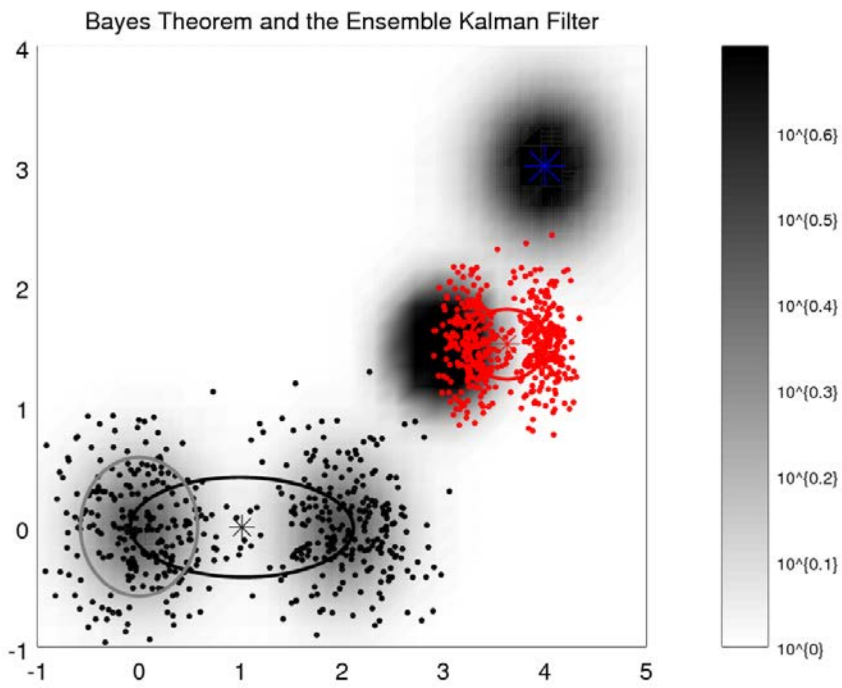
Deutscher Wetterdienst
Wetter und Klima aus einer Hand

On Ensemble and Particle Filters for Numerical Weather Prediction



Roland Potthast
Hendrik Reich, Christoph Schraff
Andreas Rhodin, Ana Fernandez
Alexander Cress, Dora Foring
Annika Schomburg, Africa Perianez
Jason Otkin, Robin Faulwetter
Daniel Leuenberger, Hans Rüdi Künsch

DWD

Contents

- **Ensemble Systems at Deutscher Wetterdienst (DWD)**
  - **I DWD Research Environment**
  - **II VarEnKF and III KENDA (Kilometer Scale EDA)**
- **Research Projects based on Ensemble Data Assimilation**
- **IV Particle Filters for NWP**
  - **Local Markov Chain Particle Filter**

Part I: Research Environment

- **Deutscher Wetterdienst (DWD) is the national weather service of Germany**

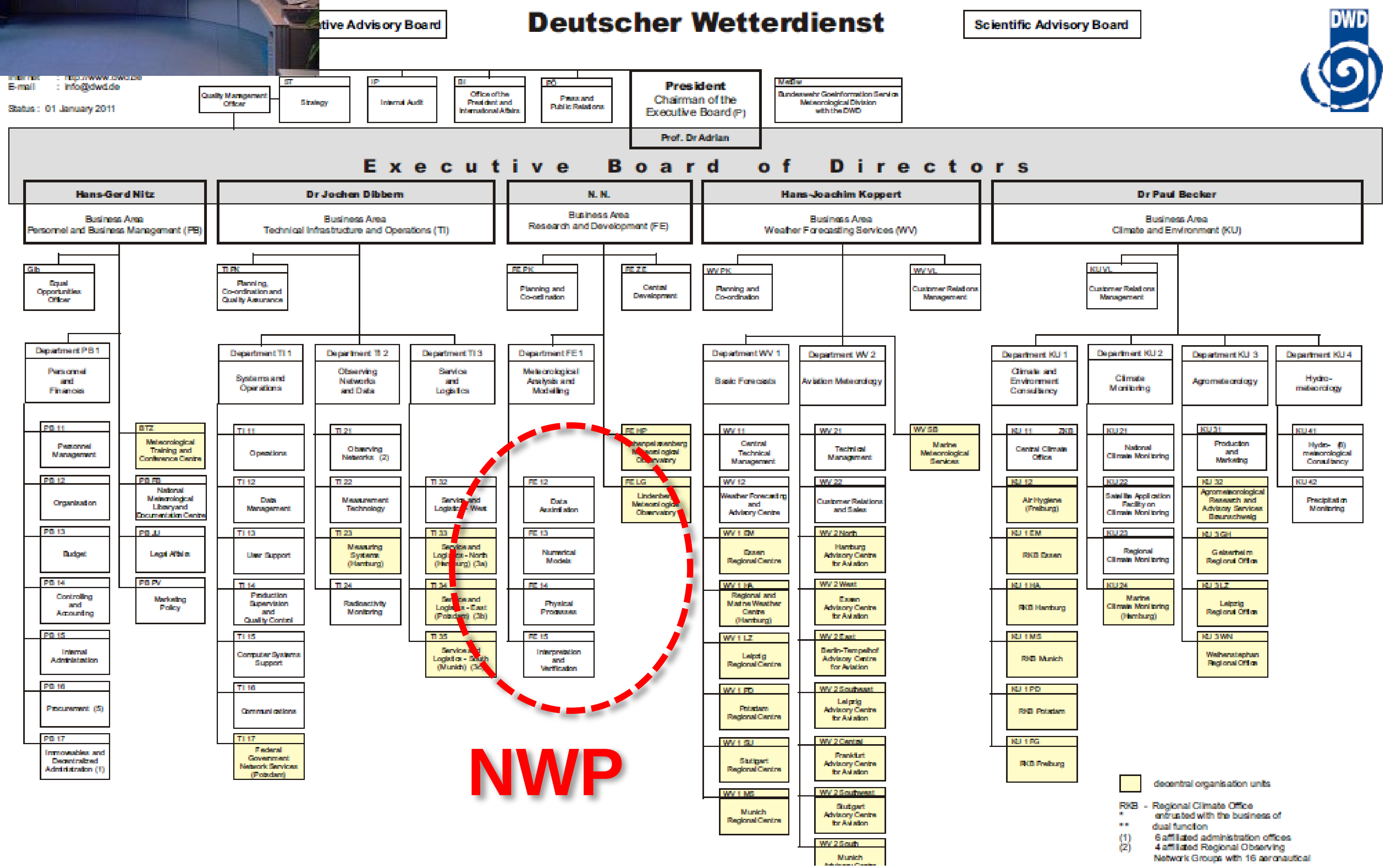
- **We are a part of the Ministery of Transport and Digital Infrastructure**

- **National and European Measurement Networks (e.g. Satellites) are controled and developed**

- **We develop our systems within a network of partnerships with other states, with research institutes and universities**

Federal Ministery BMVI

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



Deutscher Wetterdienst DWD

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

Weather Prediction and Warnings for Central Europe.

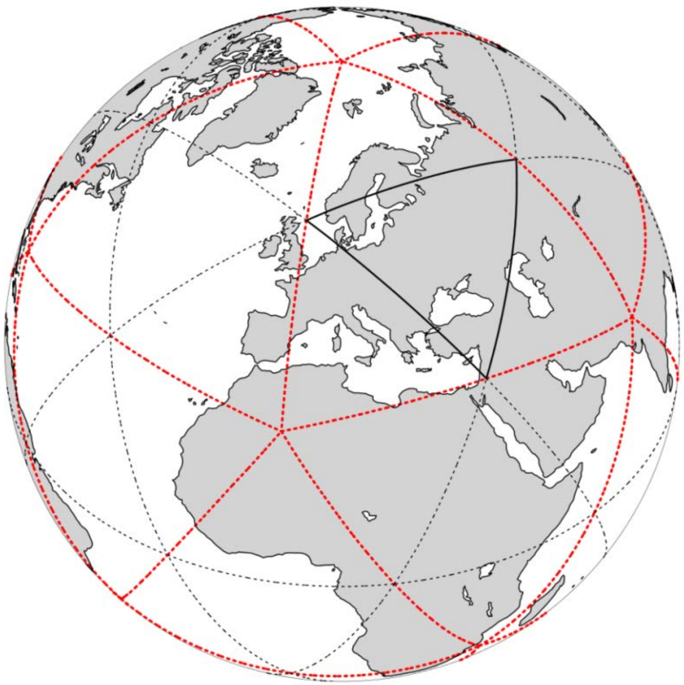
DWD operates two
CRAY Supercomputers

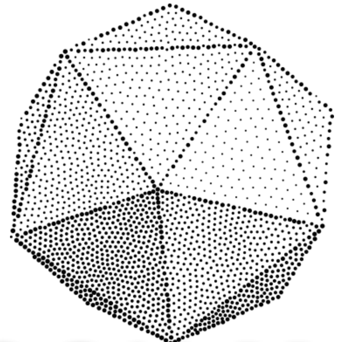
Cray XC40, Intel Xeon E5-2670v2 10C 2.5GHz/E5-2680v3 12C 2.5Ghz, Aries interconnect,

Rank 128 on the TOP 500 supercomputer list in 2014/11.

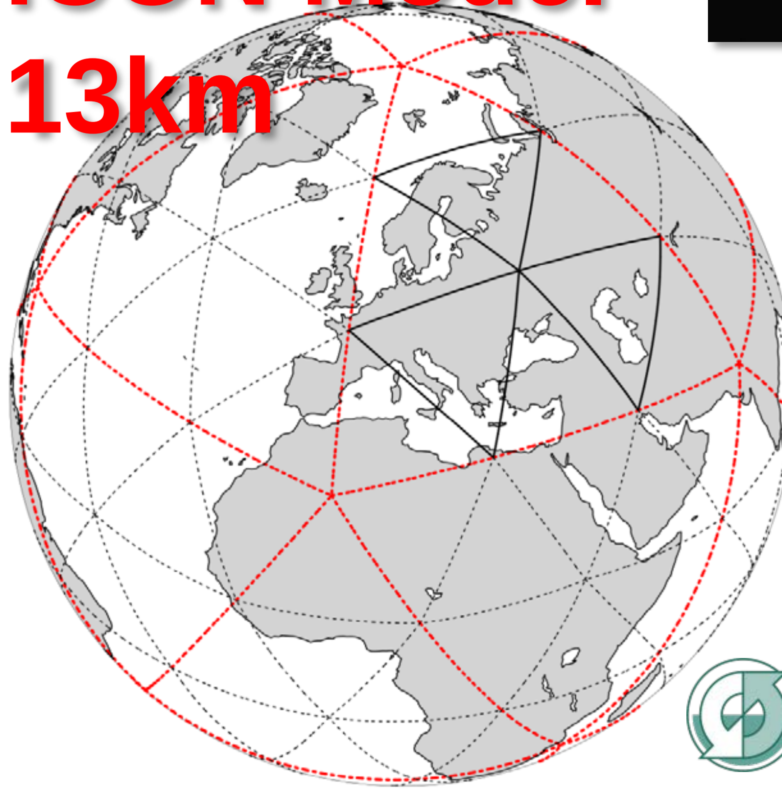
Part I: Model Chain

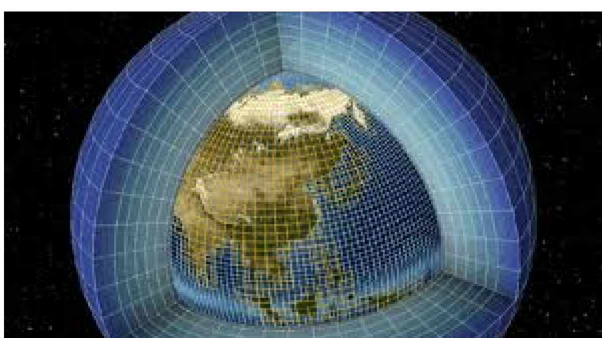
Deutscher Wetterdienst
Wetter und Klima aus einer Hand





ICON Model 13km





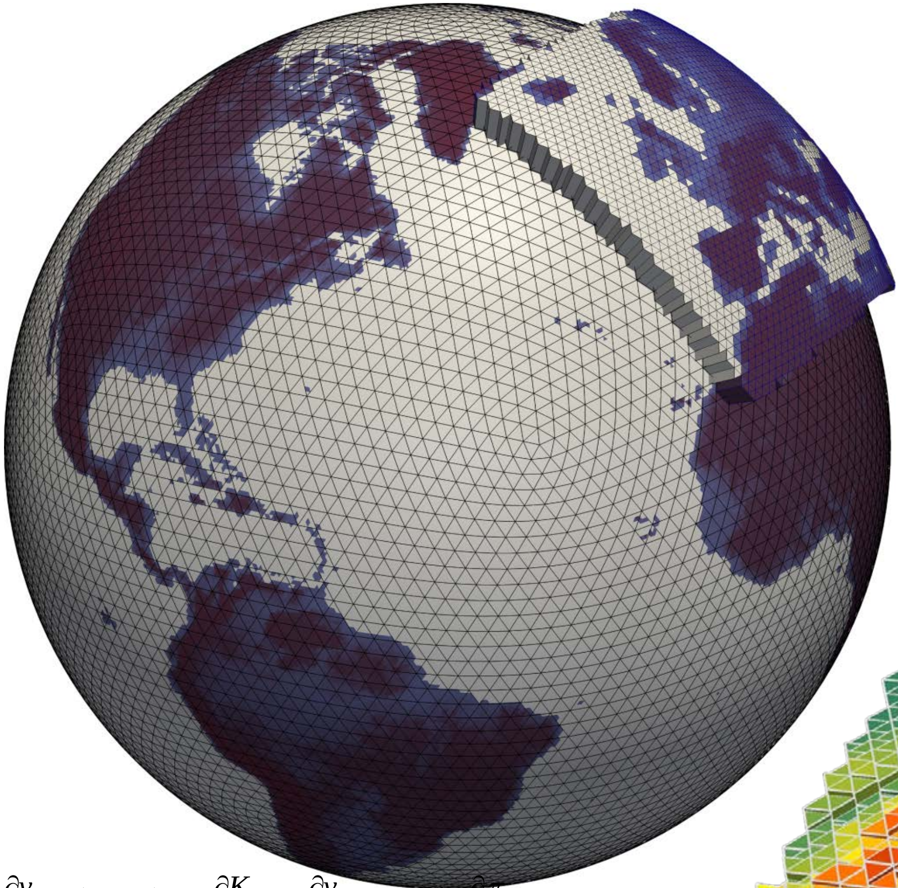
Karlsruhe Institute of Technology

Max-Planck-Institut für Meteorologie

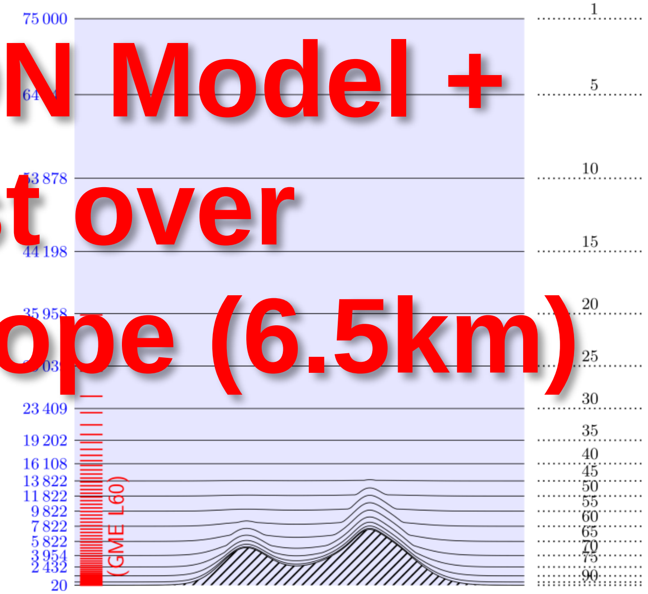
0-7 Days

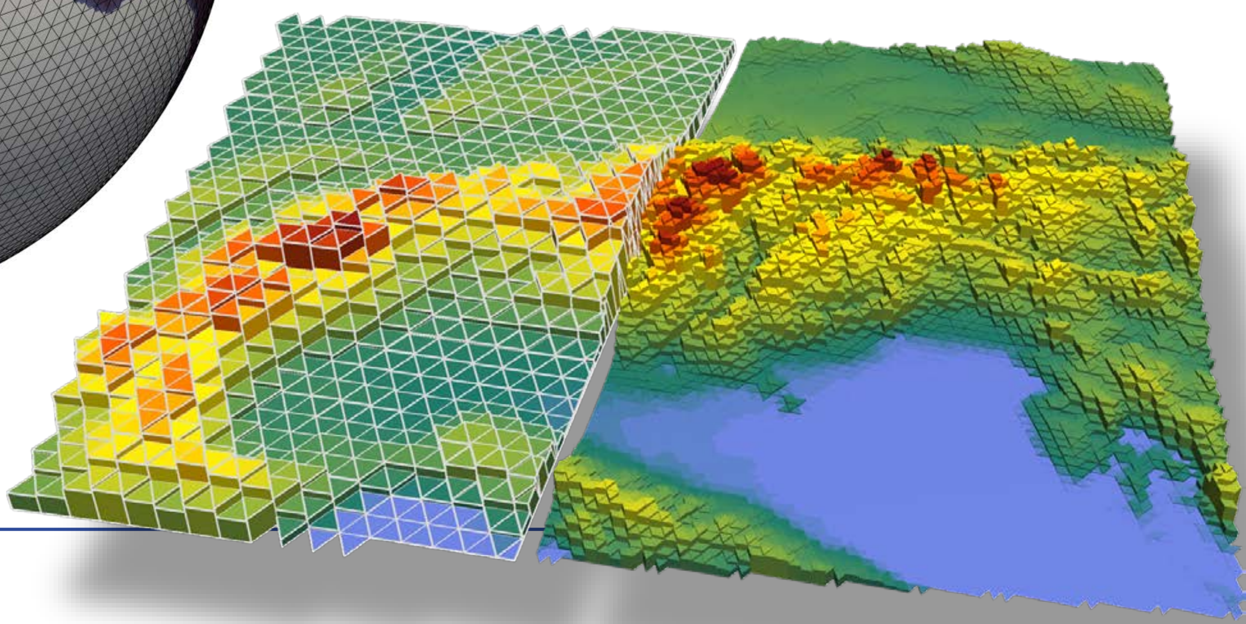
Global Modelling

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



ICON Model + Nest over Europe (6.5km)





$$\frac{\partial v_n}{\partial t} + (\zeta + f)v_t + \frac{\partial K}{\partial n} + w\frac{\partial v_n}{\partial z} = -c_{pd}\theta_v \frac{\partial \pi}{\partial n}$$

