Comparing Hybrid Data Assimilation Methods on the Lorenz 1963 Model with Increasing Nonlinearity

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Introduction

Why Hybrid Methods?

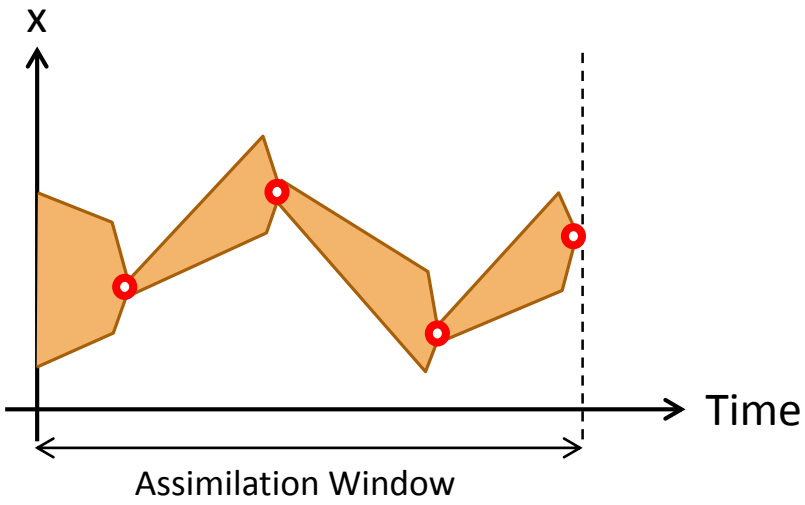
Combining Benefits of Different Methods

The Methods

Comparison of Different Methods

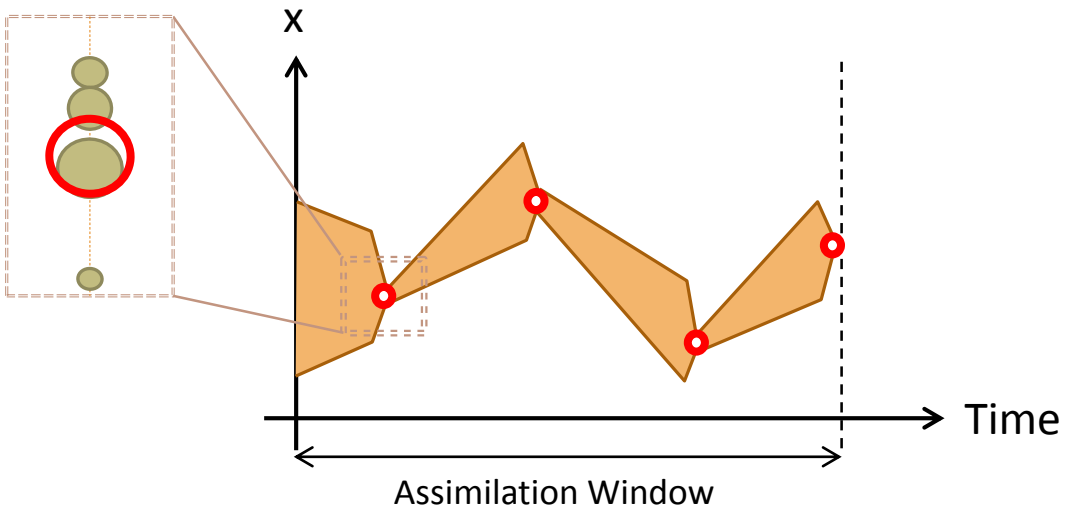
Conclusion

The Methods: EnKF



- The Ensemble Kalman Filter is a sequential data assimilation method.
- The ensemble is run forward in time until they reach an observation.
- There, linear regression is performed to produce the analysis.
- The sample covariances generated by the EnKF are flow dependent.

The Methods: ETKF

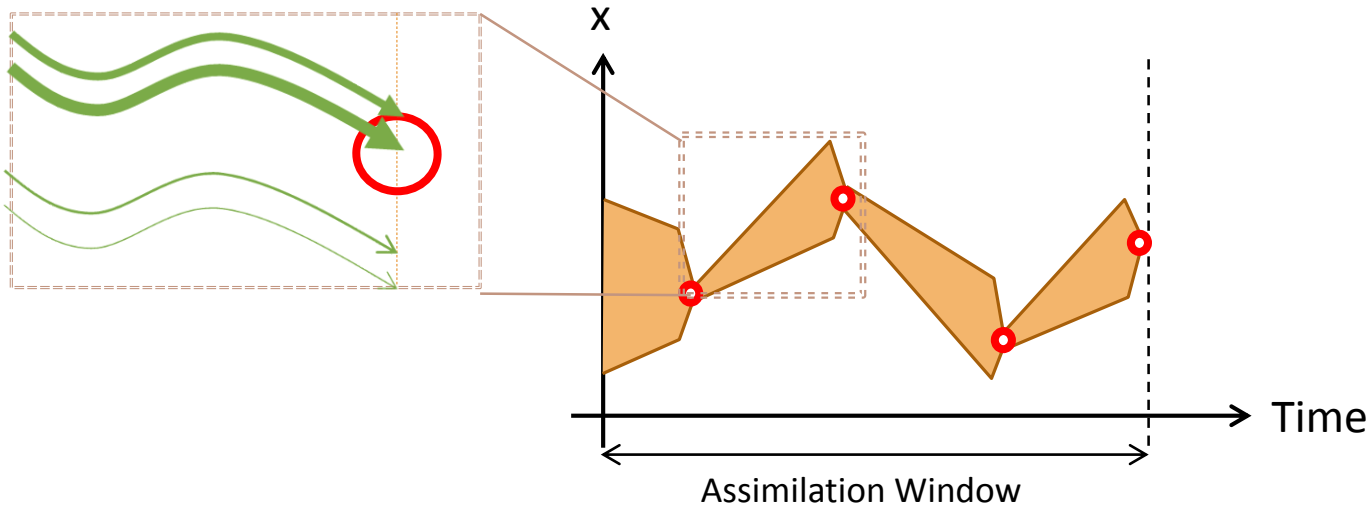


- The Ensemble Transform Kalman Filter differs from a normal EnKF in which the background ensemble of perturbations is transformed into the analysis via the equations

