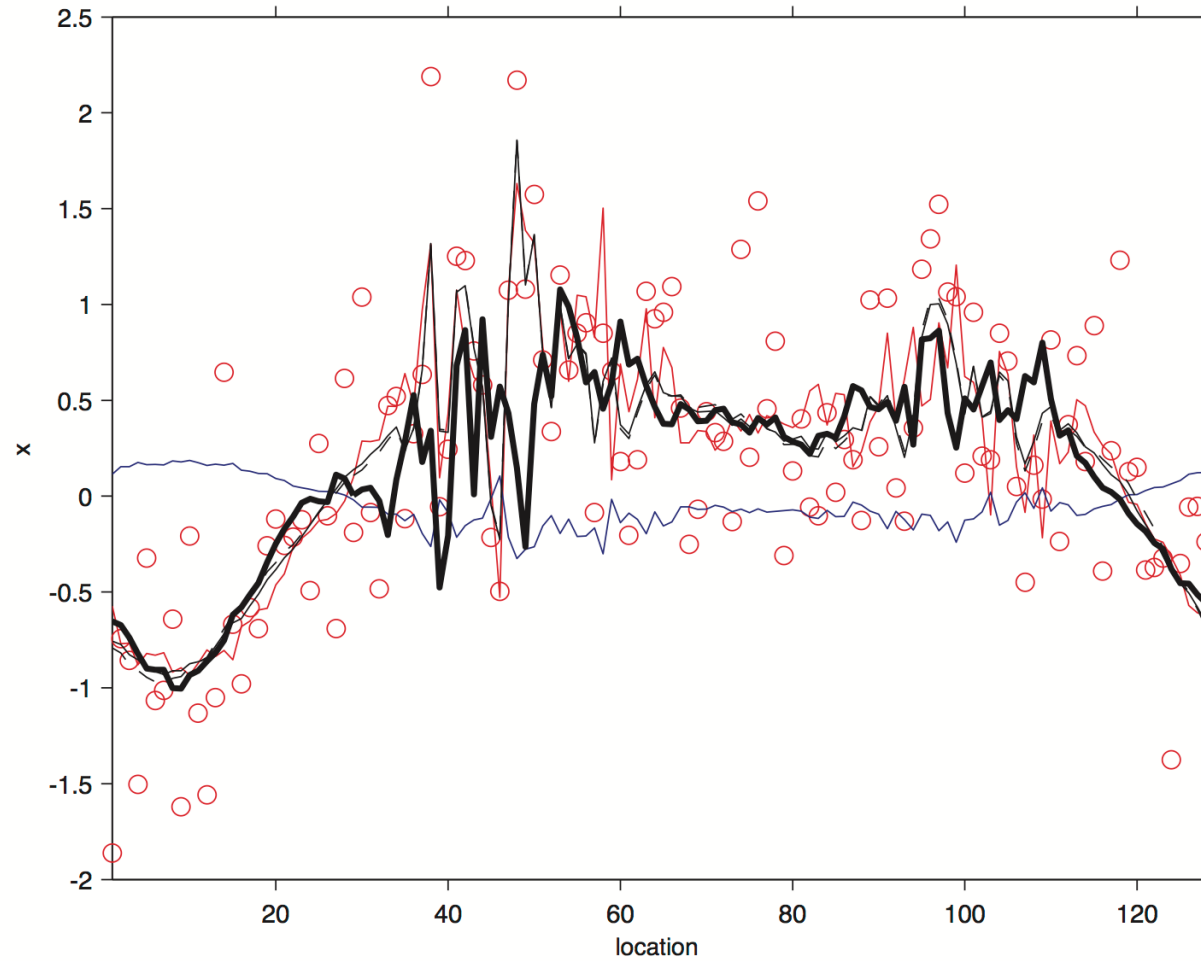
Idealization with Inhomogeneity (cont.)