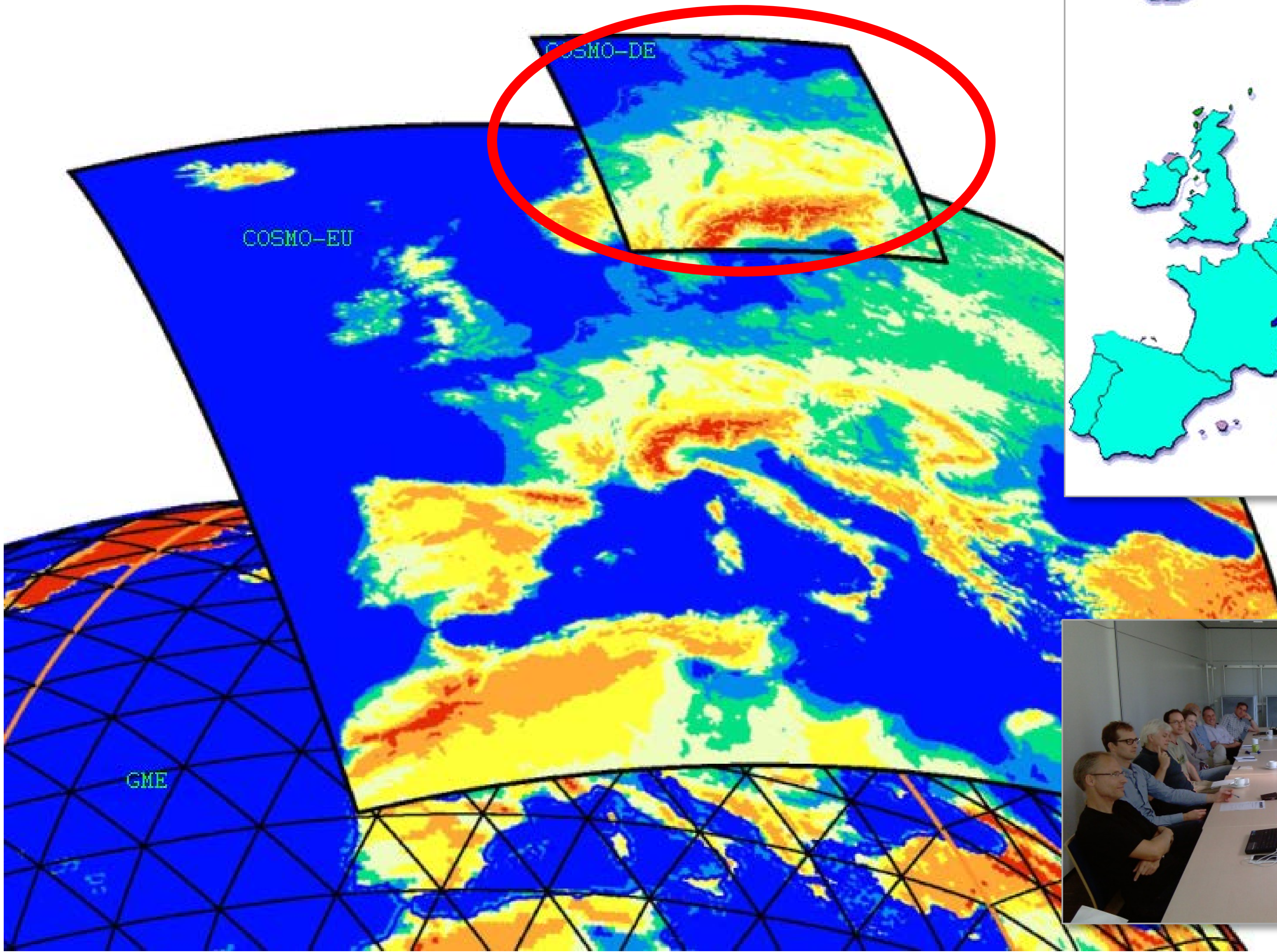
$$\frac{\partial w}{\partial t} + \vec{v}_h \cdot \nabla w + w\frac{\partial w}{\partial z} = -c_{pd}\theta_v \frac{\partial \pi}{\partial z} - g$$

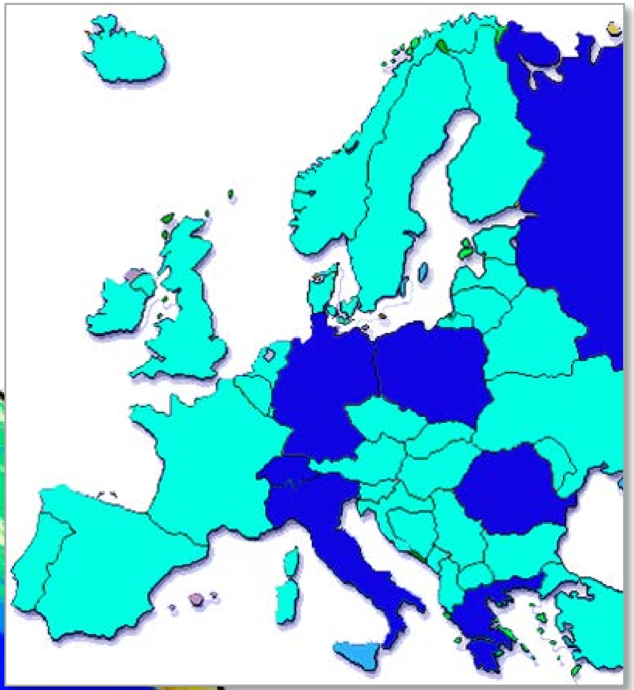
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\vec{v}\rho) = 0$$

$$\frac{\partial \rho\theta_v}{\partial t} + \nabla \cdot (\vec{v}\rho\theta_v) = 0$$

02-24 Hours

High Resolution Modelling





02-24 Hours

High Resolution Modelling

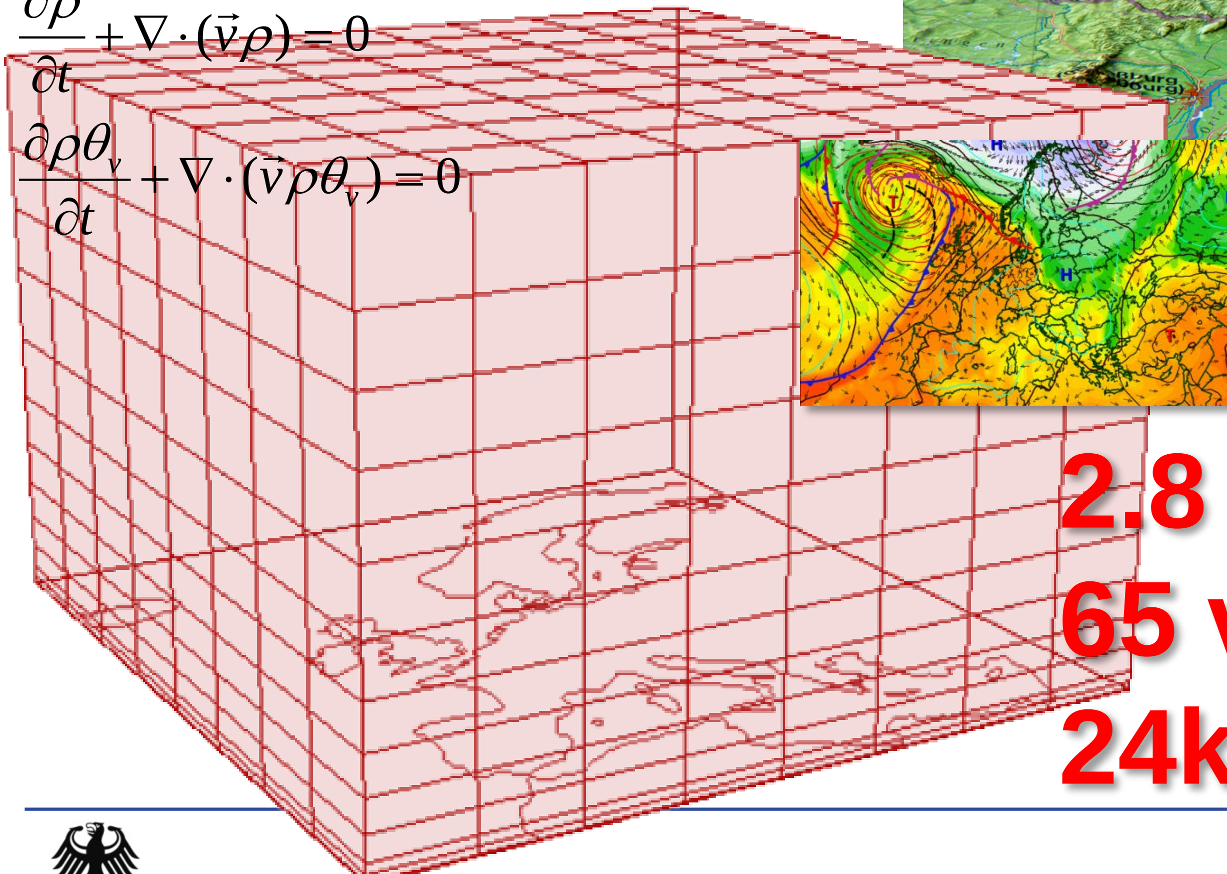
Deutscher Wetterdienst
Wetter und Klima aus einer Hand

$$\frac{\partial v_n}{\partial t} + (\zeta + f)v_t + \frac{\partial K}{\partial n} + w\frac{\partial v_n}{\partial z} = -c_{pd}\theta_v \frac{\partial \pi}{\partial n}$$

$$\frac{\partial w}{\partial t} + \vec{v}_h \cdot \nabla w + w\frac{\partial w}{\partial z} = -c_{pd}\theta_v \frac{\partial \pi}{\partial z} - g$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\vec{v}\rho) = 0$$

$$\frac{\partial \rho\theta_v}{\partial t} + \nabla \cdot (\vec{v}\rho\theta_v) = 0$$



2.8 (2.2 or 1)km
65 vertial layers
24km height

Part I: Research Network

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

HErZ
Hans Ertel Center

COSMO
International consortium

BMVI
Ministery Projects

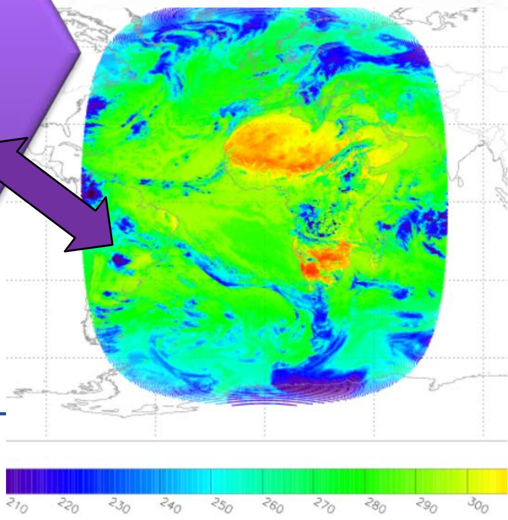
DWD

EMF
University research

IAFE
Innovation programme

Further Partners

Bundesministerium für Verkehr und digitale Infrastruktur



CONSORTIUM FOR SMALL SCALE MODELING

COSMO

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

DWD

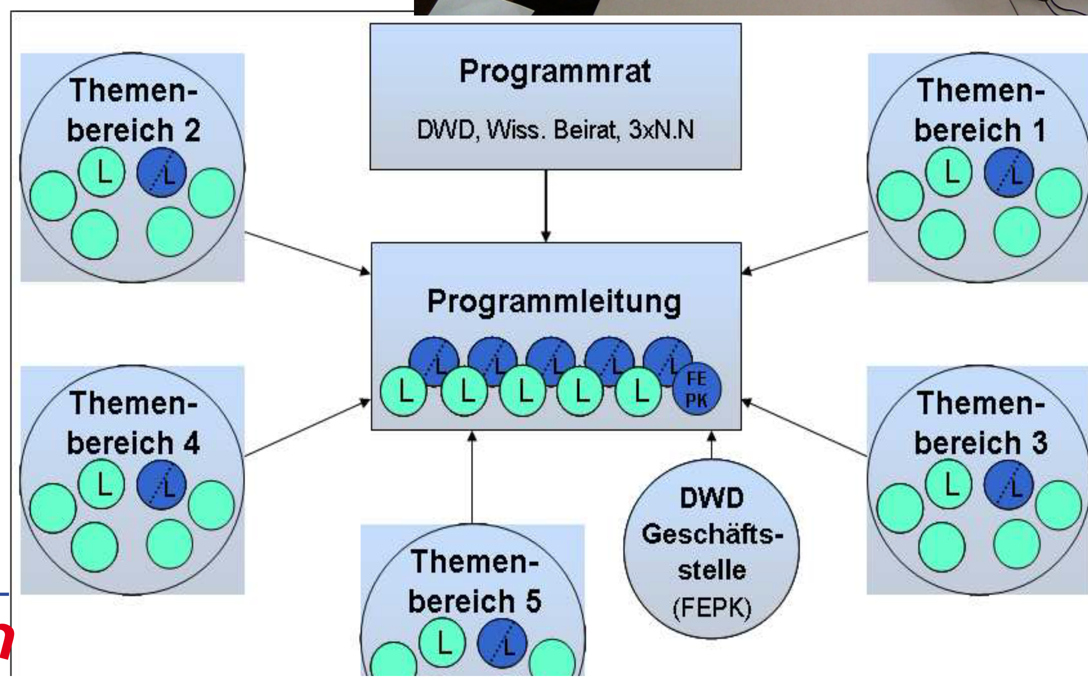
40 Countries

Cosmo User Seminar + Training Course + Symposium

Research Network 2

Hans Ertel Center for Weather Research (HErZ)

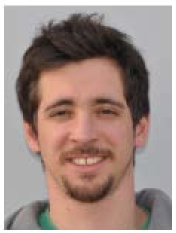
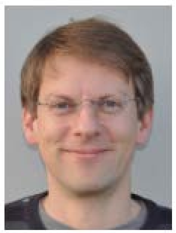
- ➔ MPI Hamburg
- ➔ LMU Munich
- ➔ Uni Bonn
- ➔ Uni Frankfurt
- ➔ Uni Berlin



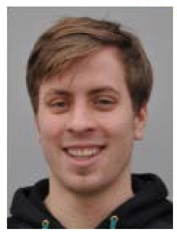
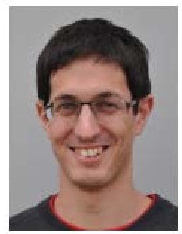
SEE Talk of Martin Weissmann

Research in HErZ-DA and associated projects

Subproject A: Assimilation of potential high-impact observations

HErZ funded

HErZ associated

Enviro… condit… of con…
- MSG … chann…
- GNNS total delay (vert. integrated water vapour)
- Novel MODE-S aircraft wind and temp. obs.

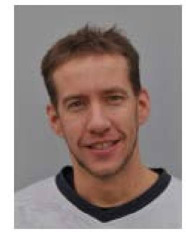
… of convection)
- w2W project on error sources

… precipitating systems)

Black: Proposed
Green: Externally funded complementary projects

…n issues
…s
… biases
…s
…mulations
…ping
- Feature-based scores
- Different assimilation settings
- Operator refinement
- Model error repr. *from B*

Subproject B: Accounting for model error in DA



HErZ finanziert

Post-Doc
Conservation in DA

HErZ associated

Work on stochastic perturbation schemes (LMU and w2W)

Subproject C: Predictability and ensemble generation

PhD
Repr. of model error, predictability

PhD
Observation impact

HErZ funded

SEE Talk of Martin Weissmann

visualization of uncertainty in 3D data

HErZ associated

related model errors

Subproject D: AMV height assignment

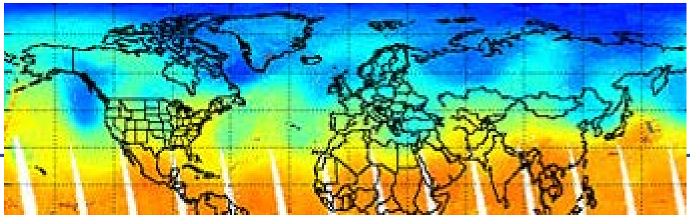
HErZ funded (2015)

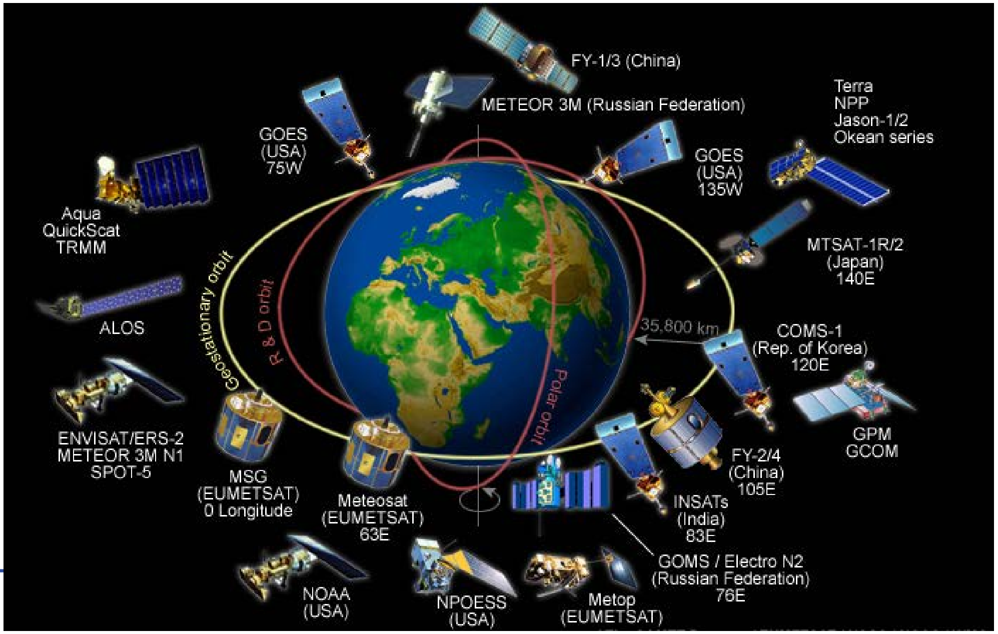
…LIPSO
…day

Research Network 4:

IAFE Innovation in Applied Research and Development

Research and Development in Data Assimilation of Satellite Data





Innovation Programme (IAFE) +Eumetsat Positions Survey

Please Apply !!!