$$\mathbf{X}^{\mathrm{a}} = \mathbf{X}^{\mathrm{b}}\mathbf{W}^{\mathrm{a}}$$

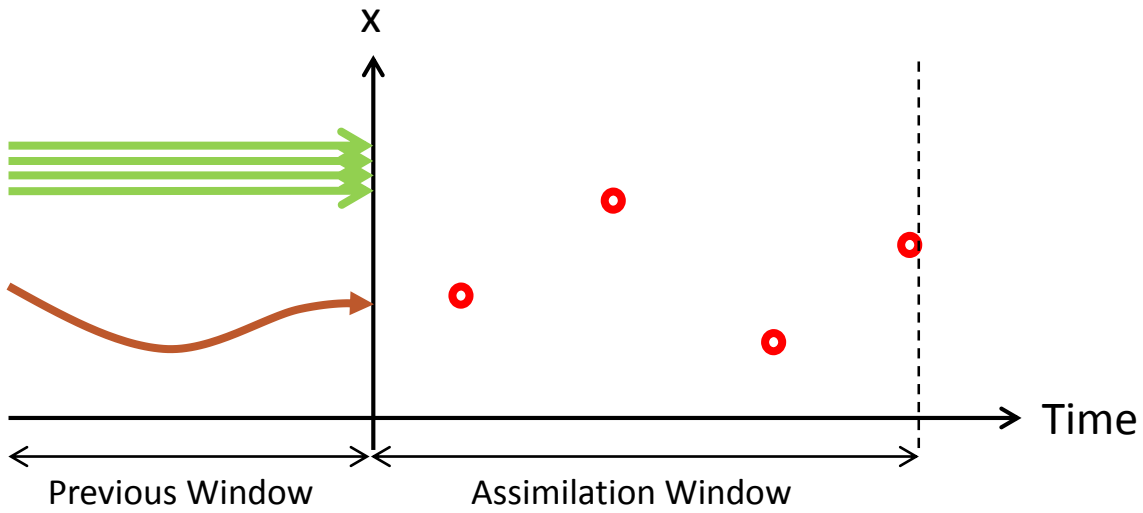
$$\bar{\mathbf{x}}^{\boldsymbol{a}} = \bar{\mathbf{x}}^{\mathbf{b}} + \mathbf{X}^{\mathbf{b}}\mathbf{w}^{\mathbf{a}}$$

The Methods: ETKS



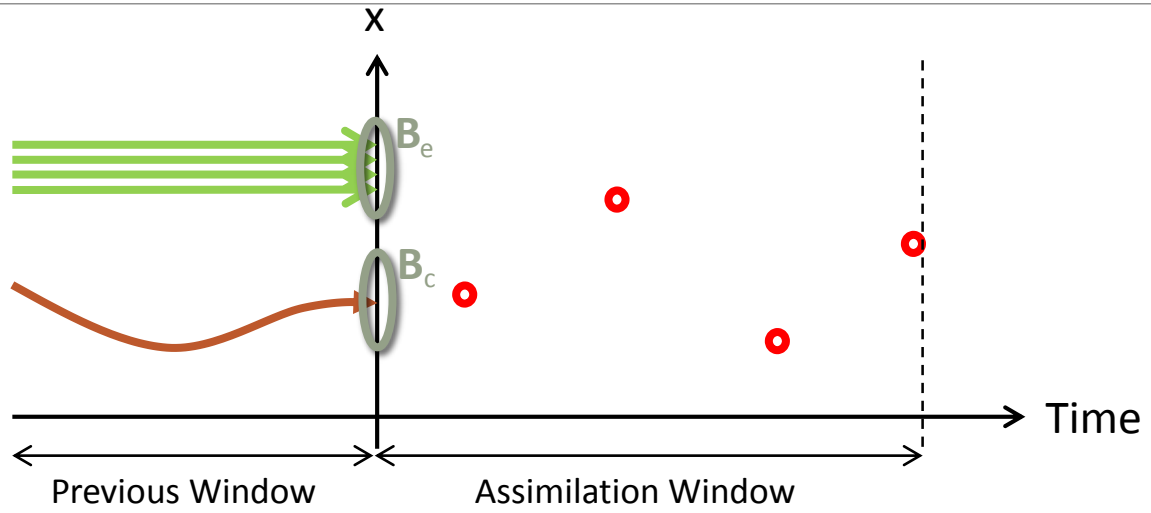
- The difference between the filter and the smoother is that the weight matrices are applied to each ensemble member between each observation and not just at observation times.

The Methods: ETKF-4DVAR



- Hybrid methods are used to combine the benefits of both variational and sequential assimilation methods.

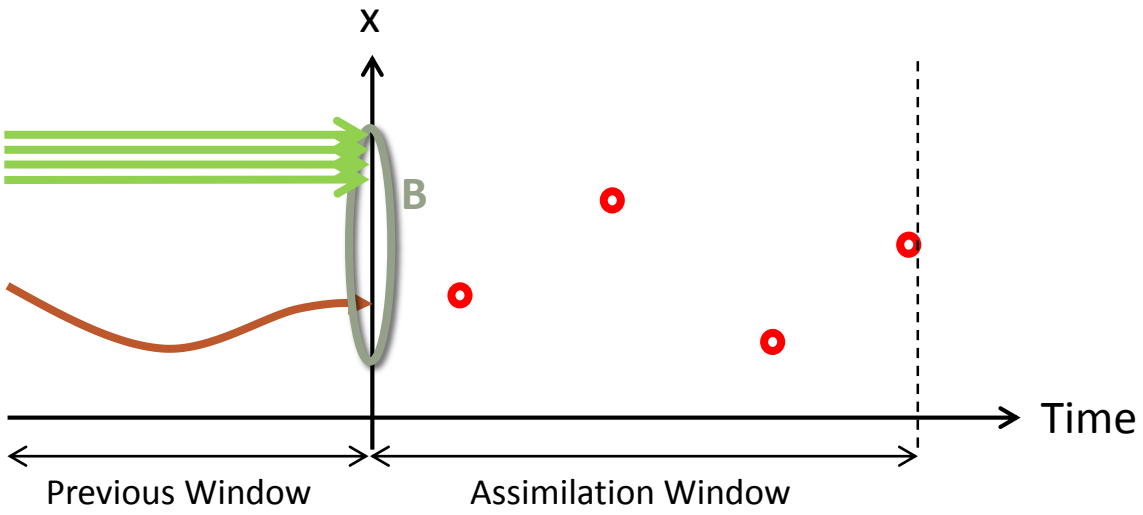
The Methods: ETKF-4DVAR



- The first examples of hybrid methods use a combination of ensemble and climatological B matrices to give an initial background error covariance matrix for the next assimilation window.