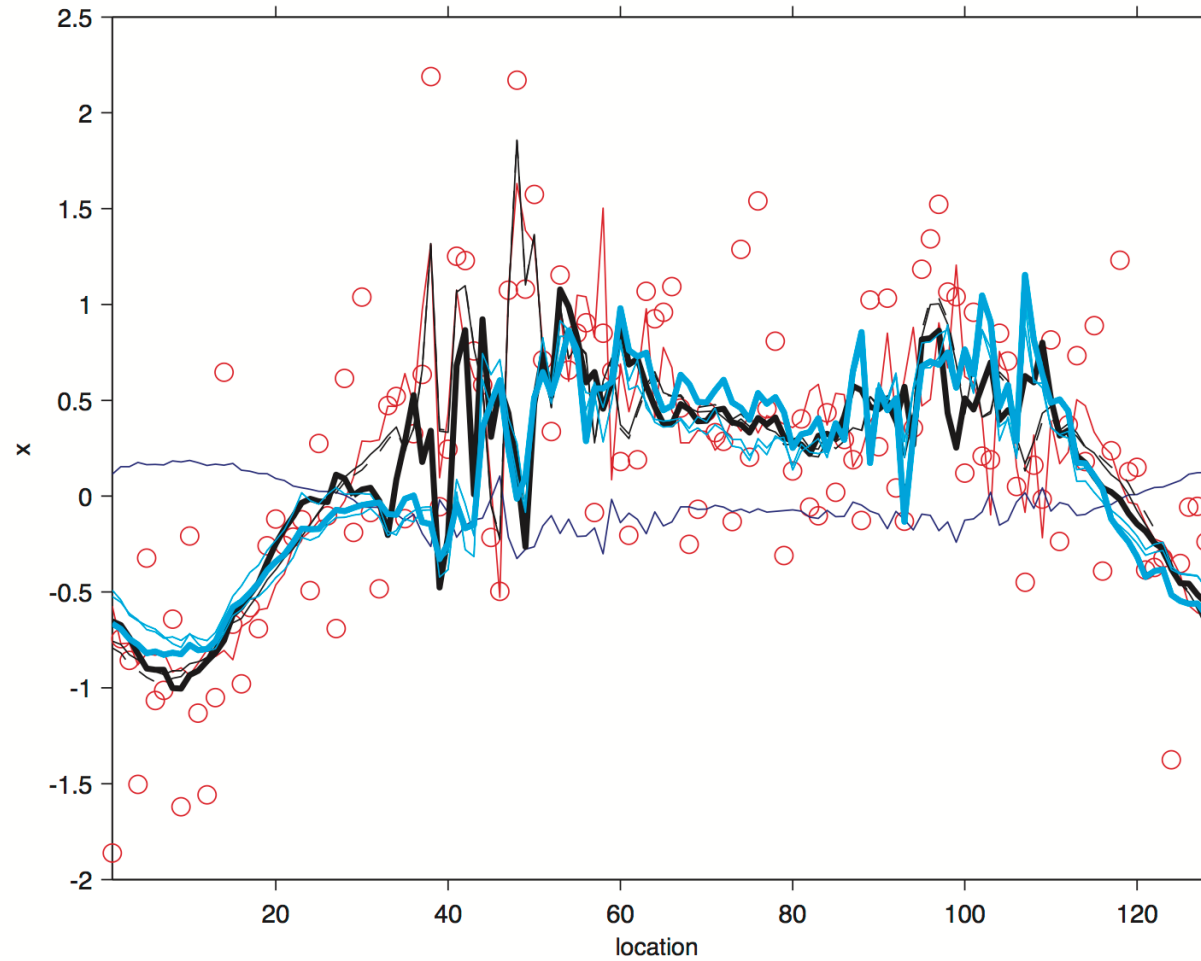
Idealization with Inhomogeneity (cont.)