Nr	Position	Length in Years
1	SAT-GNSS-HUM	4
2	SAT-IR-ICON 1	4
3	SAT-IR-ICON 2	2 (+2 tbc)
4	SAT-IR-KENDA	4
5	SAT-MW-ICON 1	2 (+2 tbc)
6	SAT-MW-ICON 2	1+3
7	EUMETSAT Fellowship	1+2

Part II: New Global Model ICON

- **Operational since Jan 20, 2015**

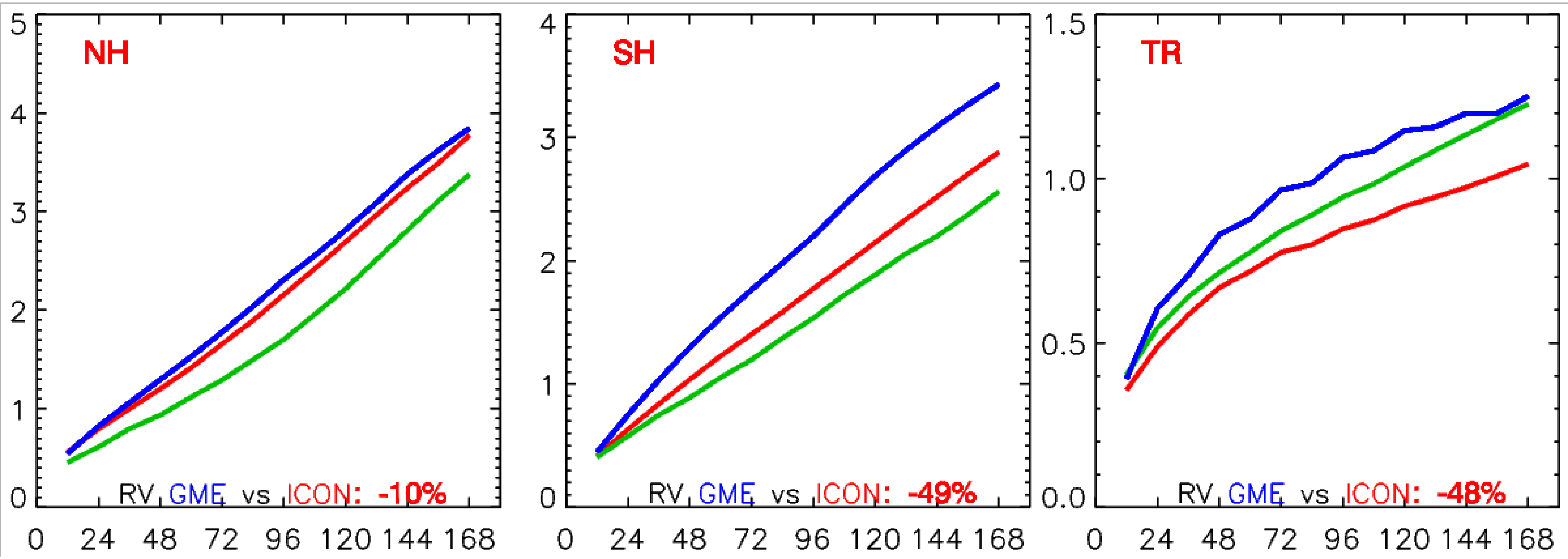
- **Non-Hydrostatic**
- **13km resolution globally**
- **75km height**
- **90 vertical levels**

- **Still a lot in the pipeline!!**

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

Temperatur in 700 hPa, RMSE in K

Blau: GME, rot: ICON, grün: IFS



Deutscher Wetterdienst
Wetter und Klima aus einer Hand

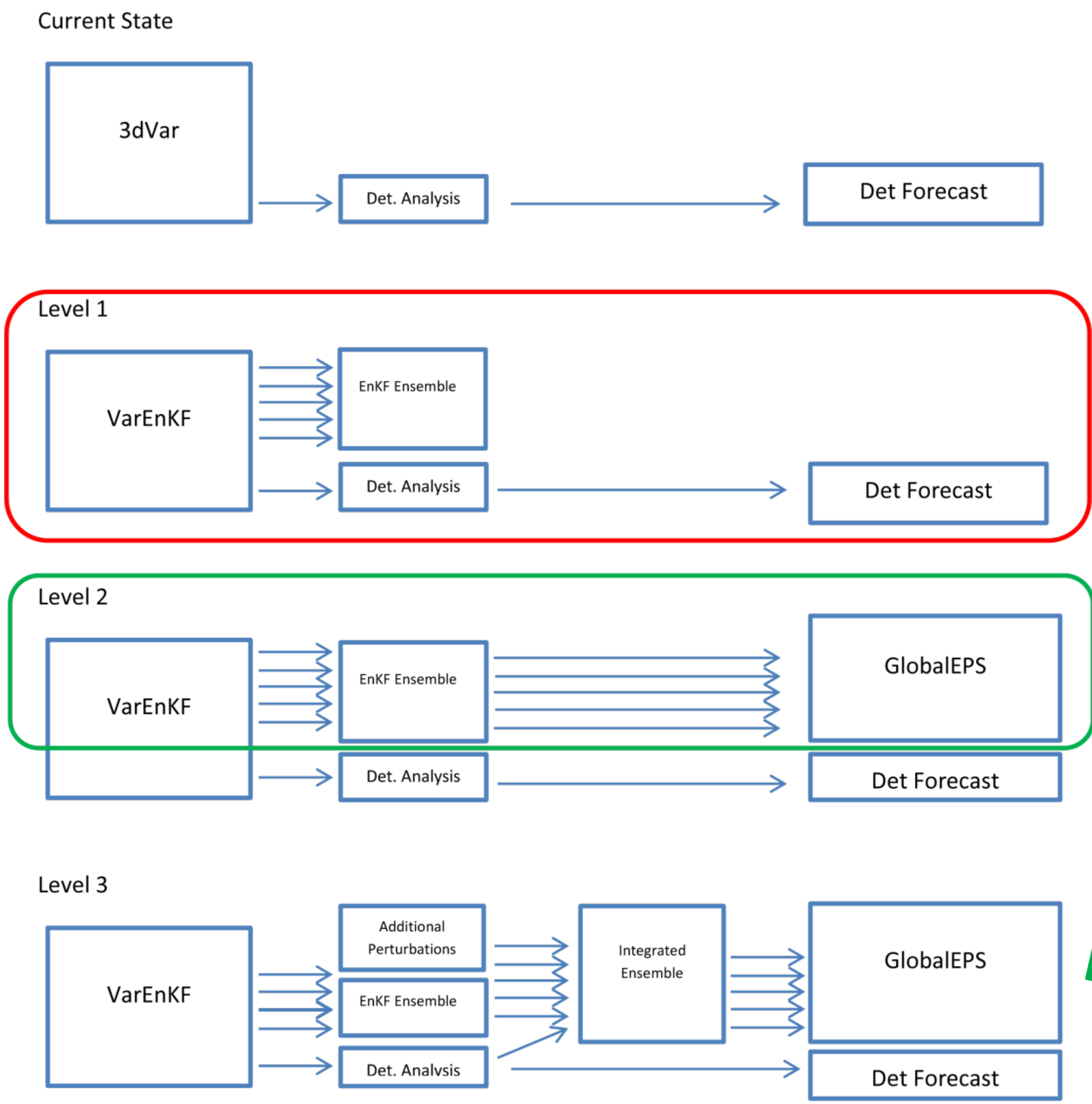
SEE Talk of
Andreas Rhodin /
Ana Fernandez

Part II: Global
Ensemble Data Assimilation (EDA)

- VarEnKF
- 40 Members
- 1 Deterministic

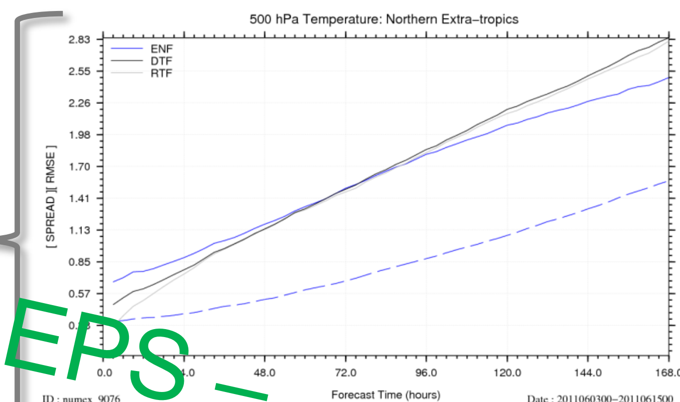
Global EDA (VarEnKF) Development

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



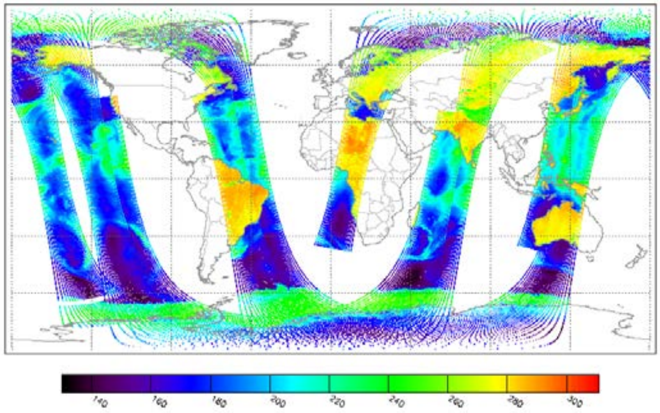
SEE Talk of Andreas Rhodin / Ana Fernandez

Deterministic Forecast



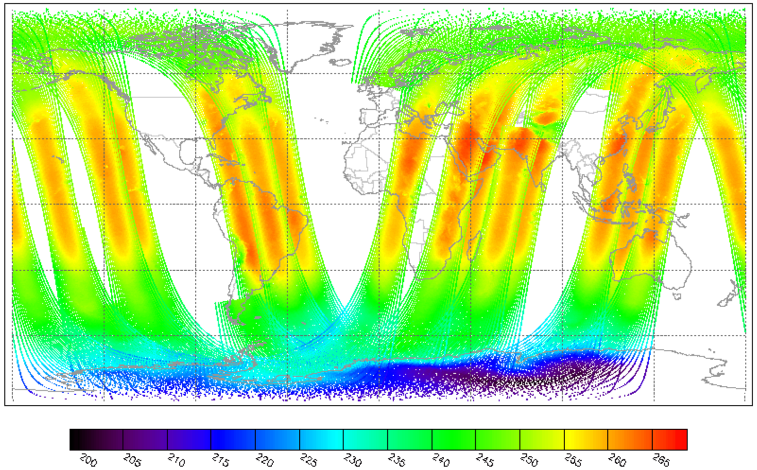
EPS – Ensemble Prediction

Global EnKF + EPS for ICON, VarEnKF



1. **Full System** with all current observation systems ***running*** *in BACY experiments (80/80, 80/40km)*

2. Currently: verification against own analysis **better or comparable** to current 3DVAR system



3. Work in progress on **spread** in different regions (upper troposphere, Europe, …)

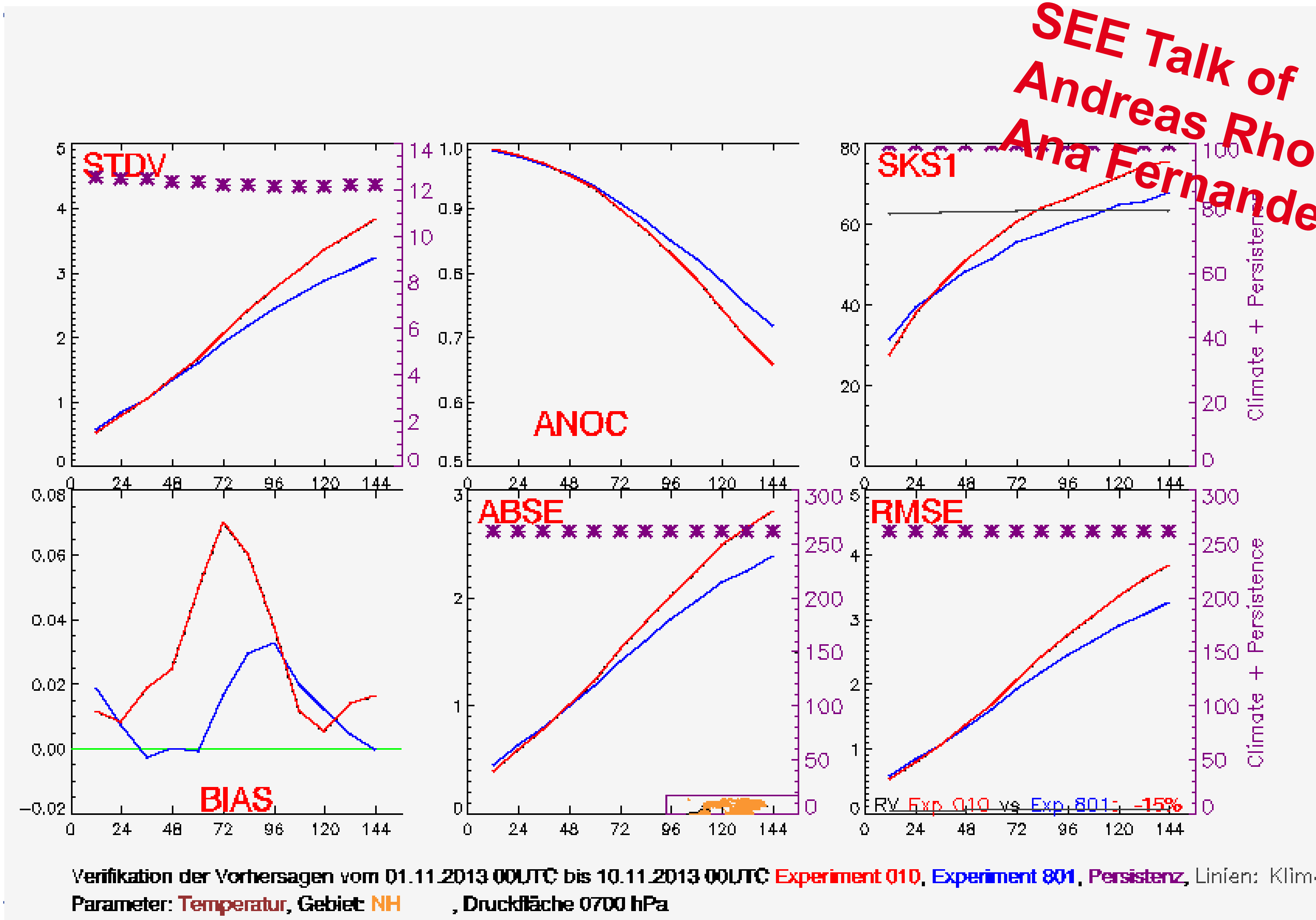
4. Adaptive localization **calibration** is ongoing

5. **Archive/Storage** challenges remain severe

Global EDA (VarEnKF) Development

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

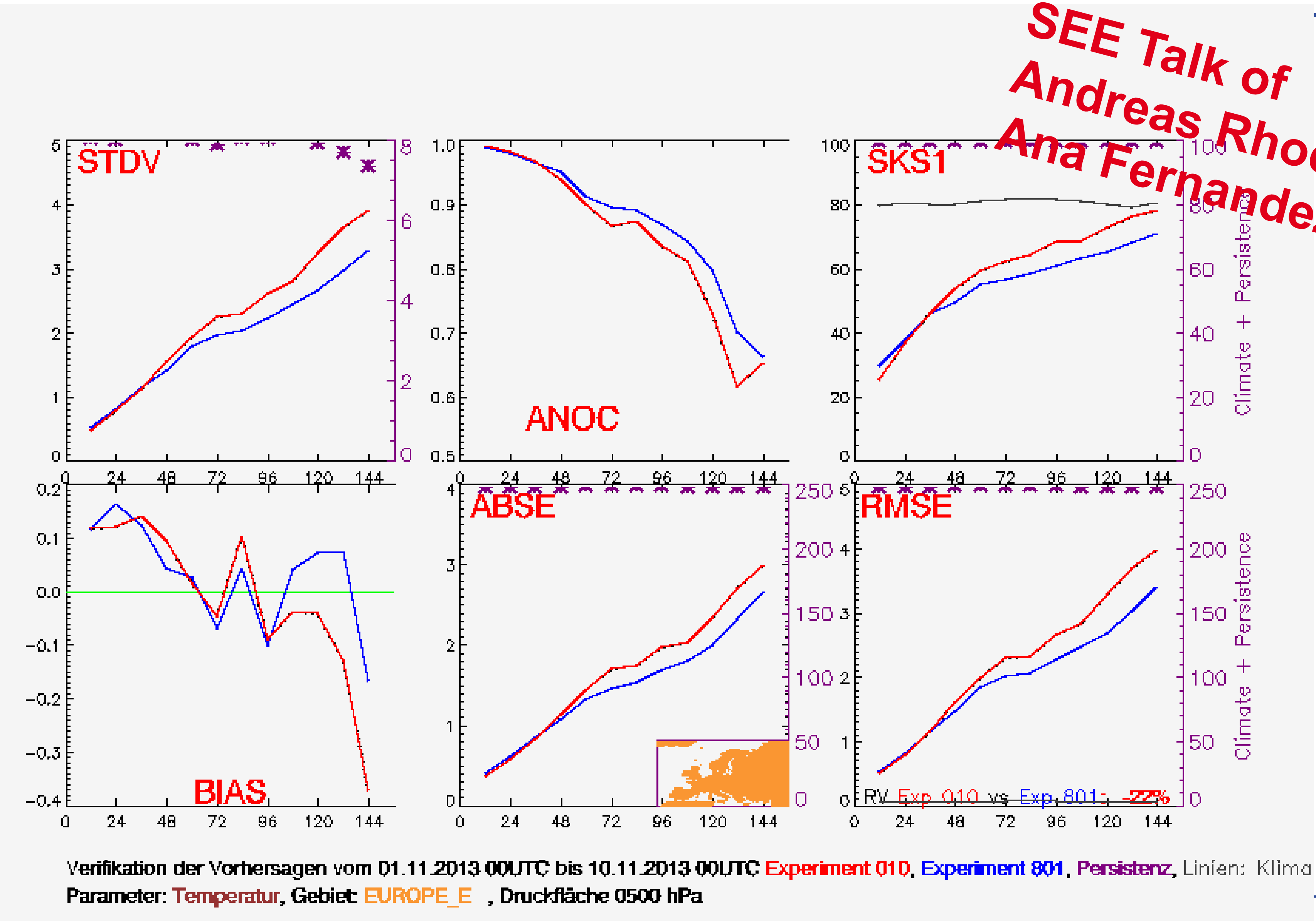
SEE Talk of Andreas Rhodin / Ana Fernandez