- At the end of each assimilation window, the background error covariance matrix is calculated from the ETKF.

The Methods: ETKF-4DVAR

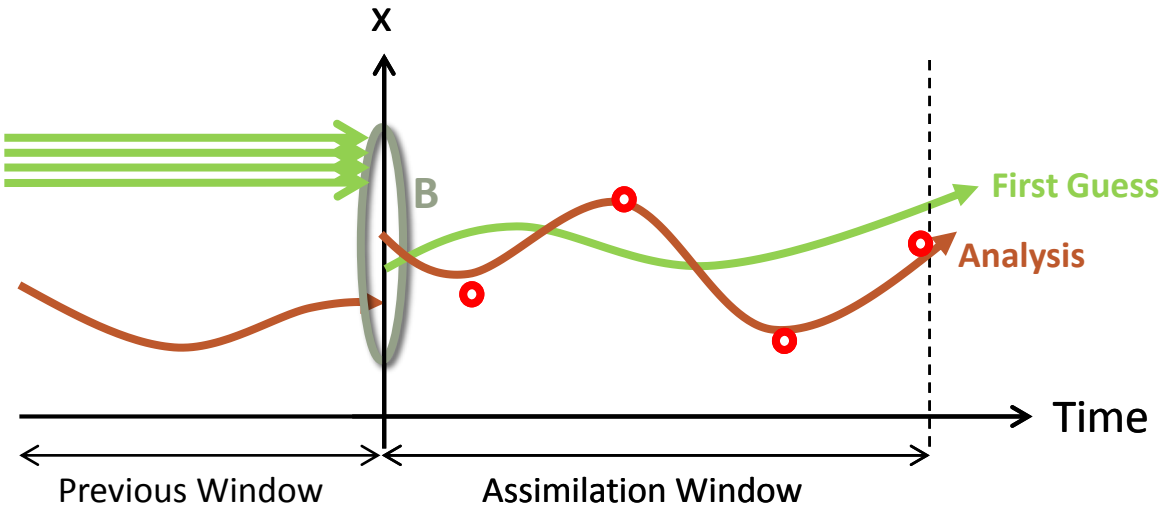


- A hybrid covariance can be constructed using the equation

$$\mathbf{B} = \beta\mathbf{B_c} + (1-\beta)\mathbf{B_e}$$

- This updated **B** matrix is then used in the current assimilation window.

The Methods: ETKF-4DVAR



$$\mathbf{B} = \beta\mathbf{B_c} + (1-\beta)\mathbf{B_e}$$

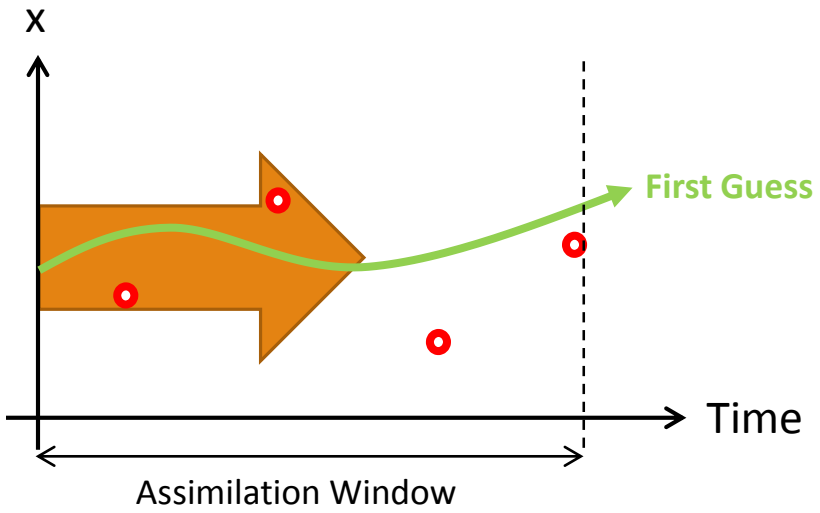
- The different values of β are used for different hybrid methods

$\beta = 0$ 4DVAR-BEN

$\beta = 0.5$ ETKF-4DVAR

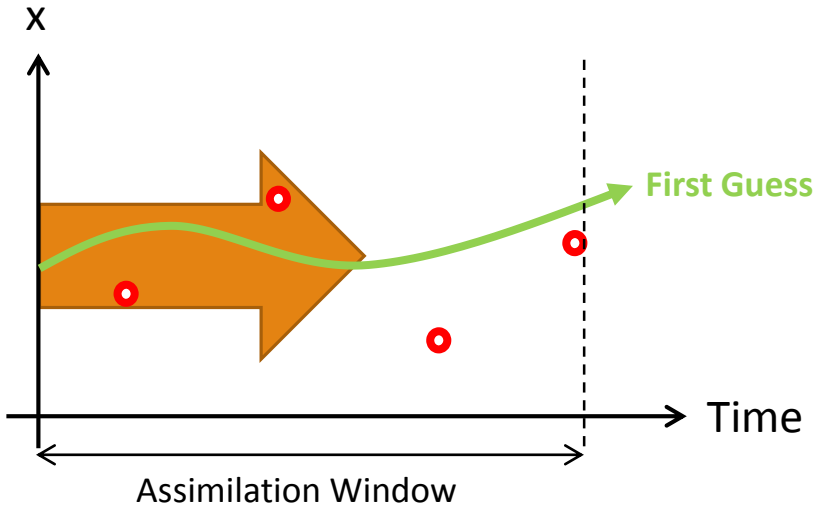
$\beta = 1$ 4DVAR

The Methods: 4DENVAR



- 4DENVAR is a variational method based on 4DVAR framework.
- First, an ensemble of model trajectories is generated and run forward in time over the assimilation window.