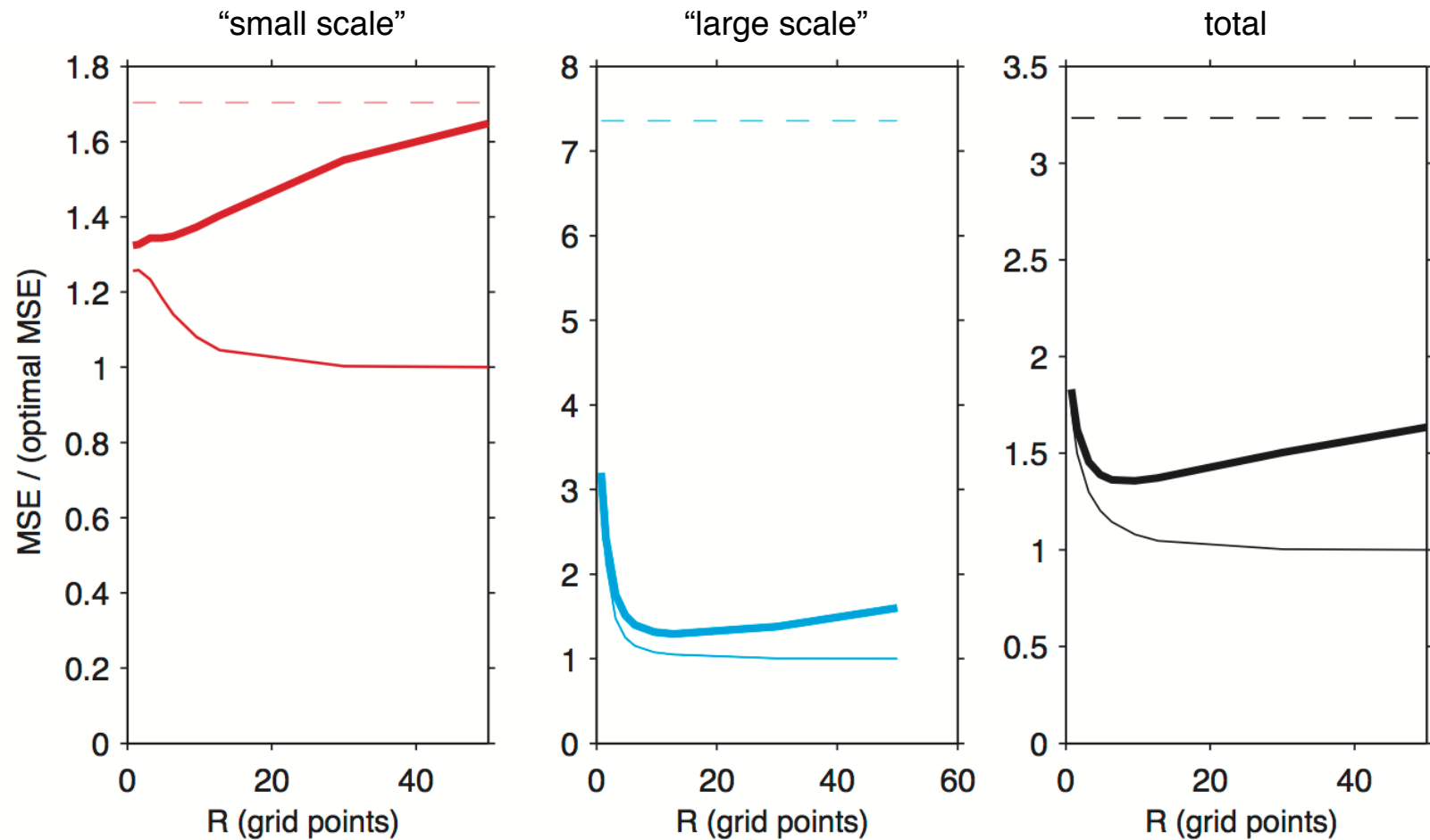
Idealization with Inhomogeneity (cont.)



Idealization with Inhomogeneity (cont.)



Idealization with Inhomogeneity (cont.)



Linear Transformations and Ensemble DA _____

In simple, two-scale example, linear transformations are potentially helpful.

Spectral transform: $\tilde{\mathbf{x}} = \mathbf{S}\mathbf{x}$ with $\mathbf{S}$ = discrete Fourier transform

- ▷ $\text{cov}(\tilde{\mathbf{x}})$ is diagonal
- ▷ localize sample covariance of $\tilde{\mathbf{x}}$ by discarding all off-diagonal elements
- ▷ c.f. Buehner and Charron (2007)

Averaging: $\tilde{\mathbf{x}} = \mathbf{L}\mathbf{x}$ = spatial average $\langle \mathbf{x} \rangle$ concatenated with $\mathbf{x} - \langle \mathbf{x} \rangle$.

- ▷ $\text{cov}(\tilde{\mathbf{x}})$ has off-diag elements $\propto N_x^{-1}$
- ▷ using $\langle \mathbf{y} \rangle$ to update $\langle \mathbf{x} \rangle$ is nearly optimal
- ▷ c.f. Dirren and Hakim (2005), Huntley and Hakim (2010)

Linear Transformations (cont.)

Optimal $\mathbf{L}$ to precede localization?