Global EDA (VarEnKF) Development

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

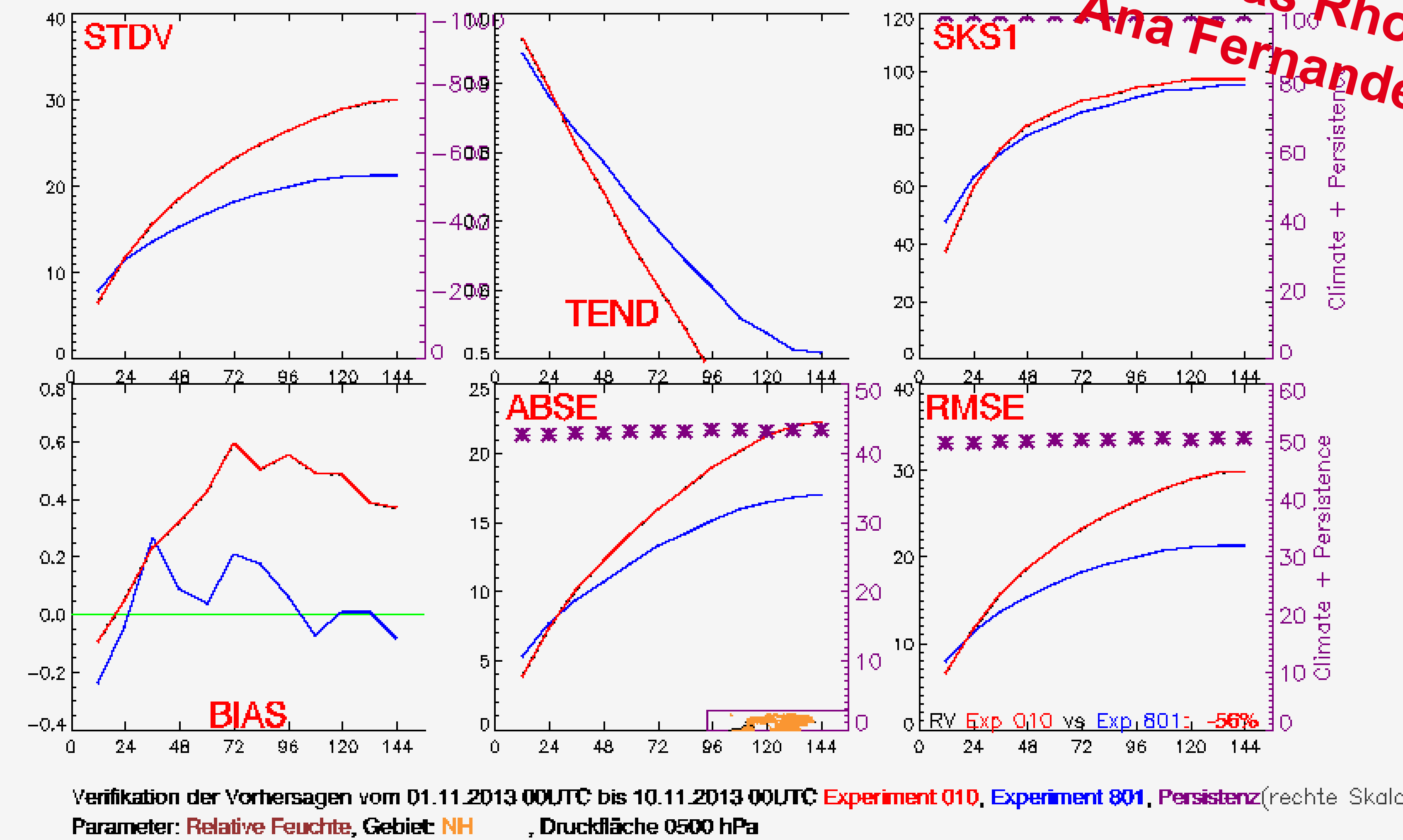
SEE Talk of Andreas Rhodin / Ana Fernandez



Global EDA (VarEnKF) Development

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

SEE Talk of Andreas Rhodin / Ana Fernandez



Global EDA (VarEnKF) Development

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

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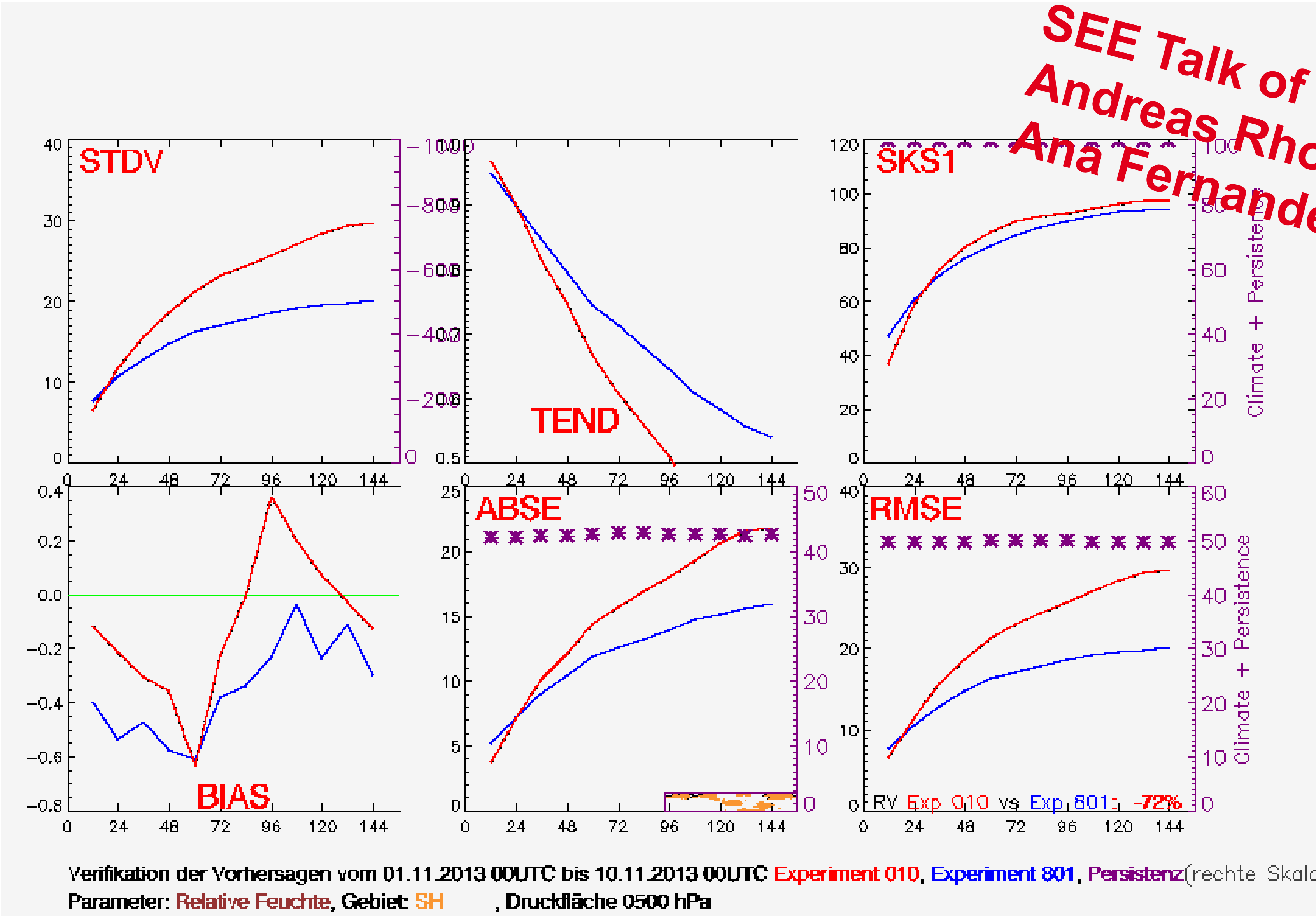

Global EDA (VarEnKF) Development

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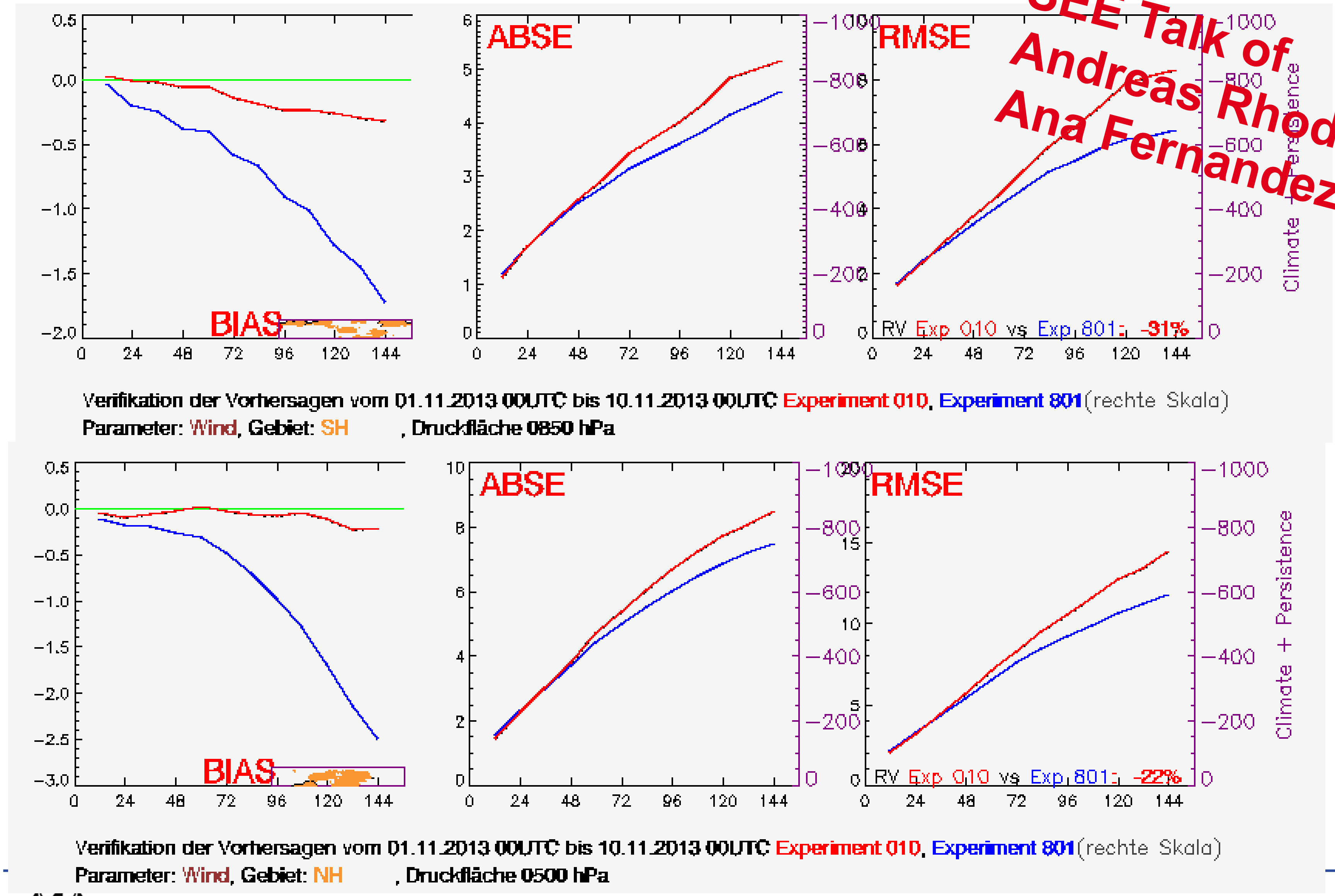
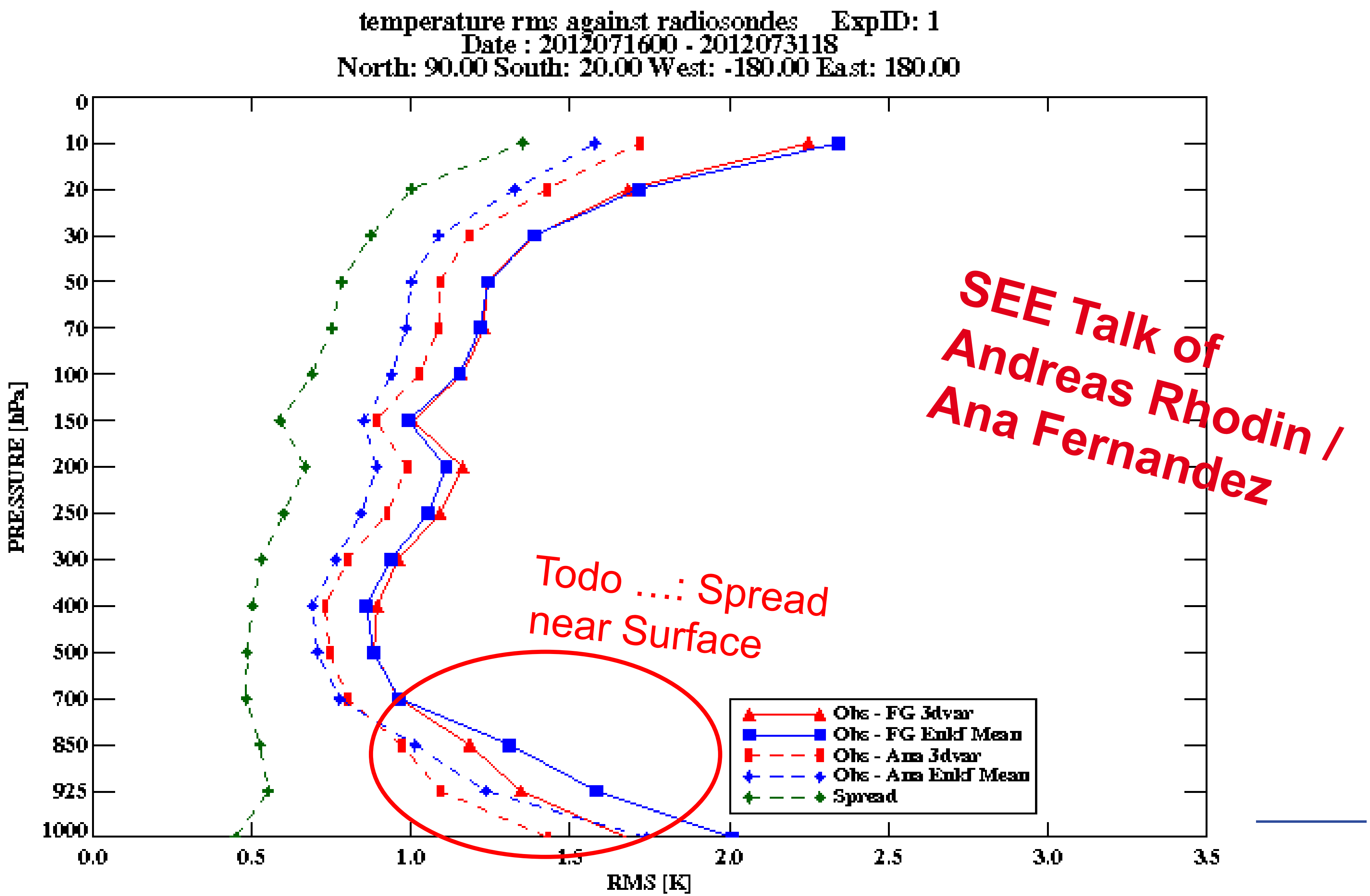
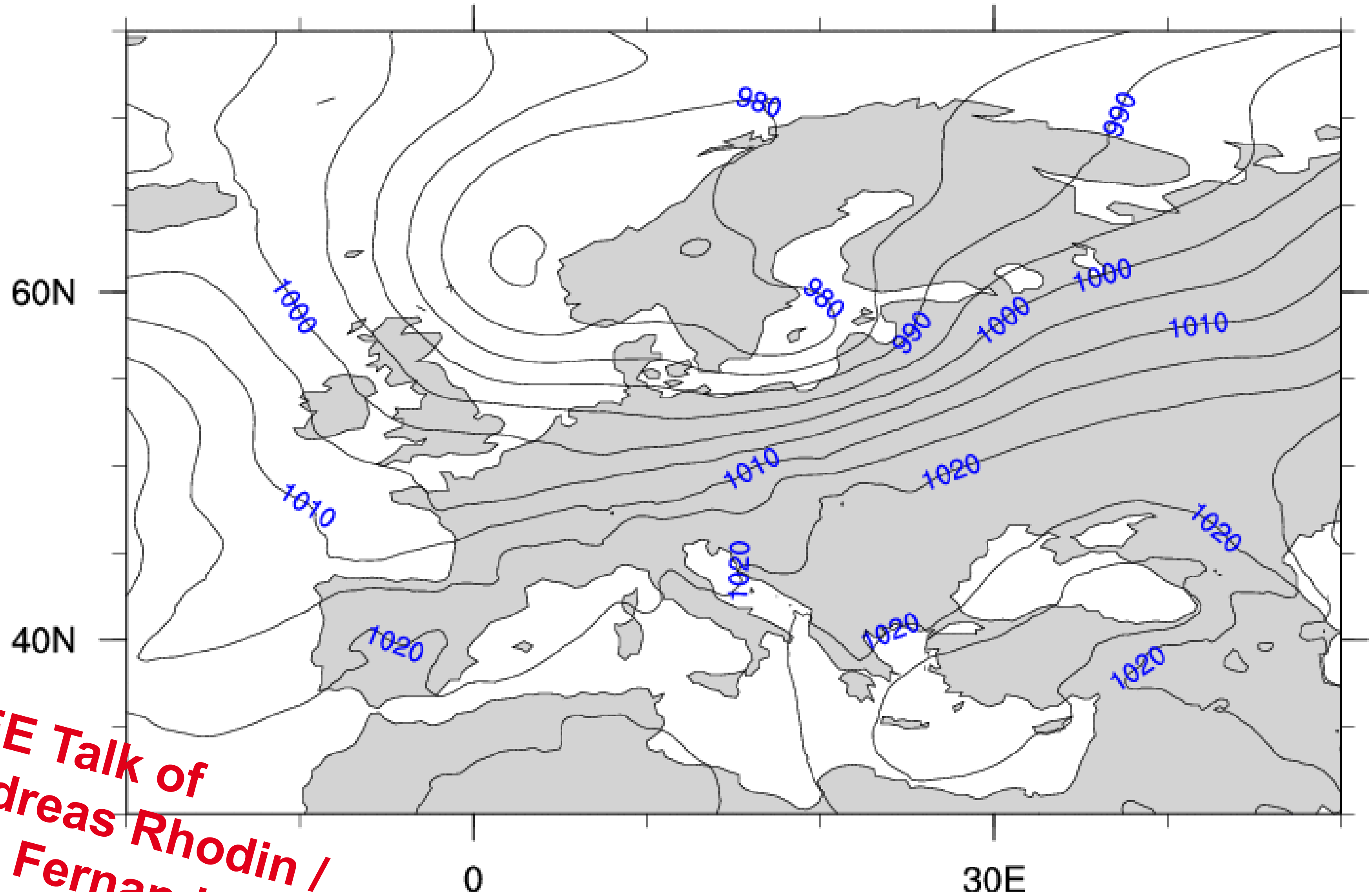



temperature rms against radiosondes ExpID: 1
Date : 2012071600 - 2012073118
North: 90.00 South: 20.00 West: -180.00 East: 180.00
PRESSURE [hPa]
RMS [K]
Obs - FG 3dvar
Obs - FG Enkf Mean
Obs - Ana 3dvar
Obs - Ana Enkf Mean
Spread
SEE Talk of Andreas Rhodin / Ana Fernandez
Todo ...: Spread near Surface