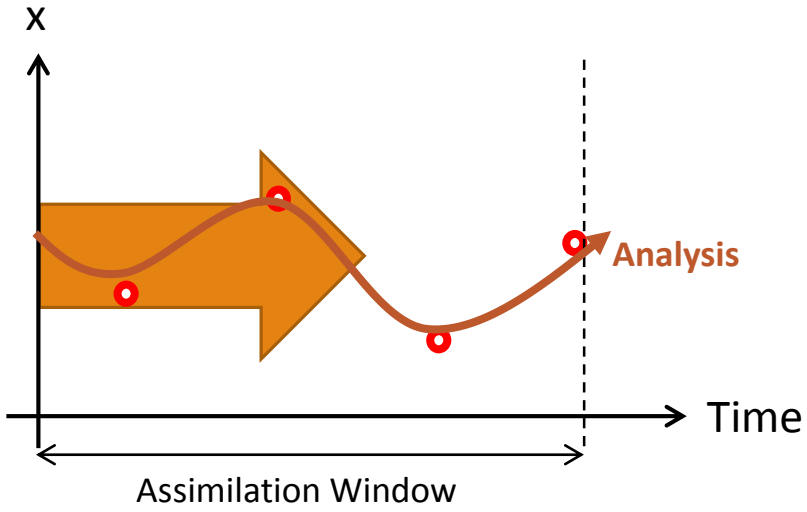
The Methods: 4DENVAR



- Using this ensemble, we can calculate cross covariances in the system.
- These cross covariances can be used to replace the adjoint model using the following transformation:

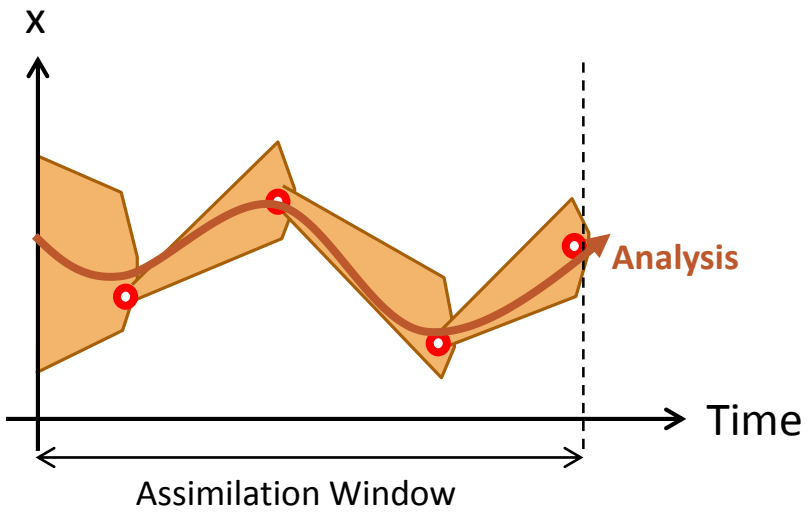
$$\nabla \mathrm{J}(\mathbf{x}_0) = (\boldsymbol{P}_0^{\mathbf{b}})^{-1}(\mathbf{x}_0 - \mathbf{x}_{\mathbf{b}}) - \sum_{\mathrm{i}=0}^{\mathrm{m}} \left(\boldsymbol{P}_0^{\boldsymbol{b}}\right)^{-1} \boldsymbol{P}_0^{\boldsymbol{b}} \boldsymbol{X}_0 (\boldsymbol{H}_{\boldsymbol{i}} \boldsymbol{X}_{\boldsymbol{i}})^{\boldsymbol{T}} \mathbf{R}_{\mathrm{i}}^{-1} \left(\mathbf{y}_{\mathrm{i}} - \mathbf{h}_{\mathrm{i}}(\mathbf{M}_{\mathrm{i}}(\mathbf{x}_0))\right) = 0$$

The Methods: 4DENVAR



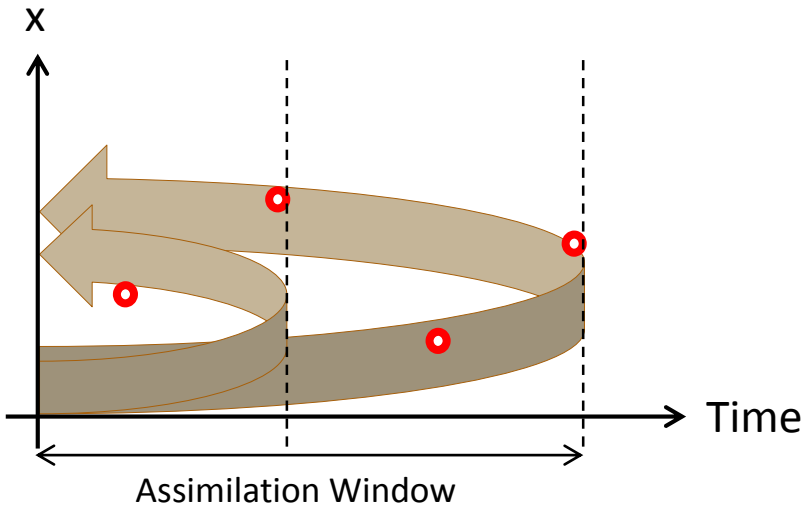
- Solving the gradient equation gives us the analysis trajectory.

The Methods: 4DENVAR



- Solving the gradient equation gives us the analysis trajectory.
- Using this analysis, we run an ETKF over the analysis window.
- This gives us an ensemble spread using P^a at this end of this assimilation window. We use this for our model trajectory ensemble spread at the beginning of the next assimilation window.

The Methods: QVSA

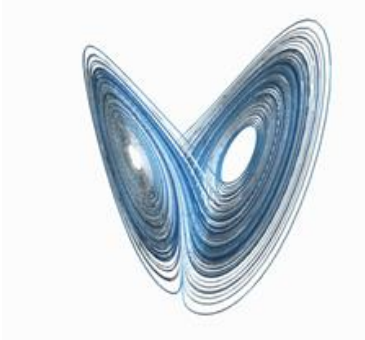


- Quasi variational static assimilation is an advanced method of finding the global minimum of the cost function.

- An assimilation window is split into smaller sections. We first minimise over the first section, giving us an approximate initial state. Next, we minimise over the first two sections using the approximate initial state as our first guess.