- ▷ Best to localize cov. matrix with known sparsity properties

Localization only in state space

- ▷ e.g. for EnVar
- ▷ $\mathbf{L} = \mathbf{E}^T$, where $\mathbf{E}$ is matrix of evecs of $\text{cov}(\mathbf{x})$

Localization for $\text{cov}(\mathbf{x}, \mathbf{y})$ too

- ▷ e.g. for serial EnKF
- ▷ seek combinations of state, and of obs, that are optimally correlated

Optimal Linear Transformation

Suppose $\mathbf{x} \sim N(0, \mathbf{P})$, $\mathbf{y} = \mathbf{H}\mathbf{x} + \epsilon$ and $\epsilon \sim N(0, \mathbf{R})$

Perform SVD of $\mathbf{R}^{-1/2}\mathbf{H}\mathbf{P}^{1/2} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^T$ and define

$$\tilde{\mathbf{x}} = \mathbf{V}^T \mathbf{P}^{-1/2} \mathbf{x}, \quad \tilde{\mathbf{y}} = \mathbf{U}^T \mathbf{R}^{-1/2} \mathbf{y}$$

Then (see e.g. Rodgers 2000)

$$\tilde{\mathbf{x}}^a = \tilde{\mathbf{x}}^f + \tilde{\mathbf{K}}(\tilde{\mathbf{y}} - \mathbf{\Lambda}\tilde{\mathbf{x}})$$

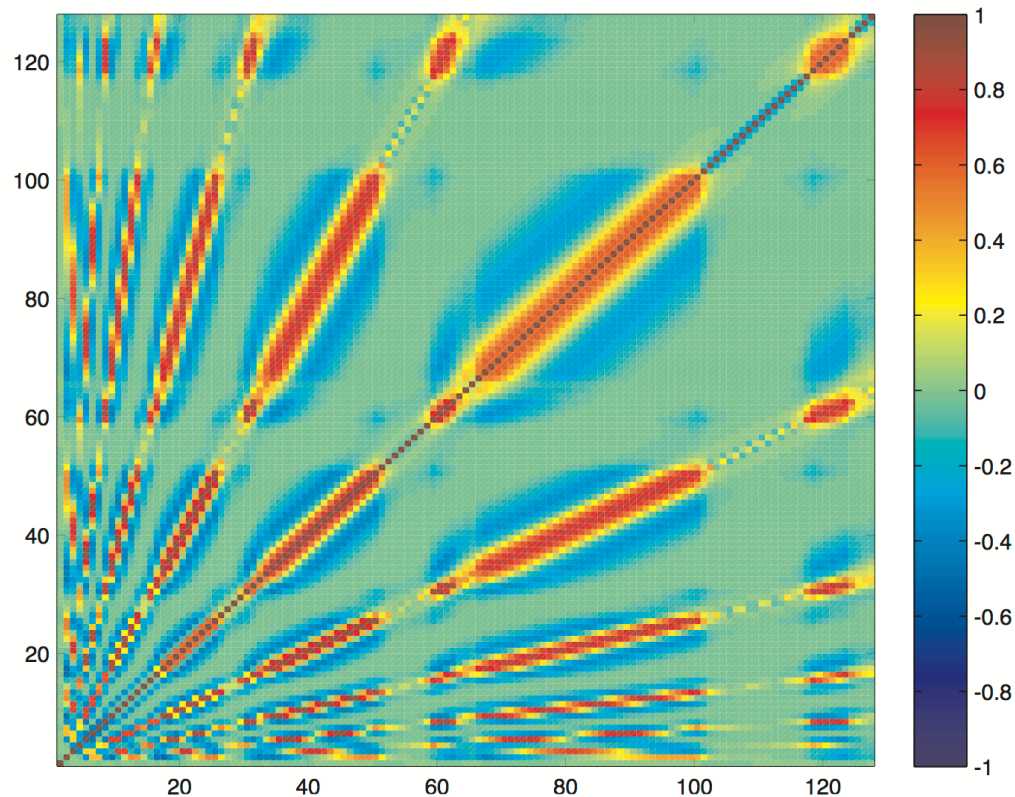
where $\tilde{\mathbf{K}} = \mathbf{\Lambda}^T(\mathbf{\Lambda}\mathbf{\Lambda}^T + \mathbf{I})^{-1}$ is rectangular diagonal. In these coordinates, each observation is related to only a single state variable.

Optimal Linear Transformation (cont.)

Unfortunately, optimal $\mathbf{L}$ depends on state covariances.