ICON Ensemble PMSL Mean Member 1

12/11/2014 (00:00) 42hr forecast hPa



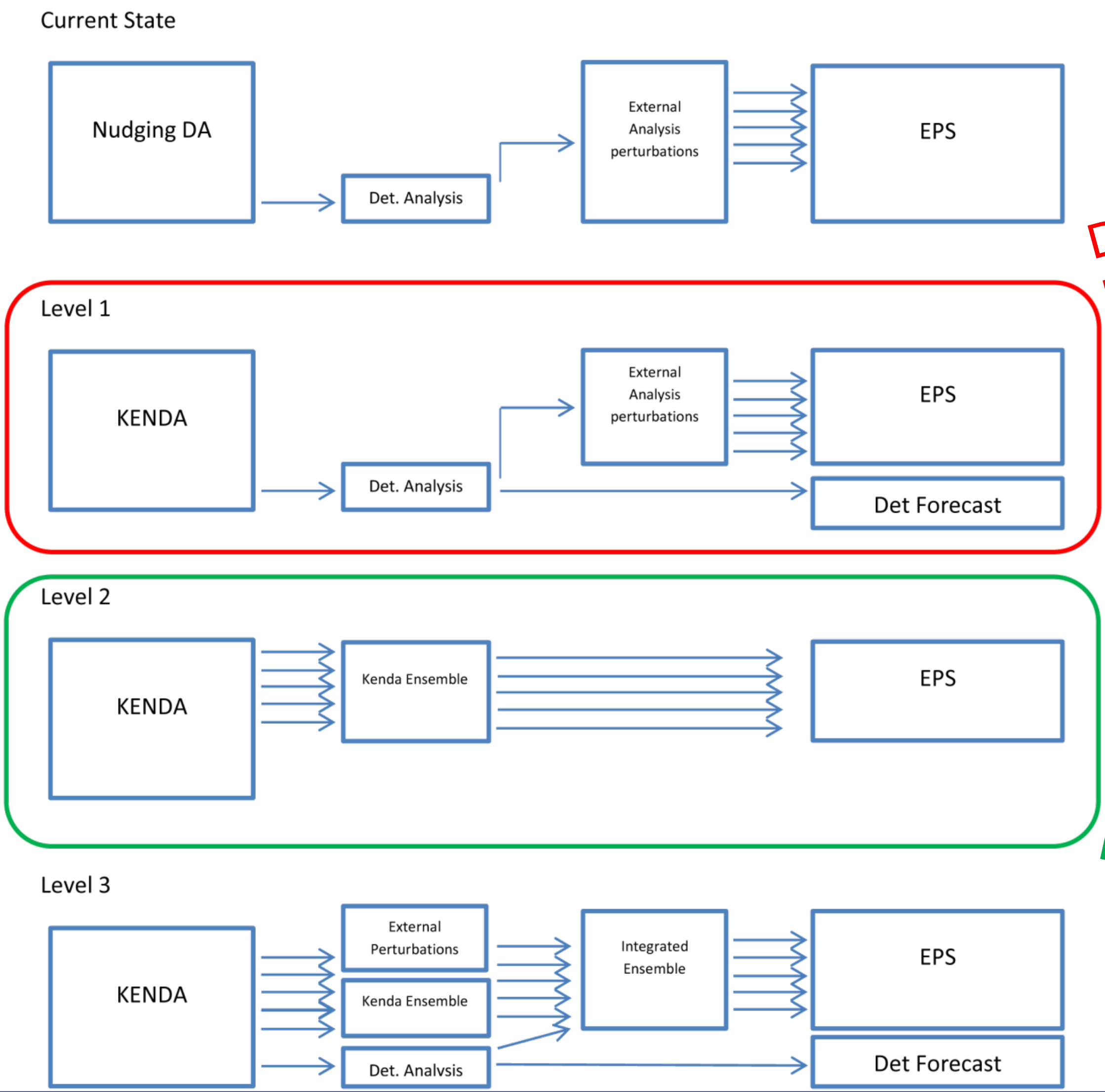
SEE Talk of Andreas Rhodin / Ana Fernandez

Part III: Kilometer Scale Ensemble Data Assimilation (KENDA)

- LETKF + DetAnalysis

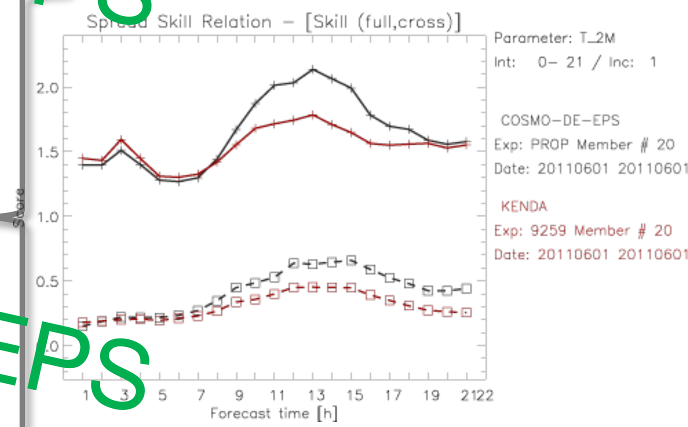
KENDA and EPS Development

SEE Talks of
Hendrik Reich,
Theresa Bick,
Martin Weissmann



Deterministic
Forecast

EPS

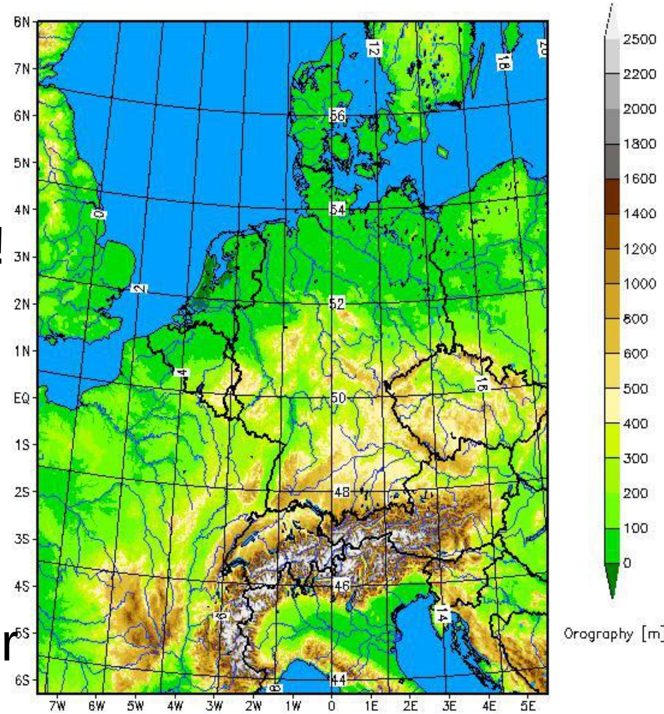


EPS

KENDA for COSMO

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

1. **Full System** with **conventional data *running***
2. Work **Latent Heat Nudging**, done, works well!
3. Further **Observation Systems** under development
   (e.g. **SEVIRI, GPS/GNSS**, Lidar, …)
4. **Longer Periods/Winter Periods** to be tested.
5. **Technical work** on operational setup (member loss) ongoing
6. **Archive/Storage** challenges remain severe
7. **Pattern Generator** and further **Refinements** (Localiyation, Covariance Inflation, …)



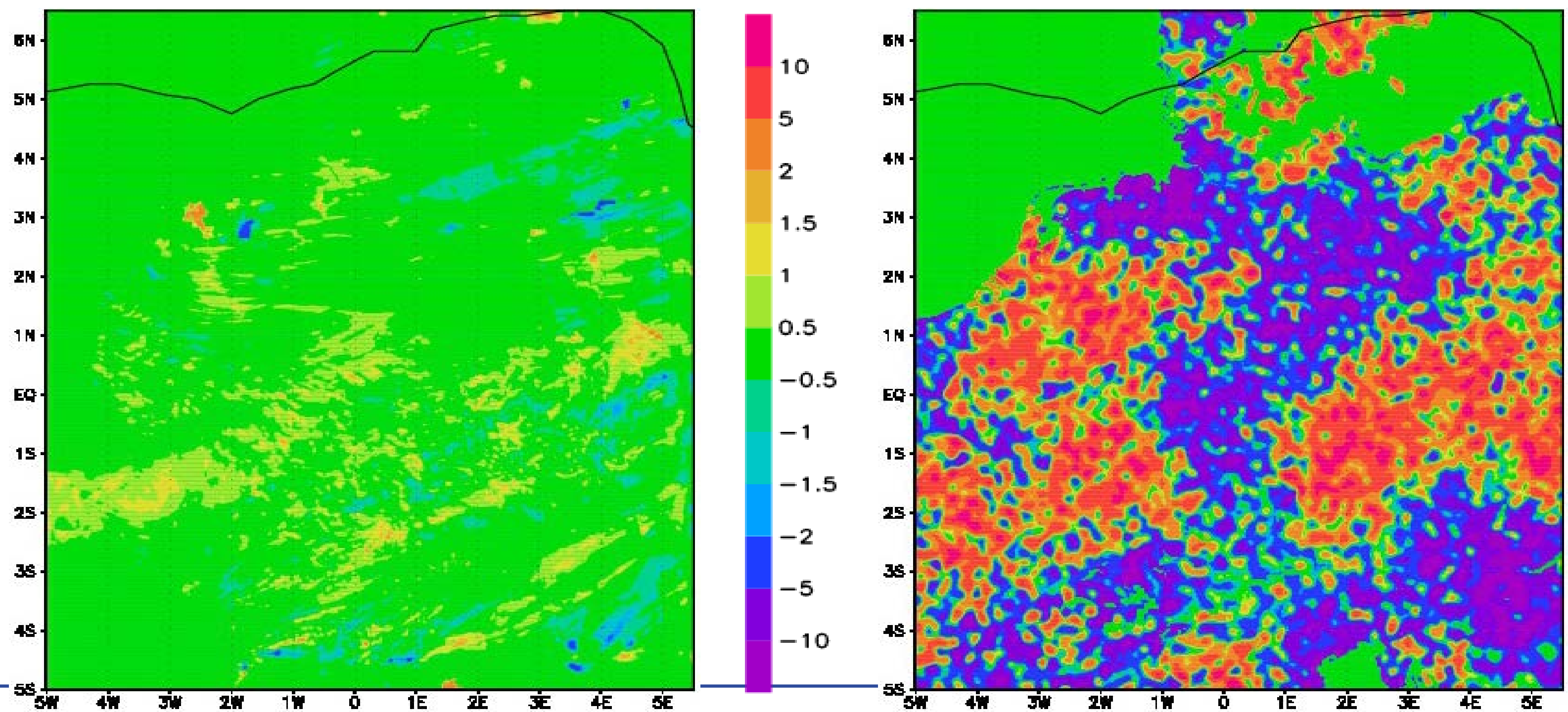
SEE Talks of Hendrik Reich, Theresa Bick, Martin Weissmann

introduction of soil moisture (SM) perturbations
(+ SST pert.)

- ✓ simple superposition of Gaspari-Cohn (~ Gaussian) functions at each analysis g.p., with random amplitude and pre-specified horiz. / temporal correlation scale(s)
- ✓ scales : 100 km + 10 km ; 1 day ;   std dev of amplitude: 0.1 soil moisture index

spread of soil moisture (WSO), layer 3 (3 – 9 cm), after 5 days

cycling without perturbations   cycling with SM perturbations



impact of including LHN in LETKF DA cycle (with ICON soil): score chart

	LETKF **LHN-det** vs. no-LHN		LETKF **LHN-all** vs. LHN-det	
variable	RMSE ETS / FSS	Bias FBI	RMSE ETS / FSS	Bias FBI
upper-air	=	=	=	=
surface	=	=	=	=
precip 0 UTC , 0.1 mm	(+)	(+)	(+)	=
precip 0 UTC , 1 mm	=	=	(+)	=
precip 12 UTC, 0.1 mm	**+**	**+**	**+**	=
precip 12 UTC, 1 mm	**+**	=	**+**	=

→ benefit from adding LHN small for 0-UTC runs, large for 12-UTC runs
→ deterministic forecast improves if LHN also added to all ens members in LETKF

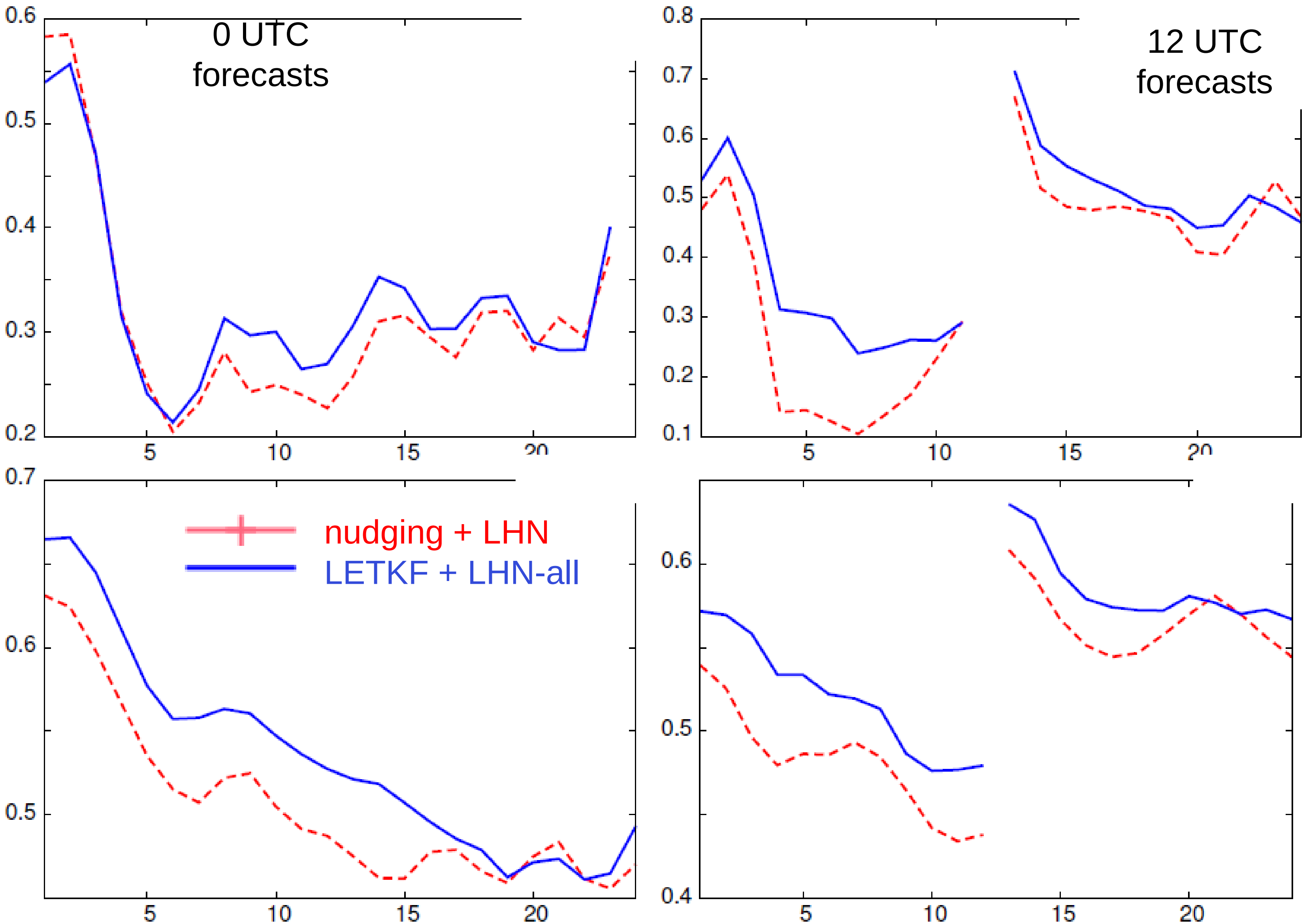
LETKF + LHN-all vs. Nudging + LHN : verification against radar precipitation

Period 1, 6 Days

FSS 11 g.p. (30 km)

0.1 mm

Period 2, 12 Days



LETKF + LHN-all vs. Nudging + LHN : KENDA score chart

Deutscher Wetterdienst
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	variable	T2012		T2014	
		rmse	bias	rmse	bias
upper air	geopotential	=	=	=	=
	temperature	=	=	(+)	(+)
	relative humidity	=	=	=	–
	wind speed	+	=	+	=
	wind direction	+	=	(+)	=
surface	2-m temperature	=	=	+	=
	2-m dew point	=	=	+	+
	10-m wind	=	=	=	=
	surface pressure	–	=	+	=
	total cloud	=	=	=	=
	low cloud	+	+	–	=
	mid-level cloud	+	+	=	=
	high cloud	–	–	–	–
radar	precip 0 UTC runs	+/(–)	+	++	++
	precip 12 UTC runs	++	++	++	++

LETKF:

→ overall comparable / better results

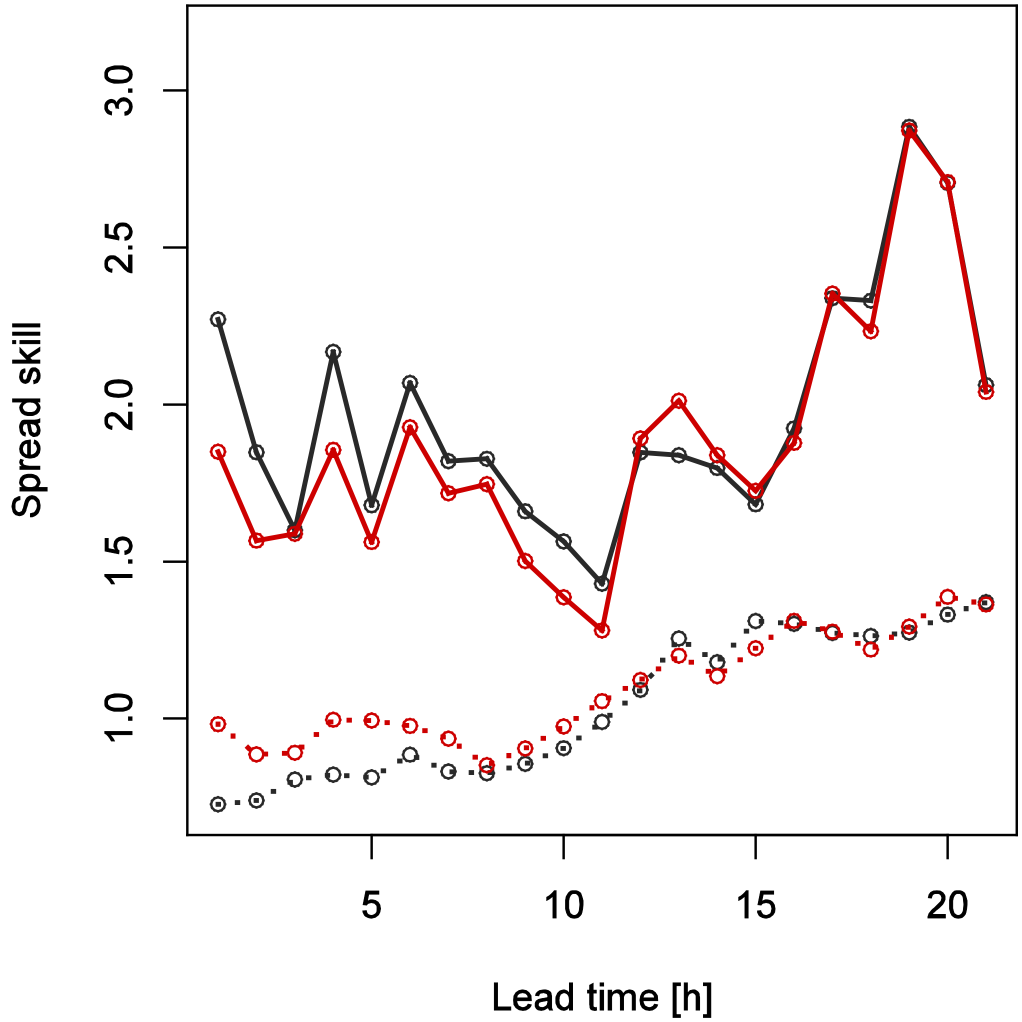
→ Mixed view for surface pressure

→ Problem with high clouds

SEE Talks of Hendrik Reich, Theresa Bick, Martin Weissmann, Yuefei Zeng, Florian Harnish, Leonhard Scheck