Comparison of Different Methods

Lorenz 1963 model

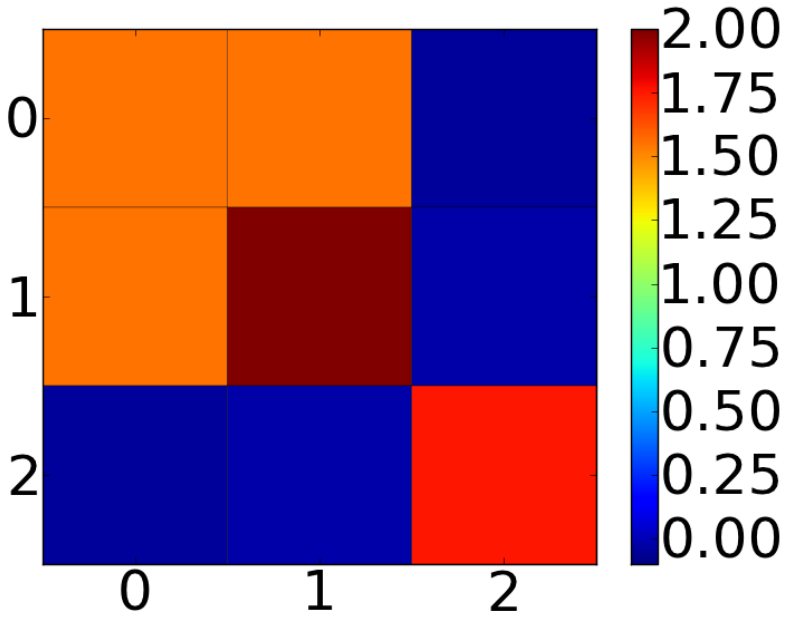


Compare the RMSE (analysis) for different observation numbers.

- Comparison of different hybrids methods with increasing nonlinearity
- 20 Ensemble Members with inflation
- Observation Variance = 1
- Over 10,000 Assimilation Windows