Approximations to $\mathbf{L}$

- When statistics are nearly homogeneous and isotropic, spectral transforms suffice
- In general, consider wavelet transform



Summary

DA for simultaneous analysis of meso- and convective scales remains important and open question

- Evidence that error at meso- and larger scales is (the?) limiting factor to progress at convective scales
- For EnKF, how to localize across range of scales?

Simple examples to illuminate fundamental issues

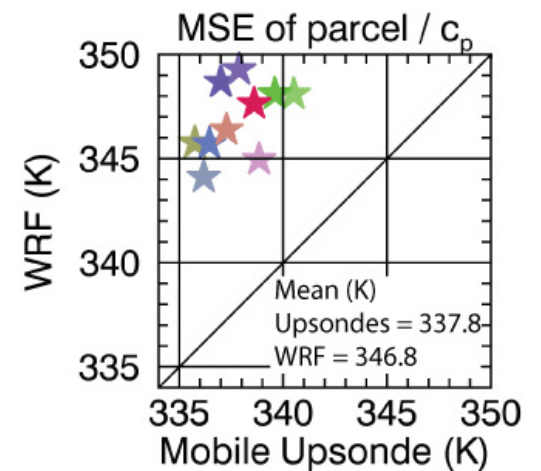
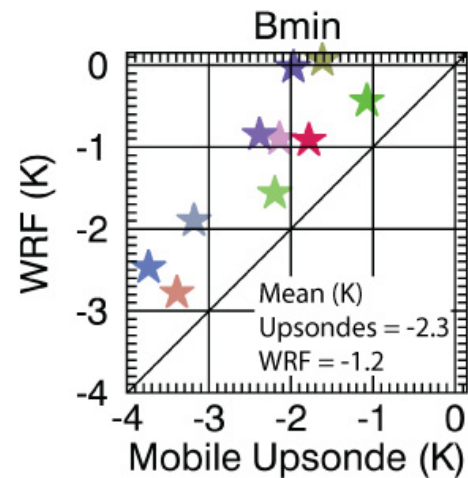
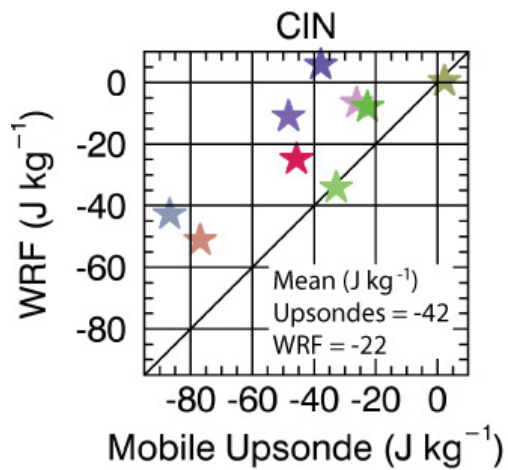
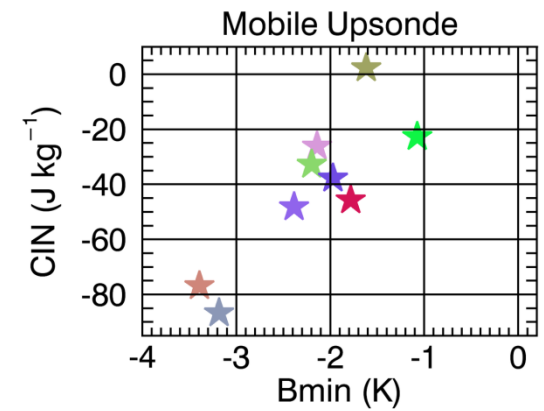
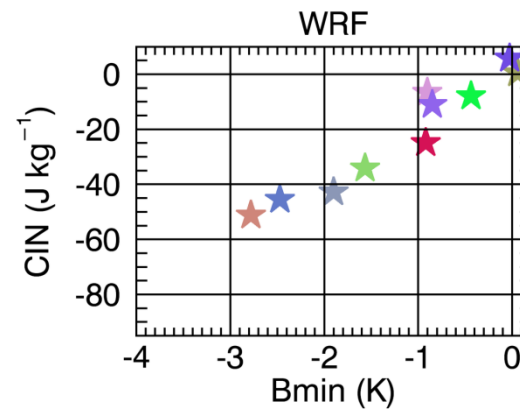
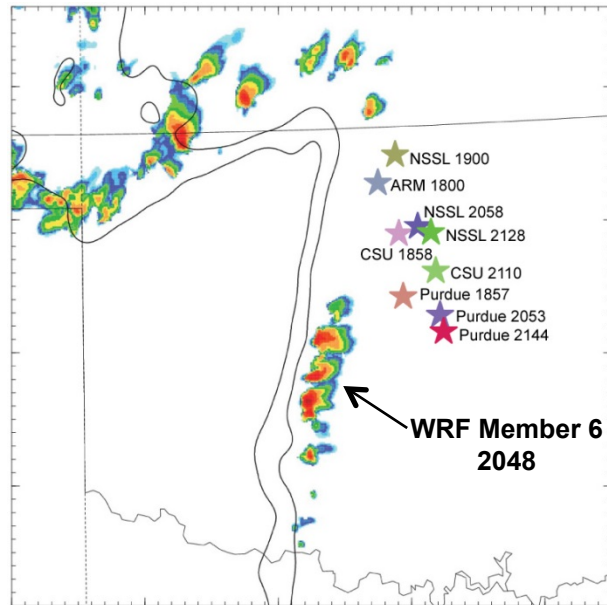
- What is correct statistical idealization for problems with multiple scales?