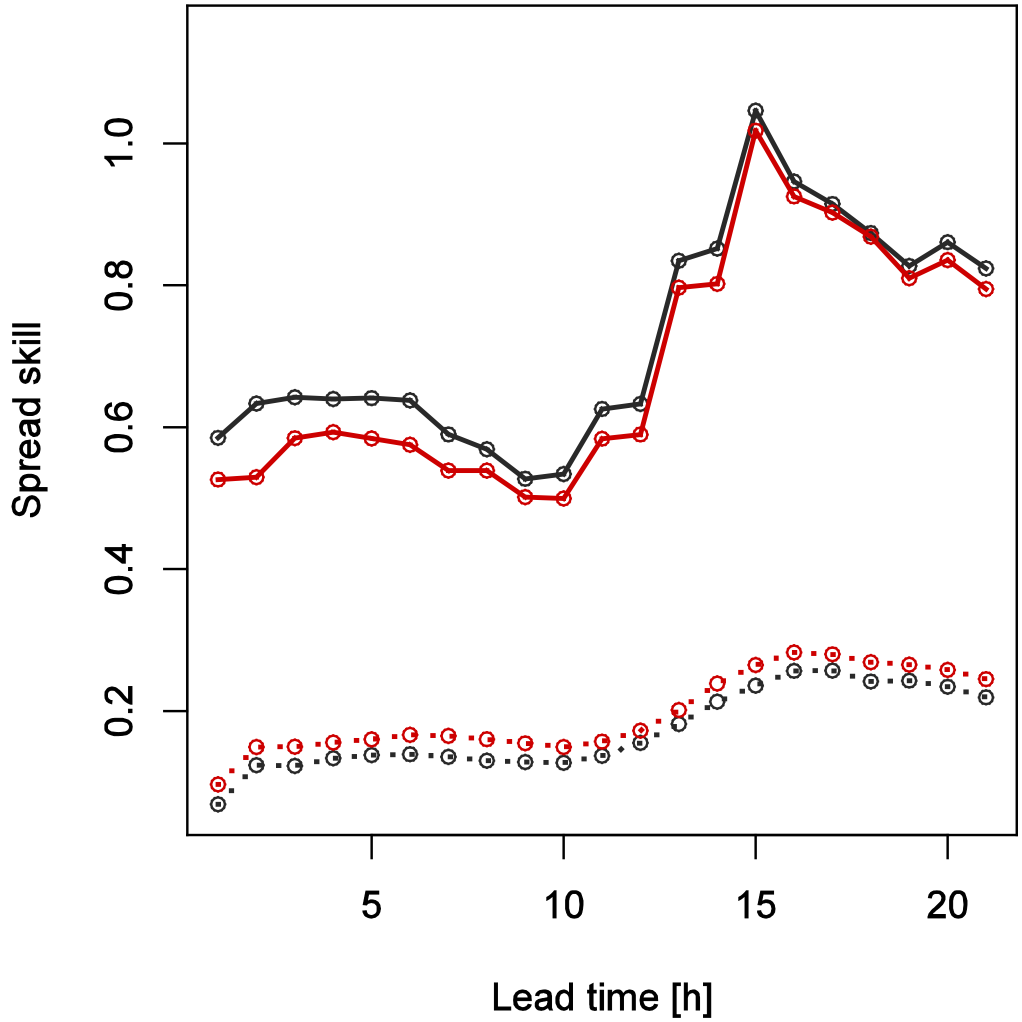
EPS 10-250 m Wind: RMSE + Spread

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



Nudging + BCEPS
KENDA

EPS Precipitation: RMSE + Spread



Nudging + BCEPS
KENDA

KENDA Project Selection

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

ISDA 2015 Thursday Session

- 9:30-10:20 Invited — [10-1] Novel observations for convective-scale weather prediction
  Martin Weissmann, (LMU, Germany)
- 10:20-10:40 — [10-2] Assimilation experiments of bright band heights
  Takeshi Enomoto (Kyoto U, Japan)
- 10:40-11:00 — [10-3] Characterization of SEVIRI satellite observations and their potential for convective-scale ensemble data assimilation
  Florian Harnisch (LMU, Germany)
- 11:00-11:20 — [10-4] Assimilation of visible and near-infrared satellite observations in KENDA-COSMO
  Leonhard Scheck (LMU, Germany)
- 11:20-11:40 — [10-5] Improved use at the Met Office of SEVIRI infrared observations in cloudy regions
  Robert Tubbs (Met Office, UK)
- withdrawn — [10-6] Assimilating radar volume data into the COSMO model using an LETKF approach
  Yuefei Zeng(LMU, Germany)
- 11:40-12:00 — [10-7] Assimilating 3D radar reflectivity with an Ensemble Kalman Filter on a convection- permitting scale
  Theresa Bick (U of Bonn, Germany)

Part IV: Particle Filter

- Can a Particle Filter work for NWP?
- Yes, the EnKF is already working (as shown above)!
- The question is how to calculate the posterior ensemble!

- DWD is developing a
  Local Markov Chain Particle Filter
- MeteoSwiss + ETH Zurich + DWD are working on an Ensemble Kalman Particle Filter

Part IV: Particle Filter

- You get a **prior distribution** p(x) by some prior ensemble

- Measurements define a **data distribution** p(y|x)

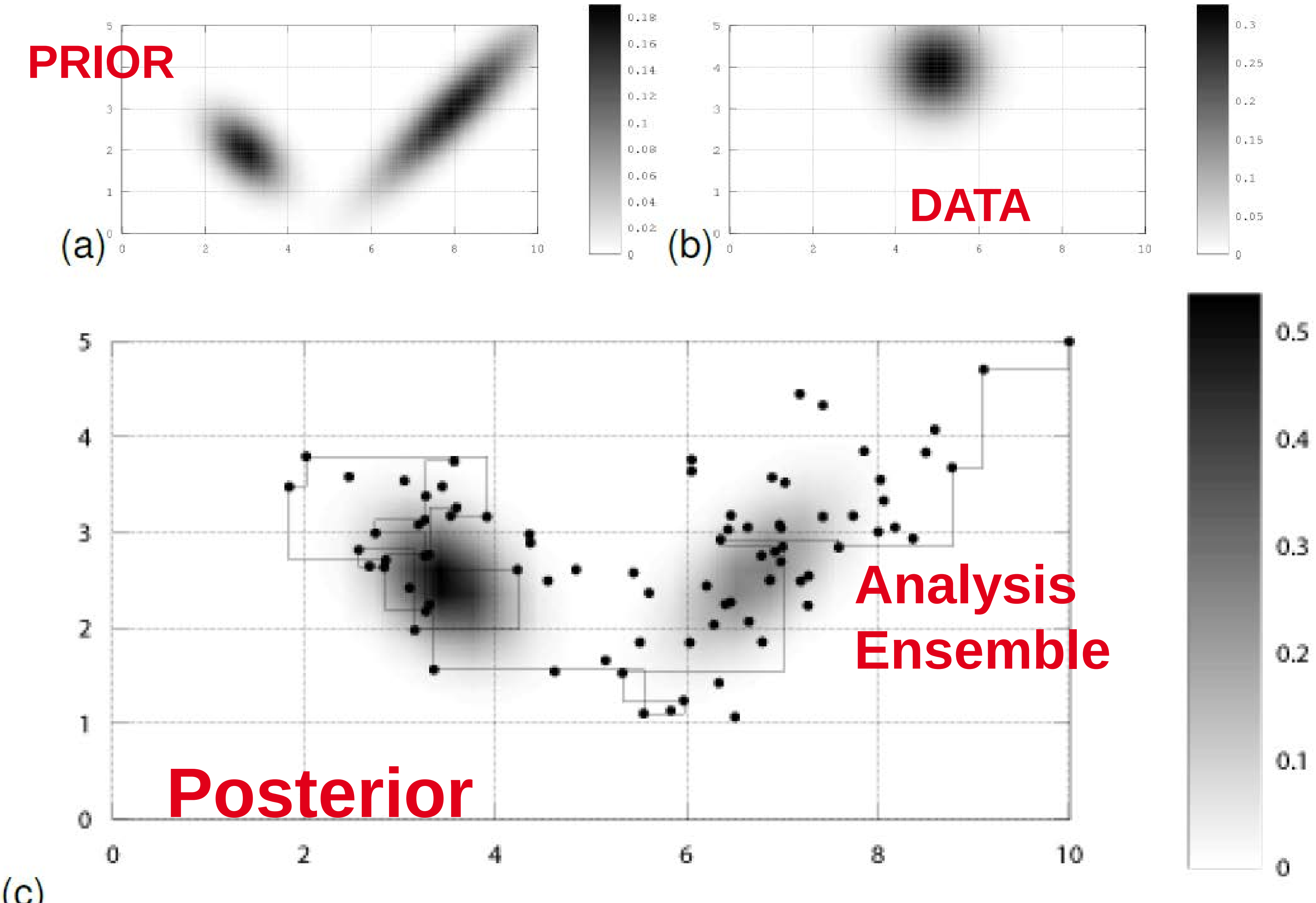
- **Bayes theorem** defines a posterior distribution by

$$p(x|y) = c\, p(x)\ p(y|x)$$

The core game is
how to get an **analysis ensemble** from p(x|y).

BAYES Data Assimilation

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



How to sample from a distribution p(x|y)?

Markov Chain Monte Carlo Methods (MCMC) generate **sequences of states** $x_1, x_2, x_3, \ldots$ which sample some particular probability distribution p. Based on this sequence particular quantities like the *mean* or the *variance* can be estimated e.g. by the estimators (1) or (2).

We understand a **particle filter** as a method which, in each step of the data assimilation, generates such a sequence of states which sample the posterior distribution as given by Bayes's theorem.

Ensemble methods are special cases of **particle methods**.

MCMC Methods

Take an open subset $\Omega \subset \mathbb{R}^n$ for X and let $p(x)$ denote some **probability density** on Ω.
We assume that $p(x)$ is in the space $L^1(\Omega)$ which is the set of all measurable functions whose absolute are integrable on Ω.
Also, by $P(x \to x')$ we denote some probability density which describes the **transition probability** of a Markov chain, i.e. the probability of a time-discrete stochastic process to draw x' at time t_{k+1} when $x \in \Omega$ was drawn in its previous step t_k.

$$P(x \to x') = k(x', x) + r(x)\delta(x' - x), \quad x, x' \in \Omega. \tag{10}$$

MCMC Methods

We employ the abbreviations

$$(Kp)(x') := \int_{\Omega} k(x', x)p(x)\, dx \quad (15)$$

$$(Rp)(x') := r(x')p(x') \quad (16)$$

for $x' \in \Omega$ and

$$A := K + R. \quad (17)$$

The probability density is *stationary* at density p, if $p_{k+1} = p_k$, i.e. if the density p satisfies the **fixed point equation**

$$\boxed{p = Ap.} \quad (18)$$

MCMC Methods

Assume that we are starting at some point $x_0 \in \Omega$ and create the sequence x_j, $j \in \mathbb{N}$, of points by drawing from the transition probability $P(x \to x')$. Then, x_1 is generated by drawing from $p_0(x) := P(x_0 \to x)$. By the Chapman-Kolmogorov equation (14) point x_2 comes from drawing from

$$\begin{aligned} p_1(x) &= \int_\Omega p_0(z) P(z \to x)\, dz \\ &= (Ap_0)(x), \quad x \in \Omega. \end{aligned} \tag{29}$$

In the same way we see that x_j is a draw from p_j defined inductively by (28). We will denote the probability distribution p_j which was starting with $p_0[x_0](x) := P(x_0 \to x)$, $x \in \Omega$, by

$$p_j[x_0](x) := \Big(A^j p_0[x_0]\Big)(x), \quad x \in \Omega, \quad j \in \mathbb{N}. \tag{30}$$

We skip the details of Lemma 1 and 2 … the result is

Theorem

Under the conditions of Lemma 2 the fixed point equation (18) has a unique fixed point p_* in $L^1(\Omega)$ and for any starting density p_0 in $L^1(\Omega)$ the sequence of densities p_j defined by

$$p_j := Ap_{j-1} = (K + R)p_{j-1}, \quad j = 1, 2, 3, \dots \tag{28}$$

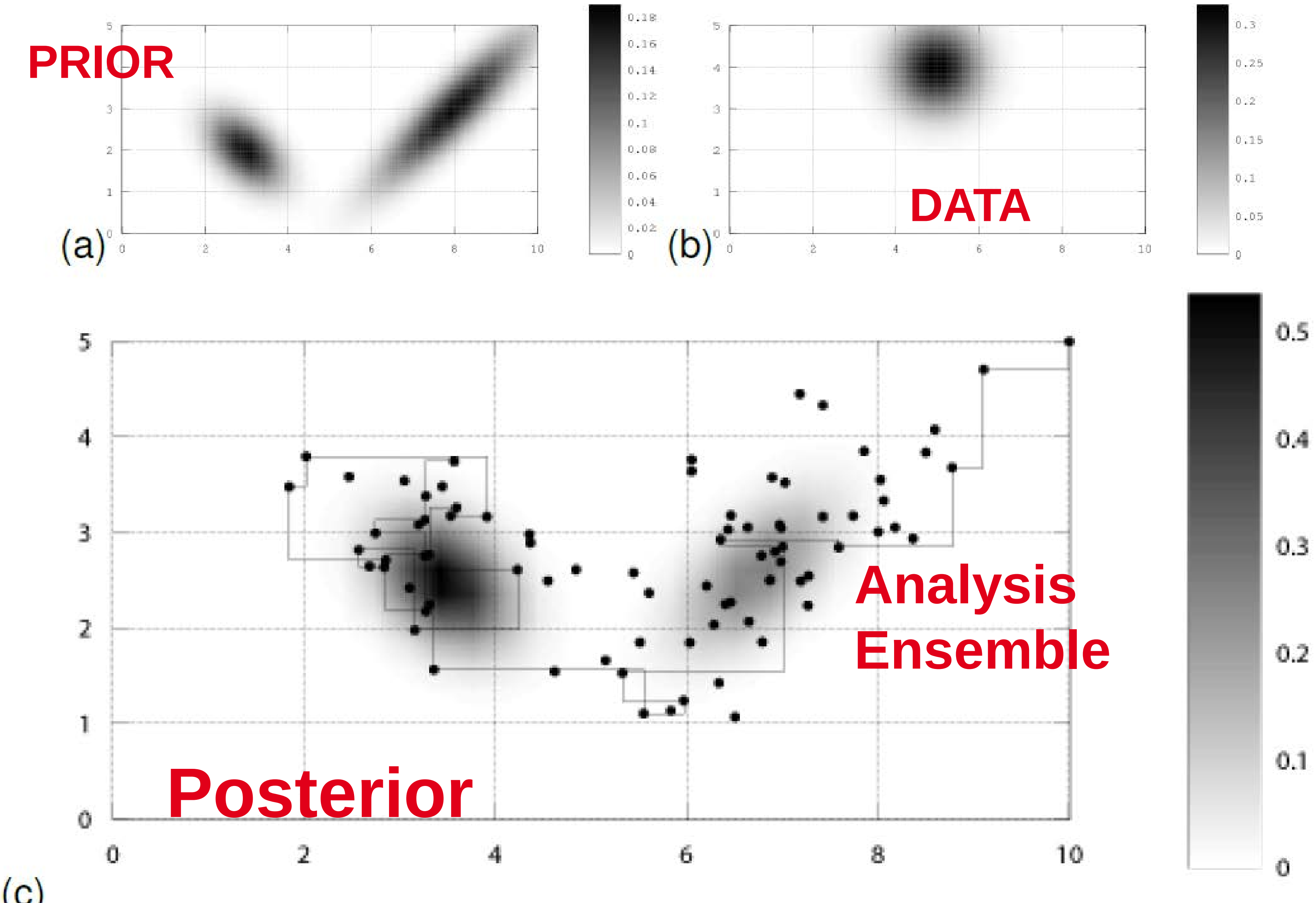
converges towards this fixed point distribution p_*.

Theorem

Choose some $z \in \Omega$ and assume that the conditions of Lemma 2 are satisfied. Then in $L^1(\Omega)$ the sequence of probability densities $p_j[z]$ converges towards the unique fixed point p_* of equation (18), i.e.

$$\|p_j[z] - p_*\|_{L^1(\Omega)} \to 0, \quad j \to \infty. \tag{31}$$

BAYES Data Assimilation



MCMC Methods, Basics

The condition

$$
\begin{aligned}
1 &= \int_{\Omega} P(x \to x')\, dx' \\
&= \int_{\Omega} k(x', x)\, dx' + r(x), \quad x \in \Omega
\end{aligned}
\tag{11}
$$

yields

$$
\boxed{r(x) = 1 - \int_{\Omega} k(x', x)\, dx', \quad x \in \Omega.} \tag{12}
$$

A helpful identity

MCMC Balance Equation

Using equation (12), we can transform (18) into

$$\begin{aligned} p(x') &= \int_\Omega k(x', x) p(x)\, dx + r(x')p(x') \\ &= \int_\Omega k(x', x) p(x)\, dx - \int_\Omega k(\xi, x')p(x')\, d\xi + p(x'). \quad (19) \end{aligned}$$

This leads to the MCMC *balance equation*

$$\boxed{\int_\Omega k(x', x) p(x)\, dx = \int_\Omega k(\xi, x')p(x')\, d\xi, \quad x' \in \Omega. \qquad (20)}$$

When we construct a transition kernel k which satisfies the MCMC balance equation, then p(x) is a fixed point of the transition equation

Metropolis Hastings Sampler

In the previous section we have studied an approach how to sample a particular probability density distributin $p(x)$, $x \in \Omega$, when a transition probability $P(x \to x')$ is given such that $p(x)$ is the unique fixed point of the *fixed point equation*

$$p(x') = \int_{\Omega} p(x) P(x \to x') \, dx, \quad x' \in \Omega. \tag{32}$$

Here, we will start with the **detailed balance equation**, which is a differential form of the MCMC balance equation (20), given by

$$k(x', x) p(x) = k(x, x') p(x'), \quad x, x' \in \Omega. \tag{33}$$

MCMC Methods

Let us start with some continuous transition probability $q(x', x)$ for the transition of x to x', which we use as a **proposal density** to suggest draws as candidates for our transition probability density $k(x', x)$, $x, x' \in \Omega$. The goal is to construct a correction $\alpha(x', x)$ such that have