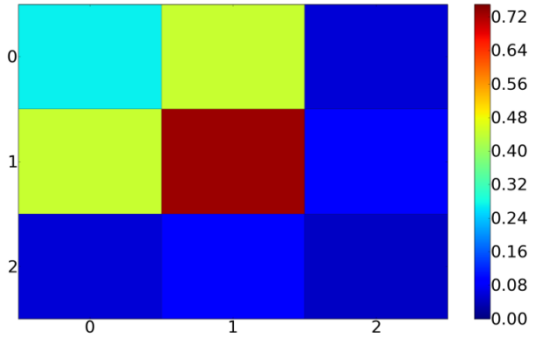
$\mathbf{B_c}$ vs $\mathbf{B_e}$

$\mathbf{B_c}$

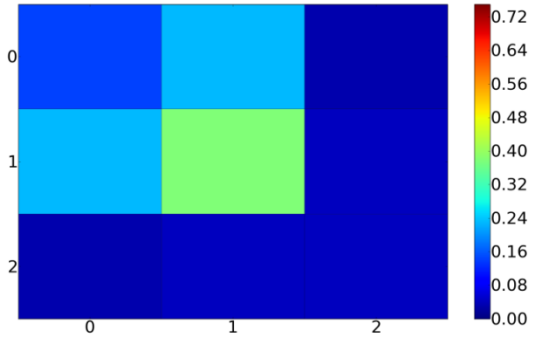


$\mathbf{B}_e$

Before

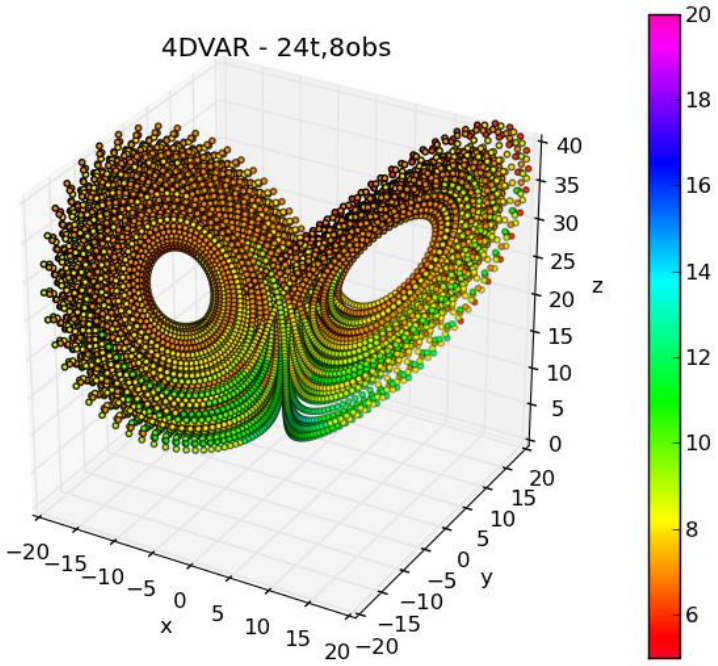


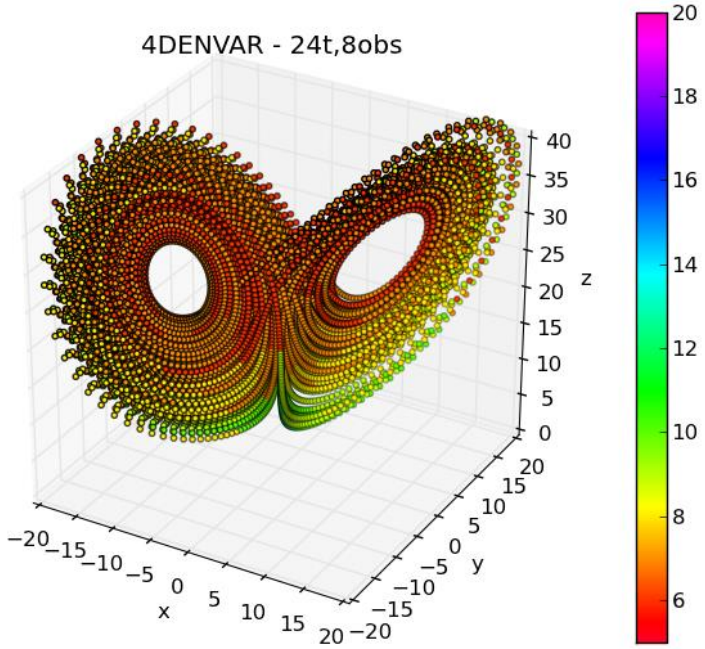
After



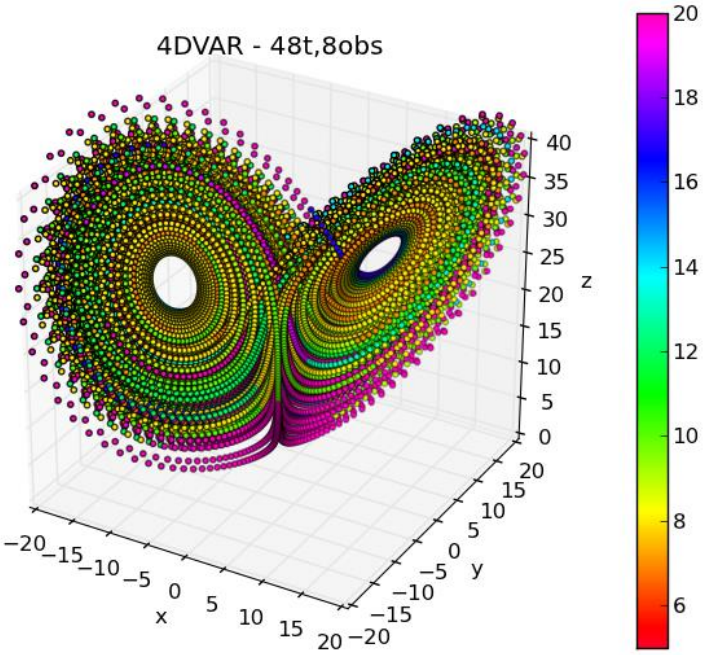
Background error covariance matrices for Lorenz 1963

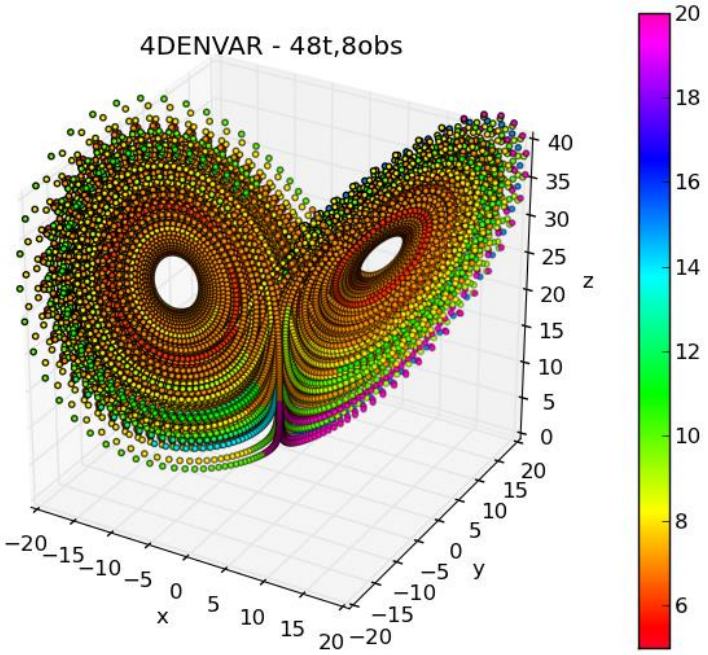
4DENVAR and 4DVAR minimisation cost.



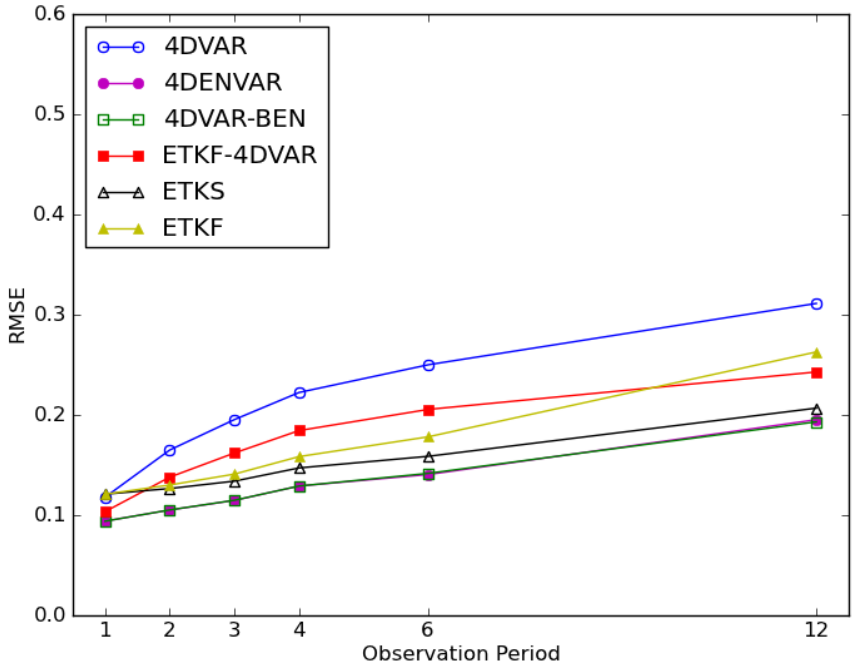


4DENVAR and 4DVAR minimisation cost.