Transform-then-localize as underlying technique for ensemble DA in presence of range of scales

- Linear transformations for best-possible localization
- Wavelet space with multi-diagonal localization as an approximation



Thermodynamic Parameter Comparisons between WRF and MPEX Mobile Upsondes



Cases (Aksoy et al. 2009, 2010)

Chosen based on:

- Diversity of convective behavior
- Availability of nearby sounding
- Isolated storms, suitable for small domain

Storm type	Date	Location (radar)
Supercell	11 April 2005	Oklahoma (KTLX)
Line, bow echo	15 June 2002	Kansas (KGLD)
Multicell	8 May 2005	Oklahoma (KTLX)

Ensemble Initialization

ICs include random temperature perturbations

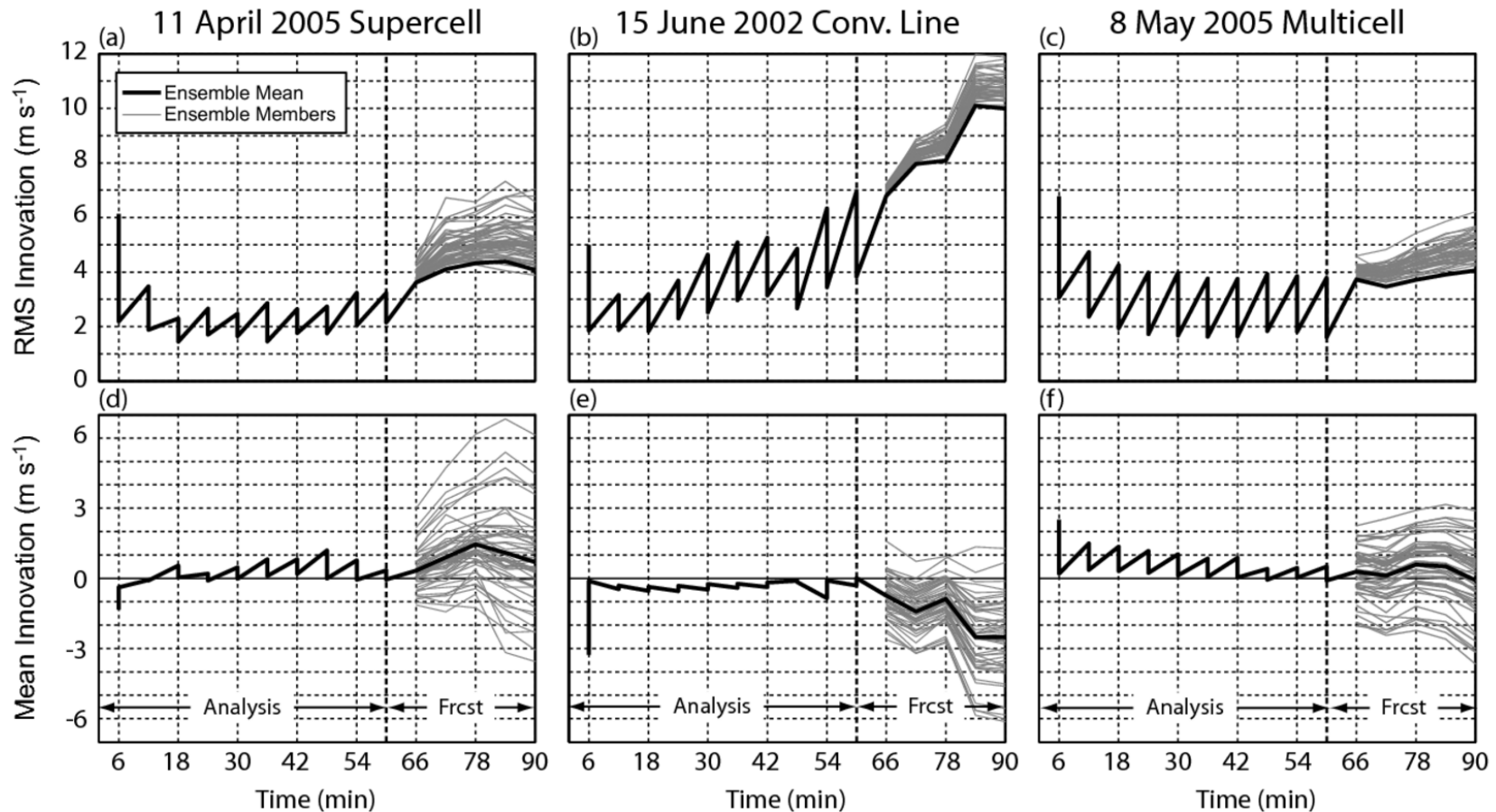
- Restricted to neighborhood of first echoes to be assimilated