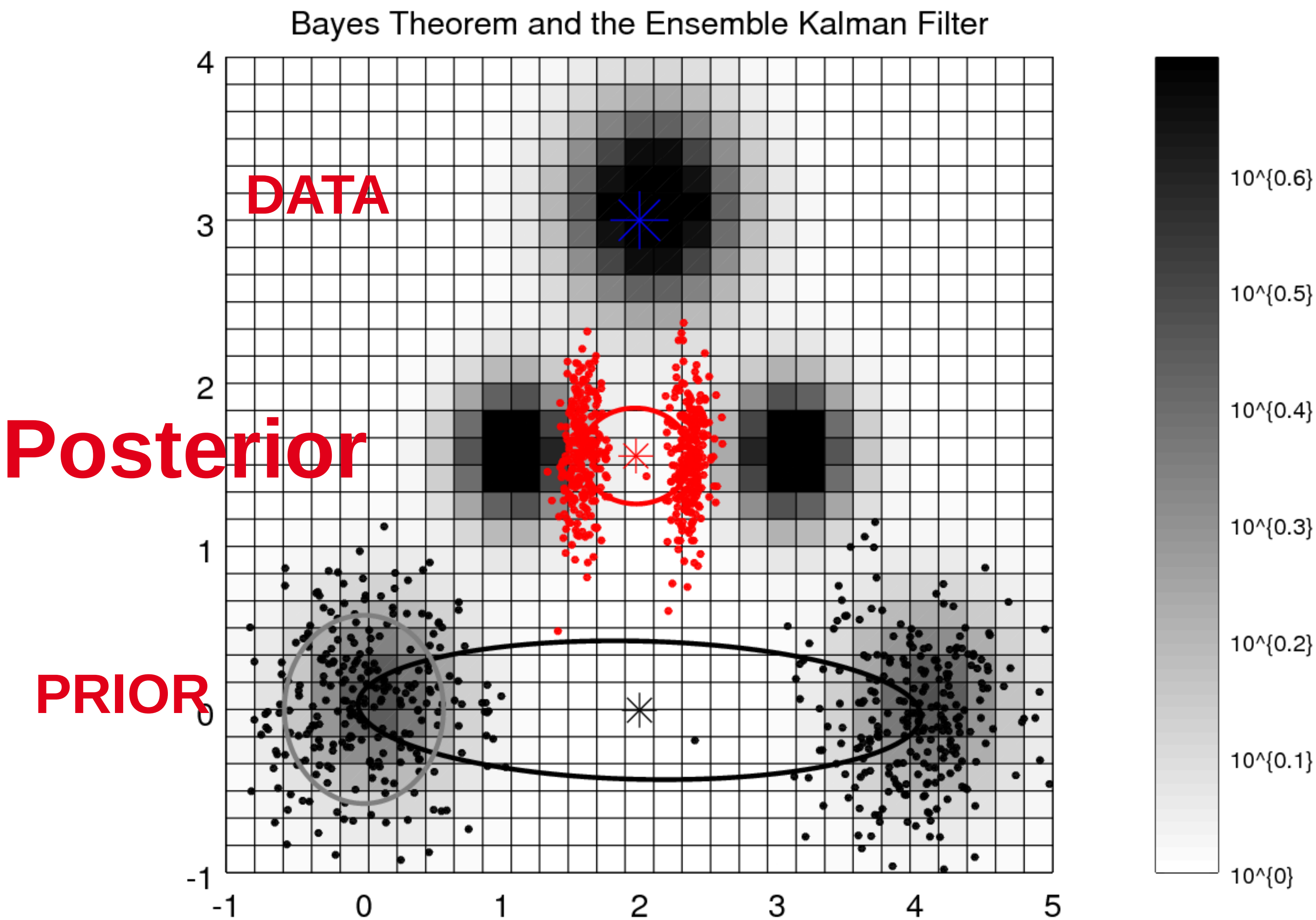
$$k(x', x) = \alpha(x', x) q(x', x), \quad x, x' \in \Omega, \tag{34}$$

is a transition kernel which satisfies the **detailed balance equation** (33). The choice

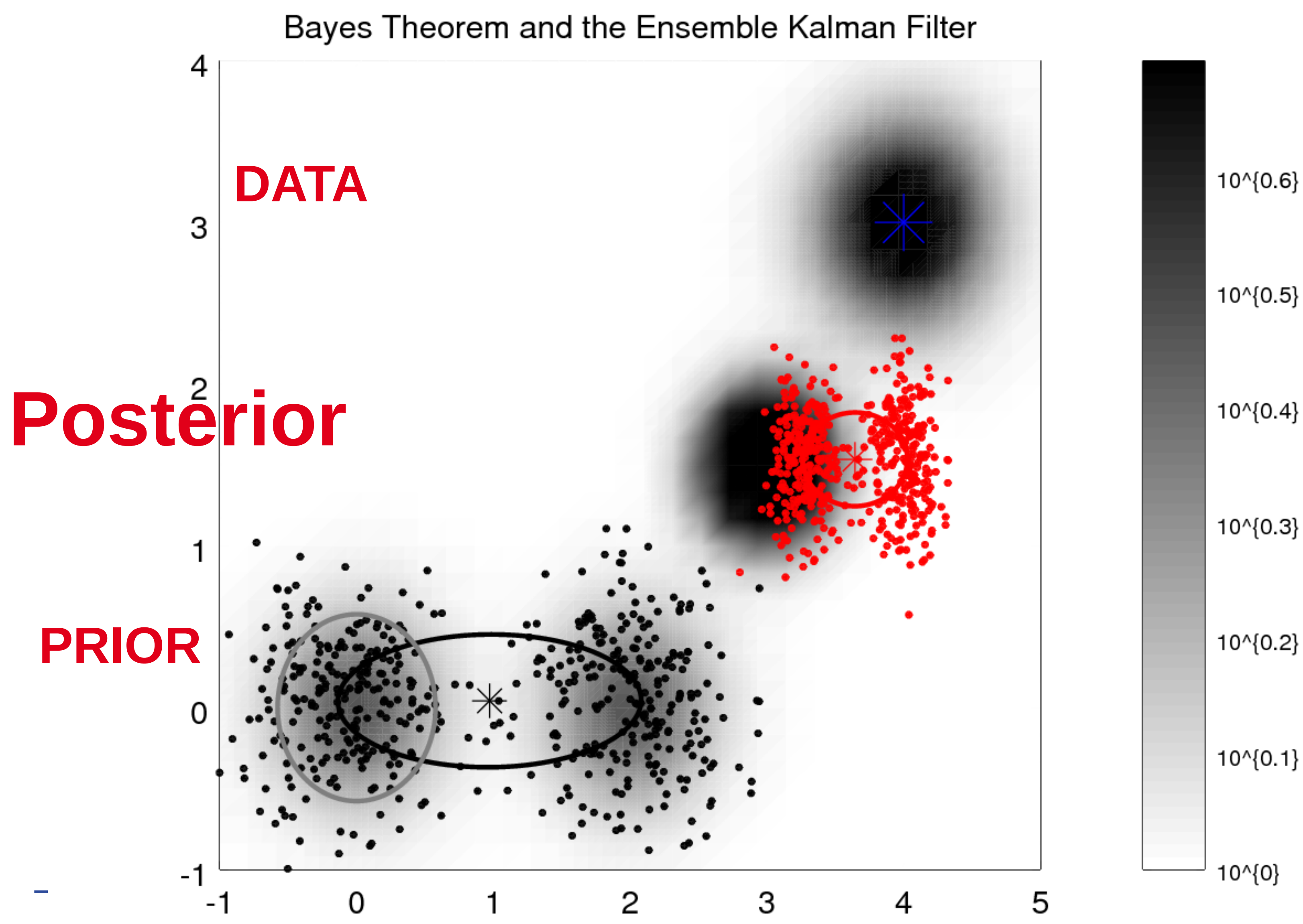
$$\alpha(x', x) := \min \left\{ 1, \frac{p(x') q(x, x')}{p(x) q(x', x)} \right\} \tag{35}$$

is known as **Metropolis-Hastings algorithm**.

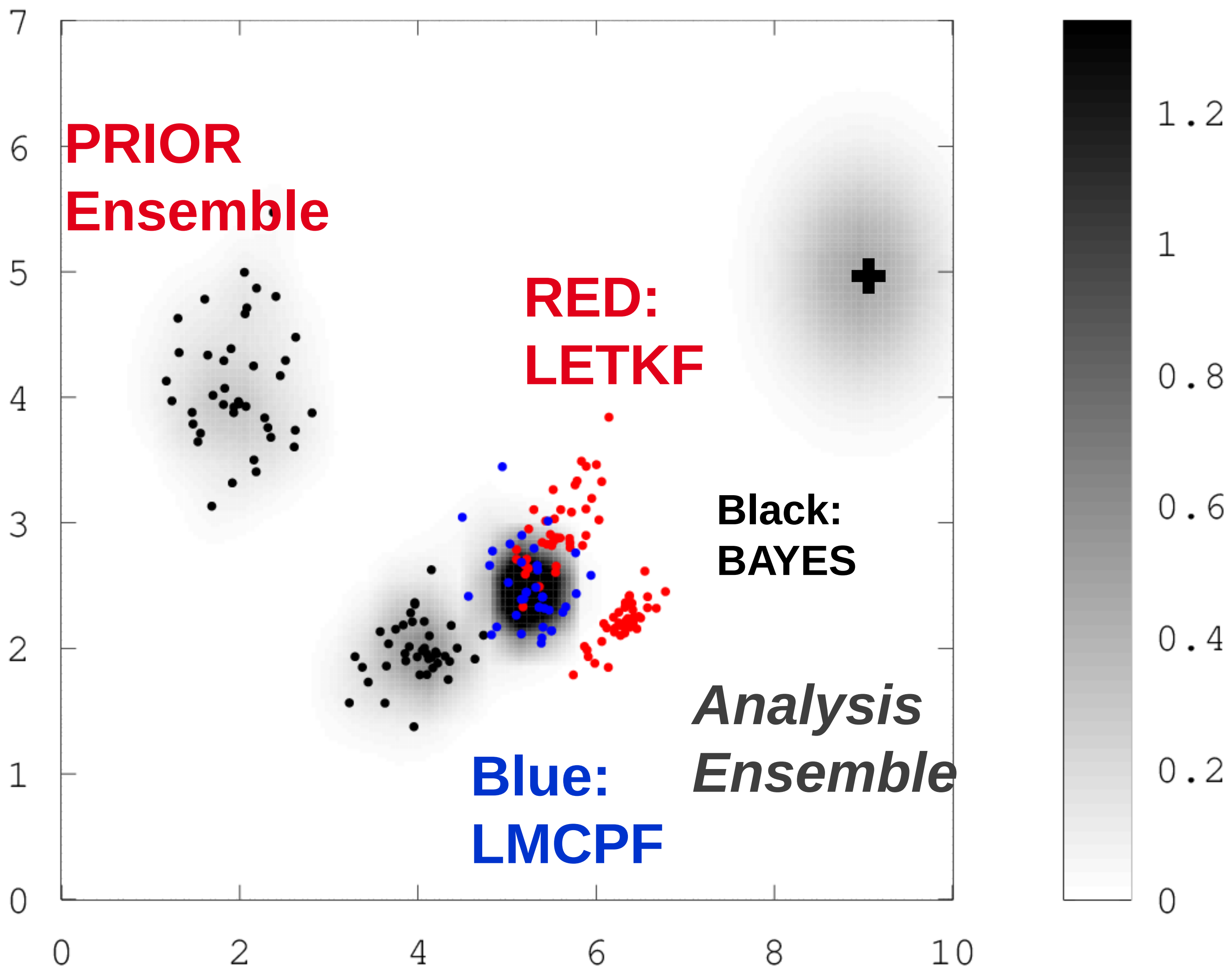
BAYES Data Assimilation

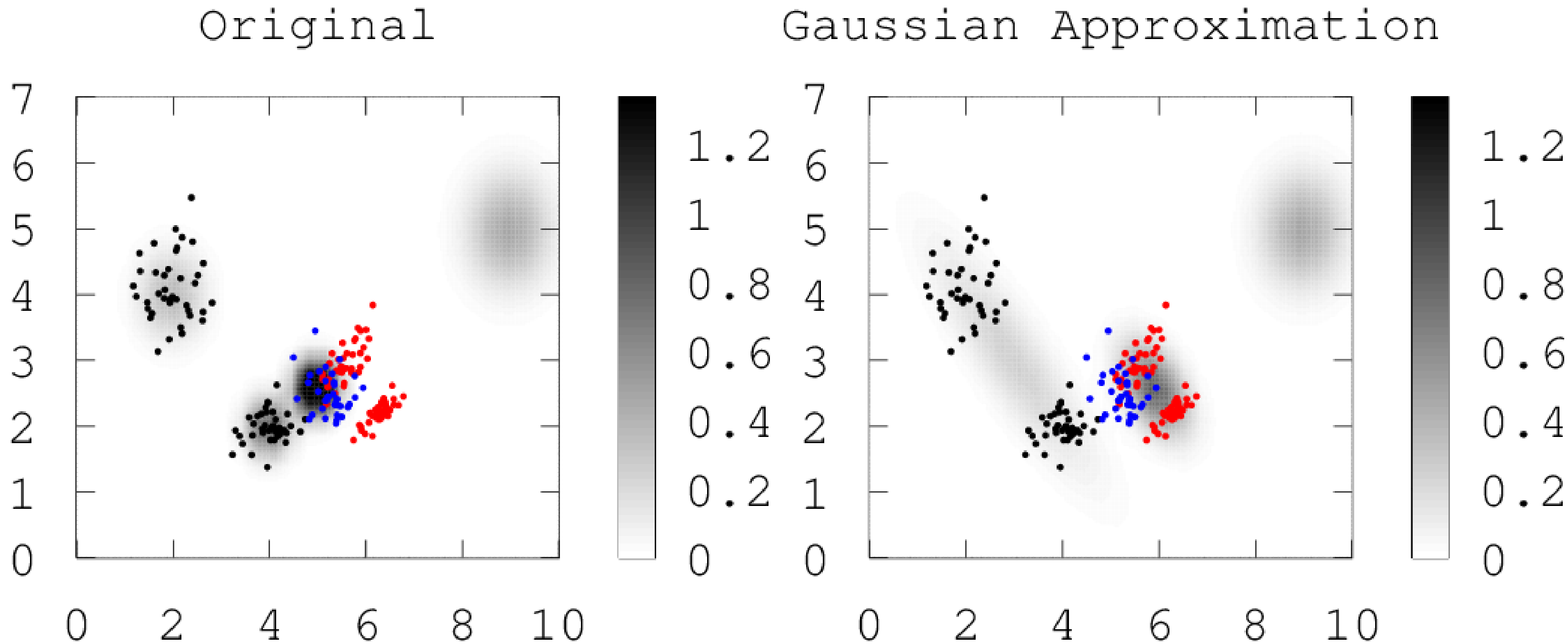


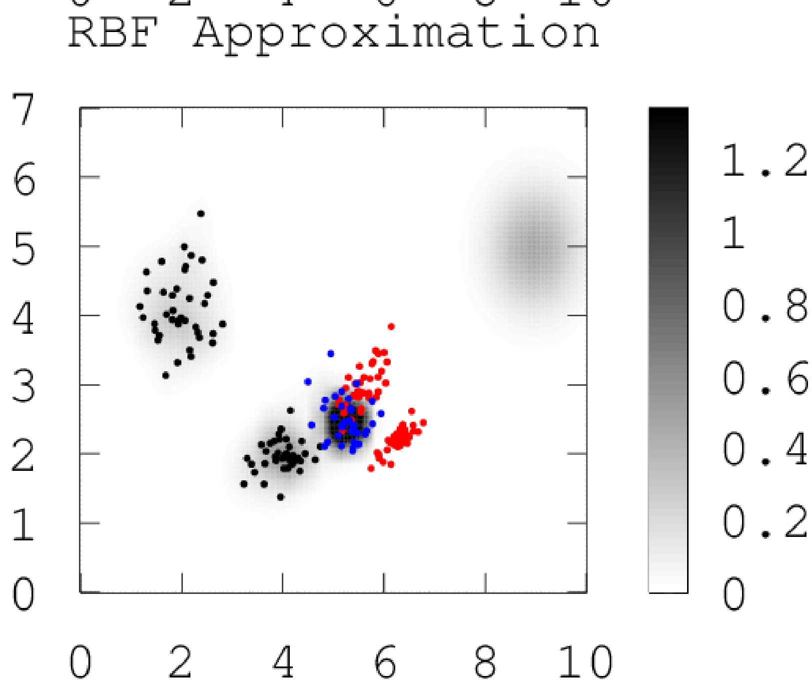
BAYES Data Assimilation



Particle Filter Example





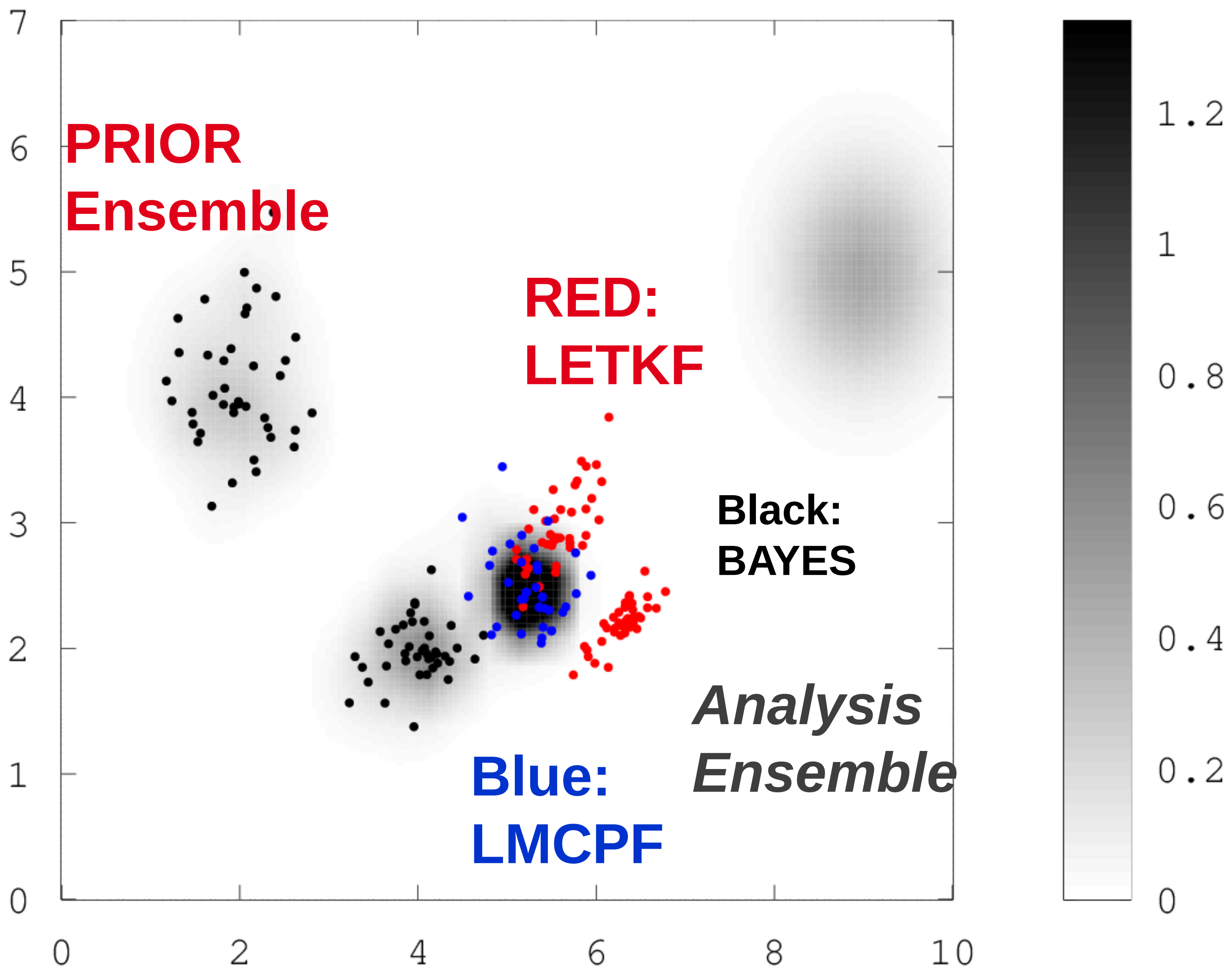


LETKF
vs.
LMCPF

The LMCPF Particle Filter

1. We carry out Bayes type data assimilation by creating an ensemble of states (the **particles** or **ensemble members**) $x^{(1)}, \ldots, x^{(L)}$ in step t_k for $k = 0$ according to some initial probability distribution $p[t_0](x)$.
2. The ensemble is propagated from t_k to t_{k+1}. It samples the background or *prior* probability distribution $p[t_{k+1}](x)$ at time t_{k+1}.
3. Given measurement data $y \in Y$ we employ Bayes Theorem to calculate the **posterior distribution** $p(x) := p^{(a)}(x) = cp^{(b)}(x)p(y|x)$. This needs the ability to calculate $p(x)$ for some given state x as well as $p(y|x)$ given y for some state x.
4. We use MCMC with **Metropolis-Hastings kernel** to draw $x^{(1)}, \ldots, x^{(L)}$ from $p(x)$. Then we proceed with Step 2.