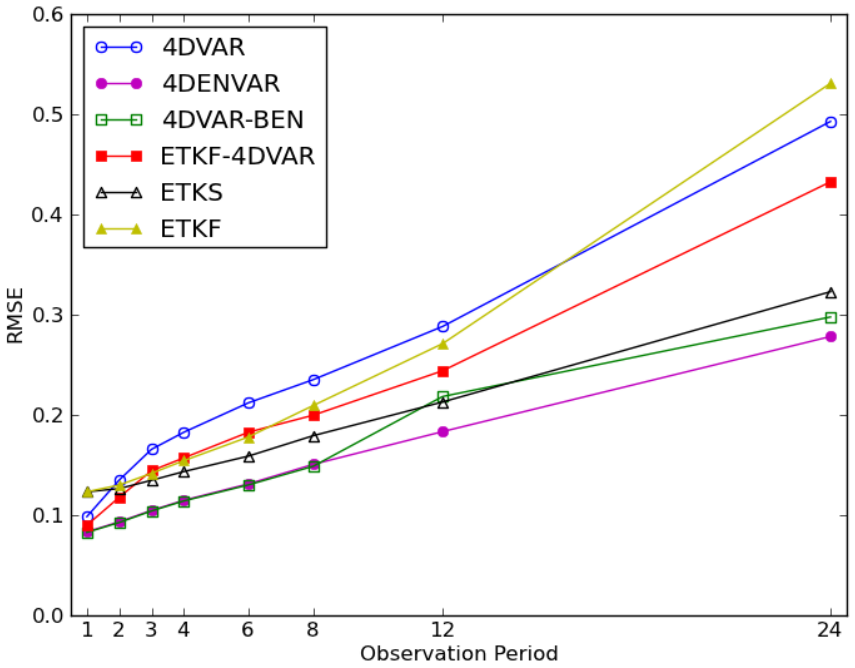




Comparison of Different Methods

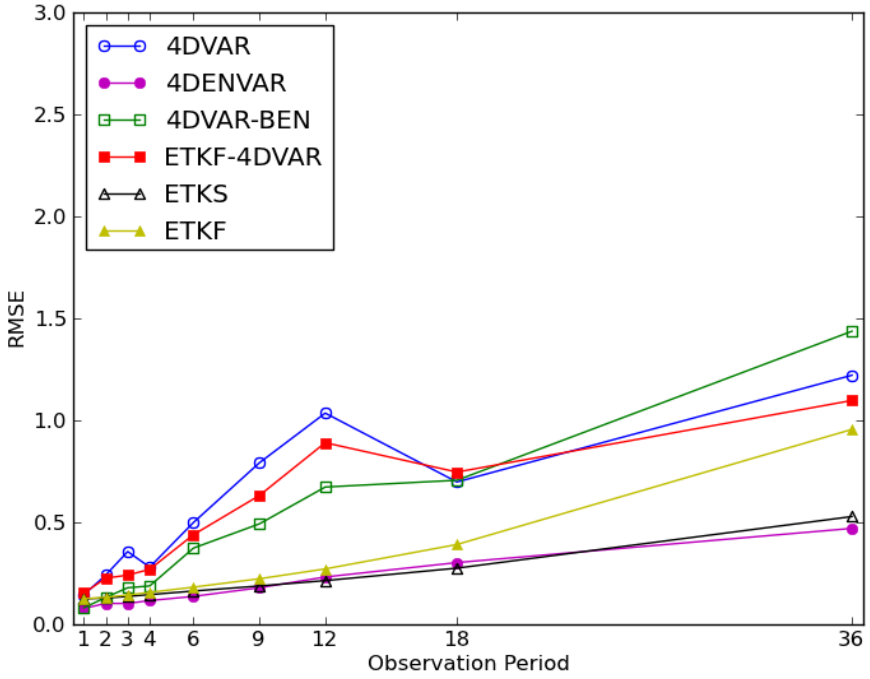


Window length: 12

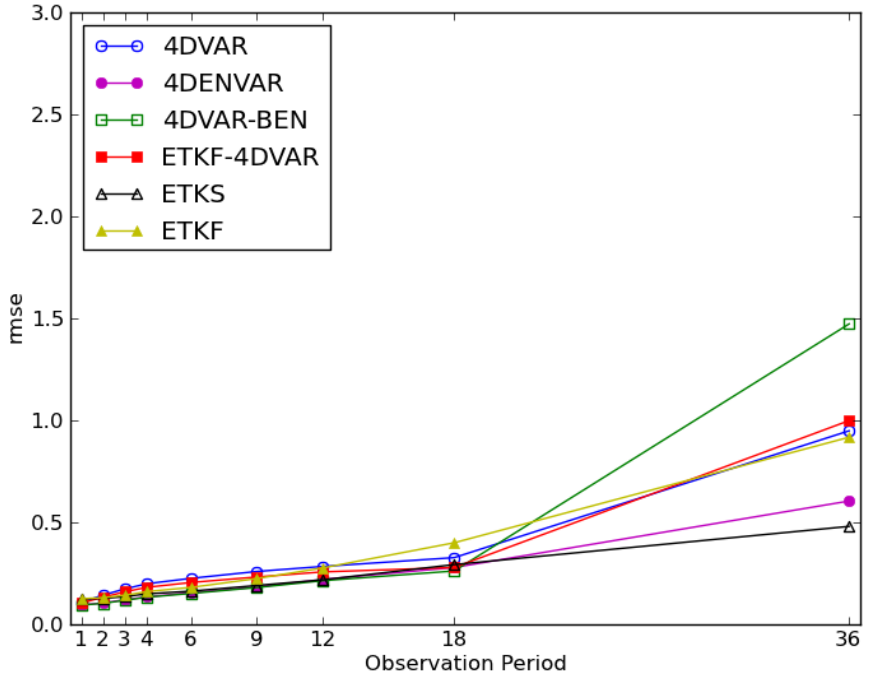


Window length: 24

Comparison of Different Methods

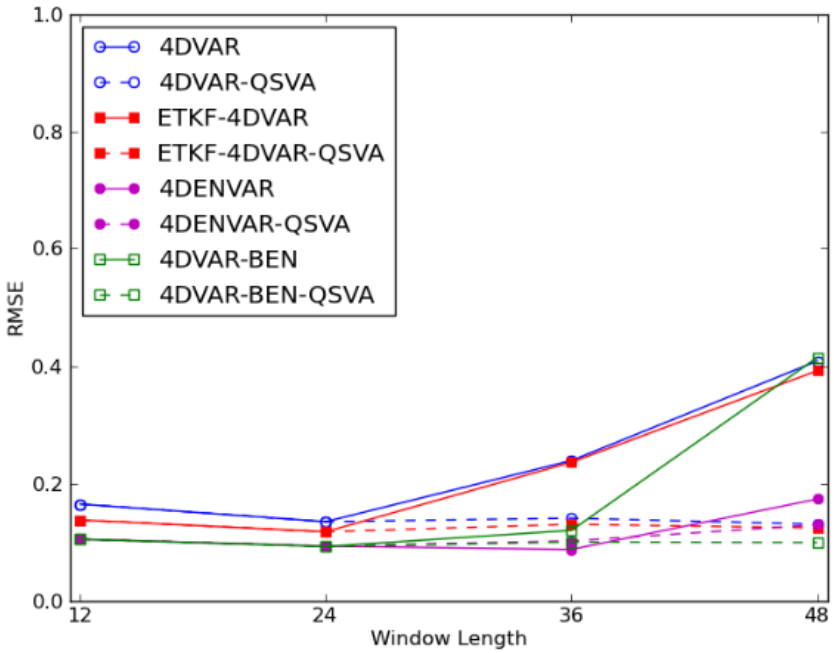


Window length: 36

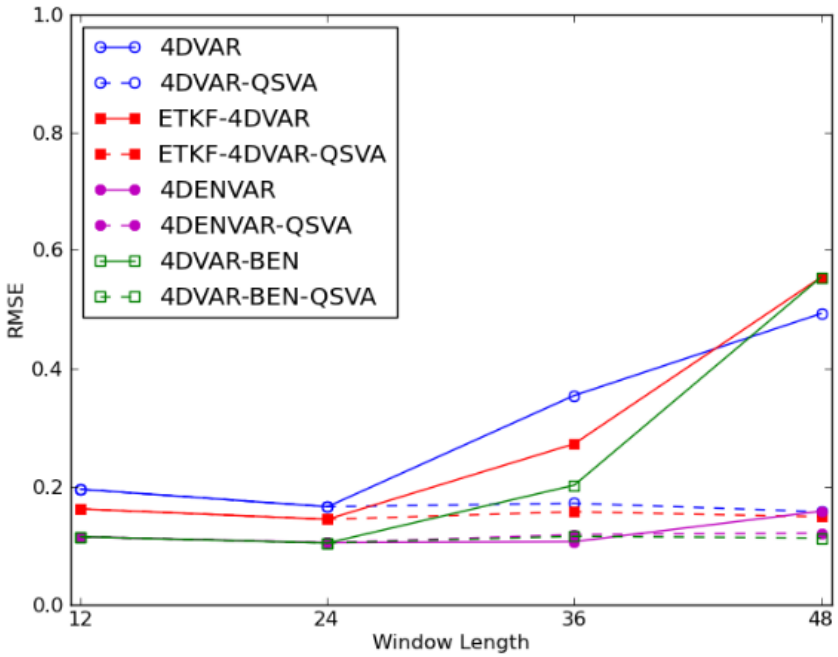


Window length: 36 + QVSA

Comparison of Different Methods

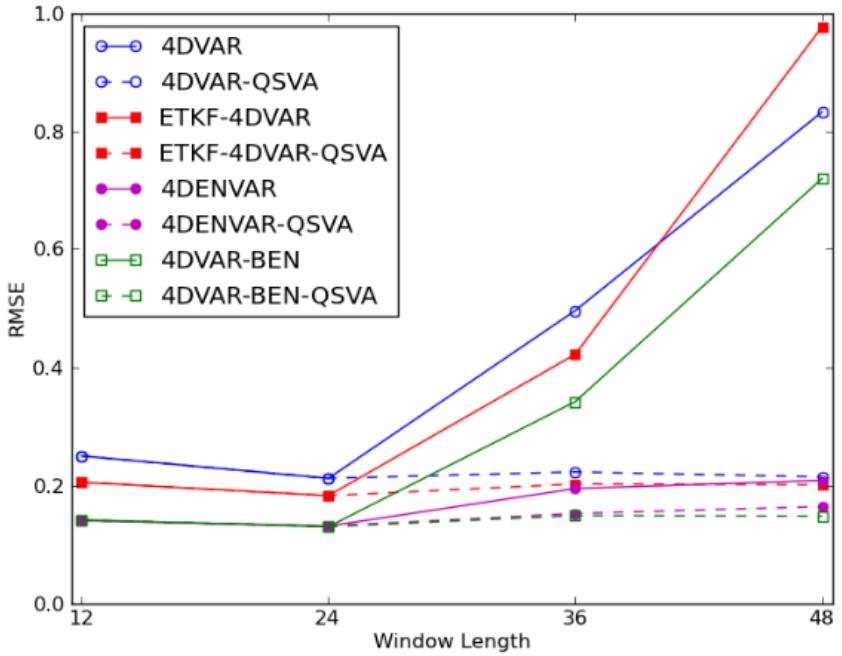


Observation Period: 2

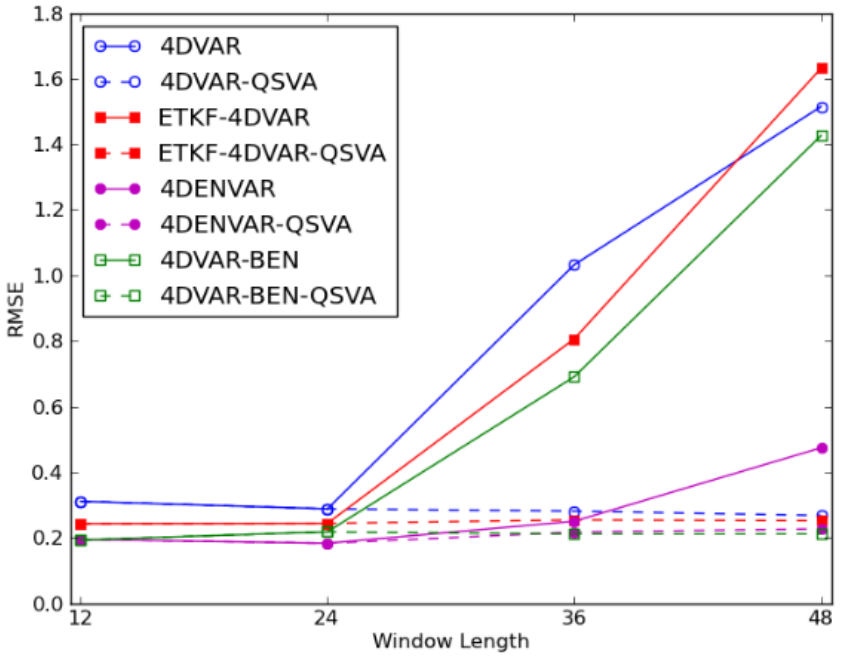


Observation Period: 3

Comparison of Different Methods



Observation Period: 6



Observation Period: 12

Conclusion

- For short assimilation windows, hybrid methods are shown to outperform traditional methods.
- As the assimilation window length increases, sequential methods become more accurate over both traditional variational and hybrid methods which use an adjoint model.
- The 4DENVAR method performs slightly better in most cases than the ETKF over shorter assimilation windows, which suggests that replacing the adjoint model by 4D-covariances from sequential methods can increase the accuracy of variational methods.
- As most forecasting centres use variational methods for NWP, we show that increasing window length may not necessarily increase the accuracy of the assimilation.