Uncertainty in sounding/environment

- (u,v) sounding for each member includes noise in three gravest vertical modes, with variance 2 m/s in each mode
- At present, not perturbing T or moisture

Obs-Minus-Forecast Statistics (wind)

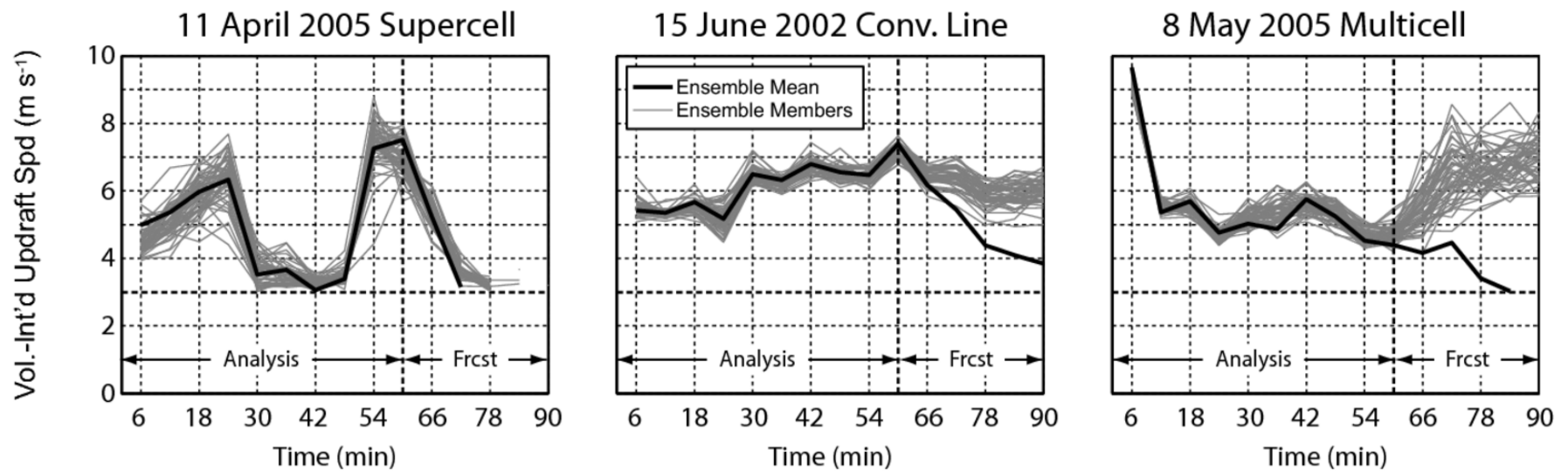
RMS: 3-6 m/s for background forecasts; area mean (bias): ~ 1 m/s



Evolution of Updraft

Does convection evolve smoothly from analysis to forecast?

- Supercell decays in forecast
- For convective line, forecast ~ steady state
- For multicell, convection intensifies and additional cells form



Aksoy et al., 2010: *Mon. Wea. Rev.*