Particle Filter Example



The LMCPF Particle Filter

- **Markov Chain Monte Carlo** Methods are a tool how to sample some given probability distribution p(x).
- We need the ability to calculate p(x) given some state x.
- Then we can employ MCMC, for example with Metropolis Hastings kernel.

- **Metropolis Hastings** is a particular way how to generate the posterior ensemble.
- The idea is to use a **proposal distribution q(x)**.
- For every proposal state a decision is made wether to take it or not.
- This depends on the Metropolis Hastings function α.

The LMCPF Particle Filter

- We can employ any **proposal distribution** q. In particular, we can use the result of the LETKF which brings us basically where we would like our particles to be - but with some wrong scaling in the directions of the LETKF-B-Matrix.
- Consequence: use a *relaxed* version of the LETKF to make sure we do not miss any states for our MCMC sampling! This is a form of **covariance inflation** for the LETKF which we have completely under our control.
- Our transition probability $q(x, x')$ can be independent of x, i.e. we can completely employ the **LETKF posterior probability distribution**! But we need the ability to calculate $p(x) = p^{(b,LETKF)}(x)$. This is no problem when we have the estimate for $B^{(b,LETKF)}$.

The LMCPF Particle Filter

- We work in the **ensemble space**

$$U = \mathrm{span}\{x^{(1)}, ..., x^{(L)}\}, \quad x = \sum_{\ell=1}^{L} \alpha_\ell x^{(\ell)} \tag{37}$$

with the background ensemble propagated from the previous step.

- We might use a **localized** version of the space, i.e. we work around some point $p \in \mathbb{R}^3$.
- The term $p(y|x)$ can be calculated when we have observations in the same way as we do it for the LETKF.
- We need to calculate the **prior probability density** $p(x)$ for x given by (37) given the ensemble members $x^{(1)}, .., x^{(L)}$ many times in each analysis step of the LMCPF.

The LMCPF Particle Filter

- To **approximate** $p(x)$ we can employ several methods. A very general approach is to attach a Gaussian to each ensemble member, i.e. to use

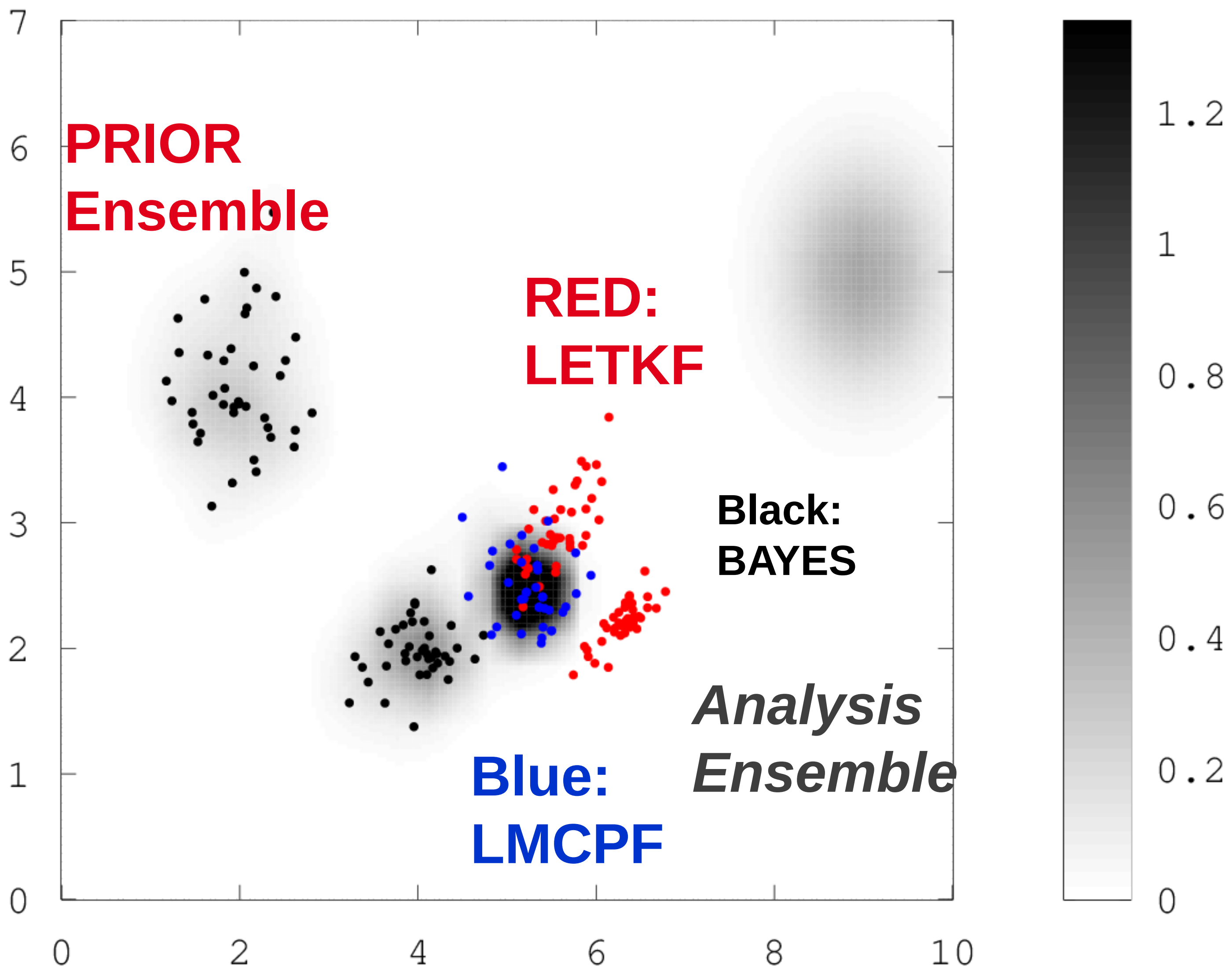
$$p(x) := \sum_{\ell=1}^{L} c_\ell e^{-\|x - x^{(\ell()}\| / b_\ell} \tag{38}$$

with constants b_ℓ and c_ℓ. Here, c_ℓ is chosen in dependence of b_ℓ such that the integral is equal to 1 for each of the Gaussian ensembles. The constants b_ℓ are **tuning constants** to get a best approximation of $p(x)$ based on $x^{(\ell)}$.

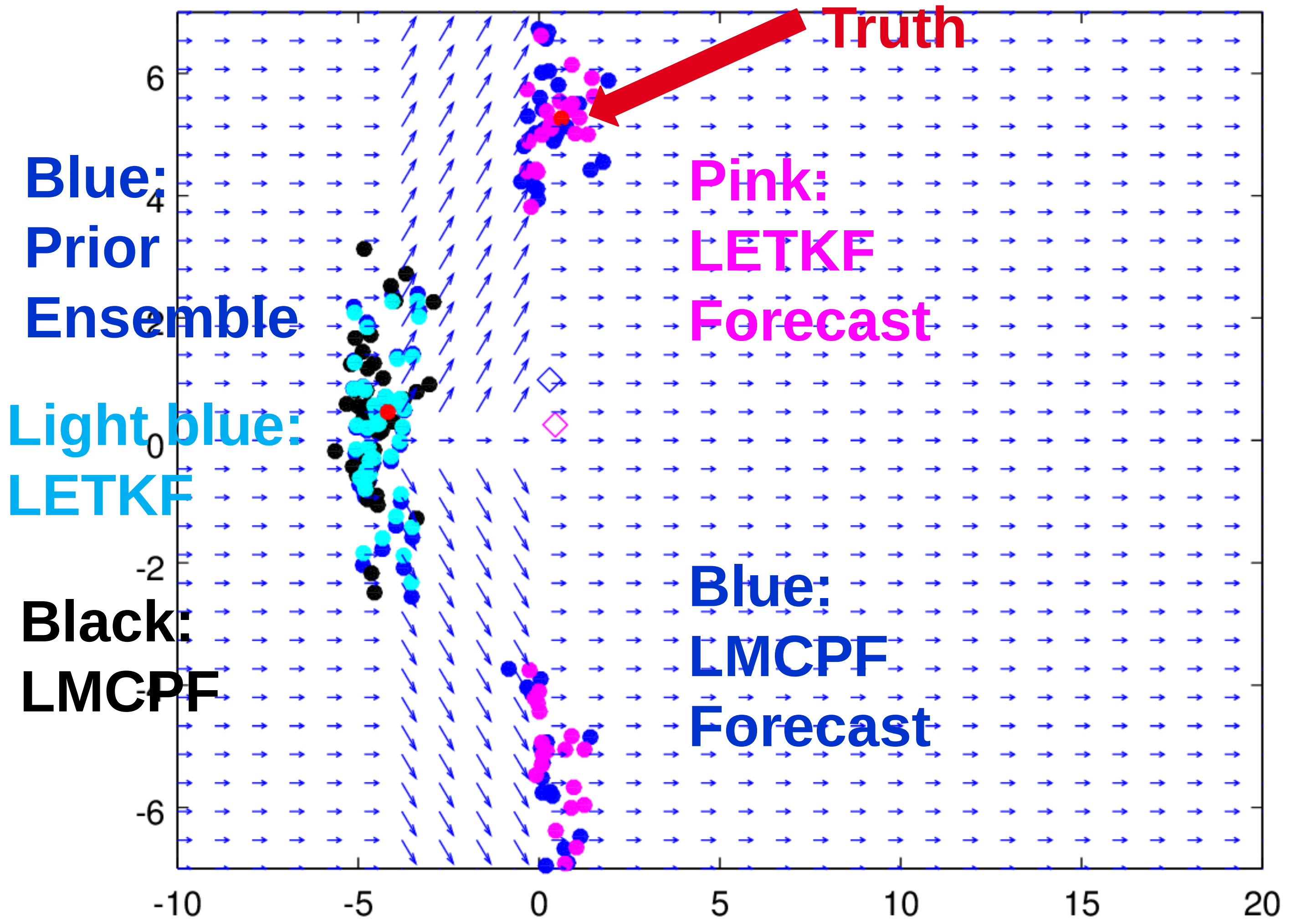
- *Some version of the* **Ensemble Kalman Filter** *can be obtained from the particular choice where we approximate $p(x)$ by a global Gaussian distribution!*

Particle Filter Example

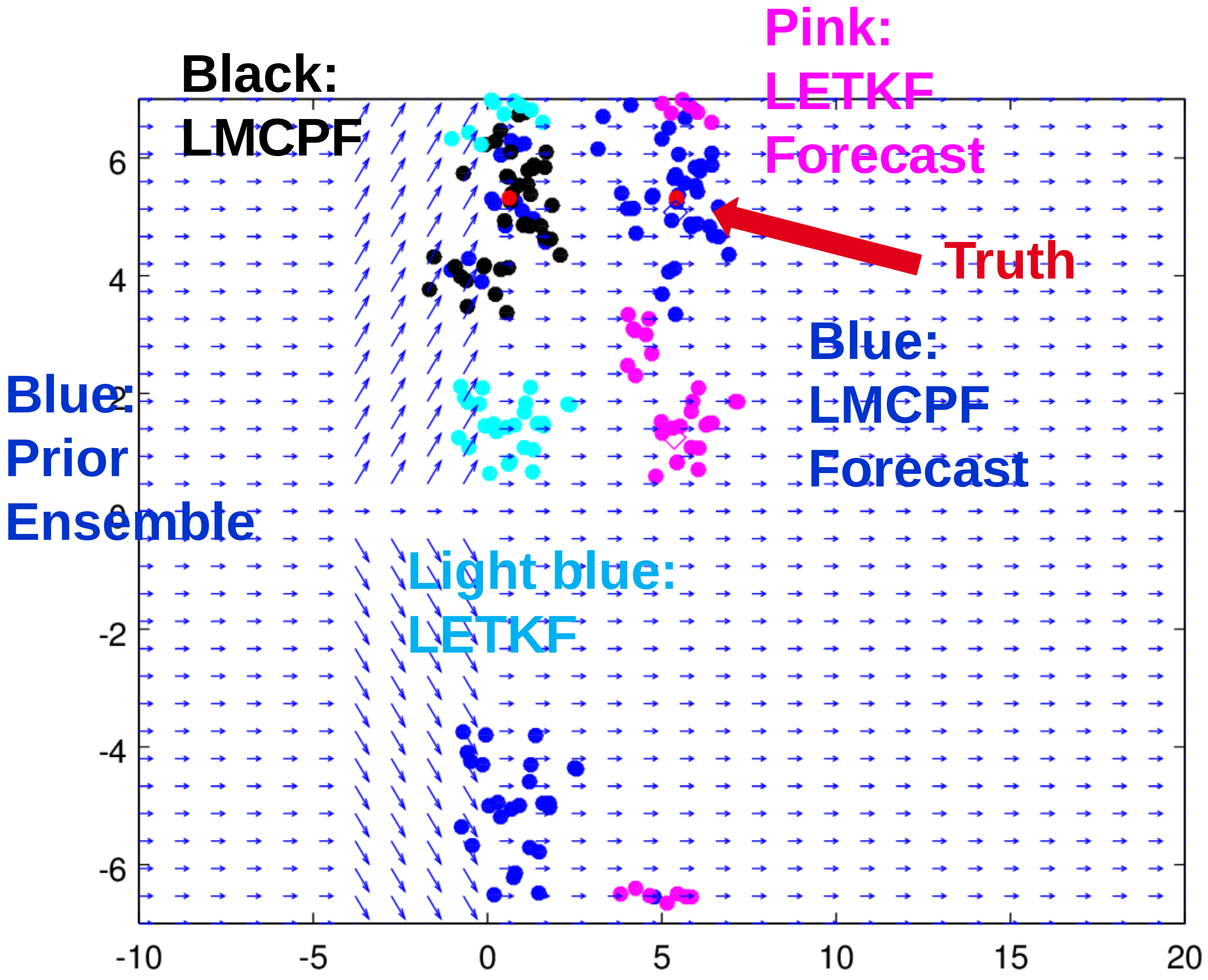
Deutscher Wetterdienst
Wetter und Klima aus einer Hand



LMCPF Example: nonlinear gate

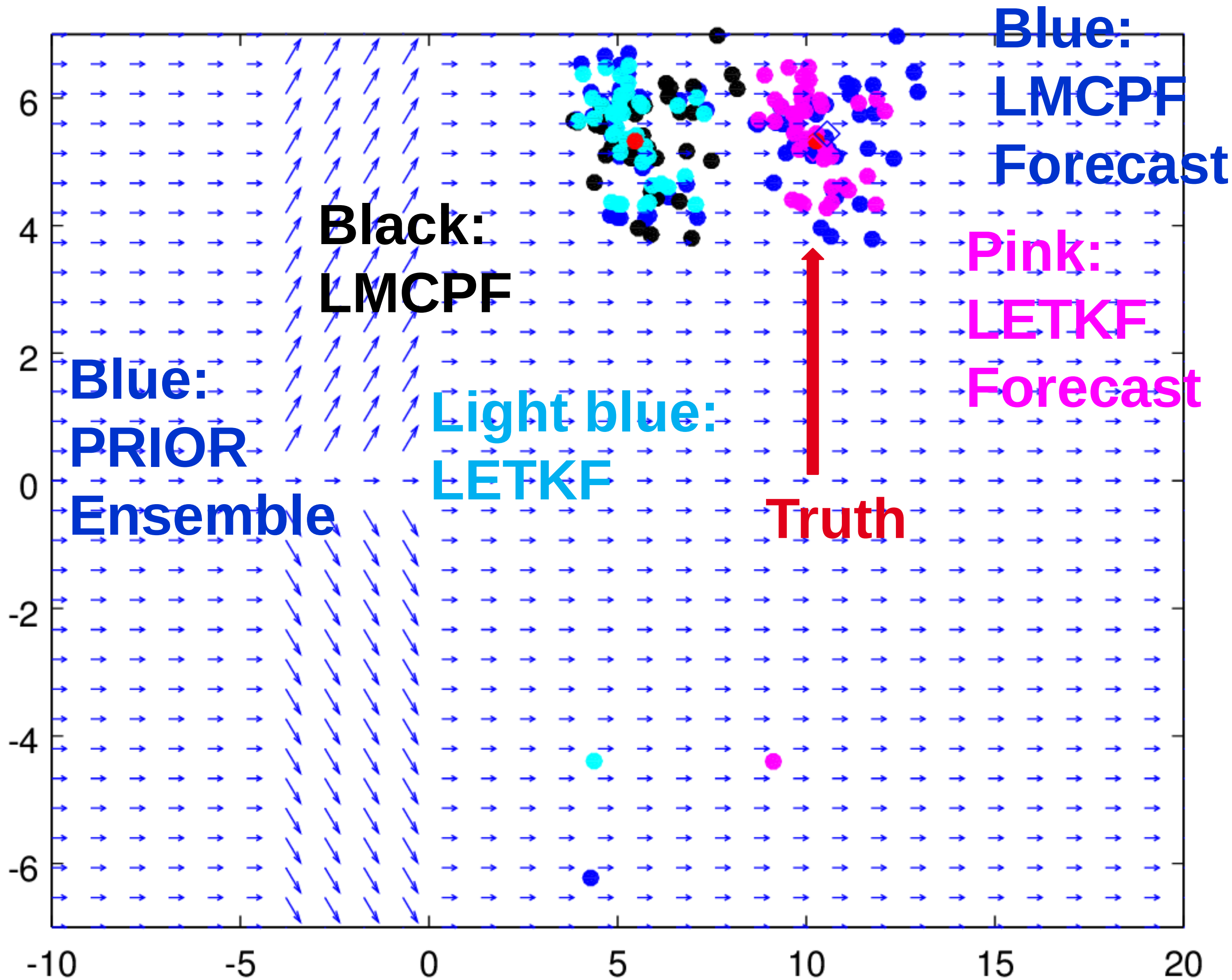


LMCPF Example: nonlinear gate



LMCPF Example: nonlinear gate

Deutscher Wetterdienst
Wetter und Klima aus einer Hand



Next Steps

- **LMCPF and**
  Ensemble Kalman Particle Filter Implementations for NWP

- **Further diagnostics on nonlinear dynamics and the behaviour of particle filters (simple statistics are not sufficient)**

- **Test different localization schemes**

- **Extensive testing for large-scale NWP systems.**

Many Thanks!