Deutscher Wetterdienst
Wetter und Klima aus einer Hand

Multi Scale VarEnKF Localisation using Wavelets

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1. DWD Global Forecast system
   - Global Data Assimilation Setup
2. VarEnKF Basics
   - Localisation
   - Transformed Representation
3. Wavelet Transformations
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   - Transformed Covariance Matrices
   - ICON Model - Icosahedral Grid with local Refinements
   - Localisation in Wavelet Representation

DWD Global Forecast system

- Operational setup:
  - 13 km ICON model (since 2014-01-20)
    provides boundary conditions to 7 km COSMO-EU
    provides boundary conditions to 2.8 km COSMO-DE
  - 3 hourly cycling 3D-Var
- Plan (until end 2015)
  - 7 km local refinement (2-way nesting)
    to replace current 7 km COSMO-EU
    provides boundary conditions to 2.8 km COSMO-DE
  - 40 km LETKF with 40 members
    provide boundary conditions to 2.8 km COSMO EnDA
    (40 member 2.8 km LETKF)
  - VarEnKF based on the 40 km LETKF

Related Talks and Posters

5-2 Global LETKF for ICON (Ana Fernandez del Rio)

P-8 Cyclone Haiyan Testcase (Ana Fernandez del Rio)

P-8 LETKF Humidity Cross Correlations (Dora Föhring)

4-2 Convective Scale LETKF for COSMO (Hendrik Reich)

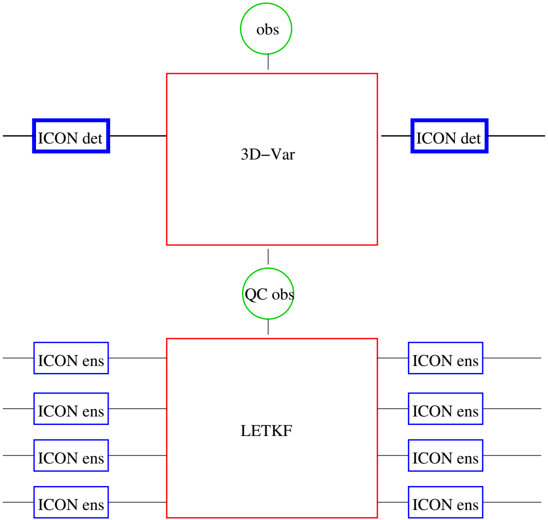
7-1 On Ensemble and Particle Filters (Roland Potthast)

here: Global VarEnKF

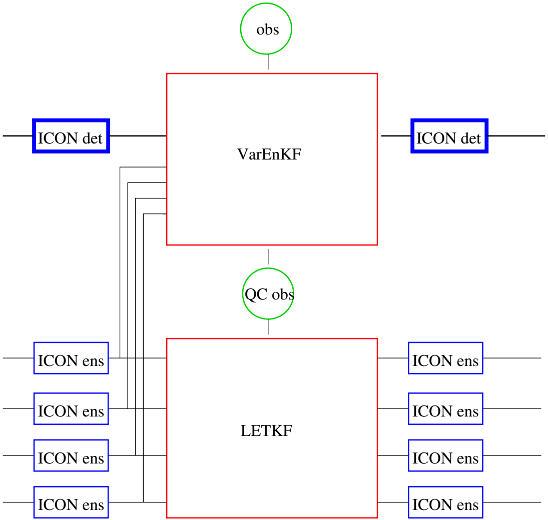
VarEnKF Status

- LETKF (Ana Fernandez del Rios talk)
  - \+ Tropics, SH, Humidity
  - – NH
- VarEnKF First preliminary results:
  - \+ Tropics, SH, Humidity
  - – NH
  - ▸ VarEnKF performs well where LETKF performs well
    - ★ Improve the LETKF first

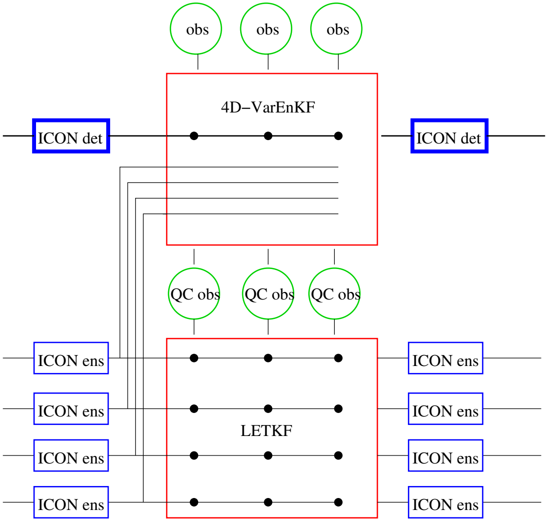
Global Data Assimilation System
3D-Var (operational) + LETKF (pre-operational)



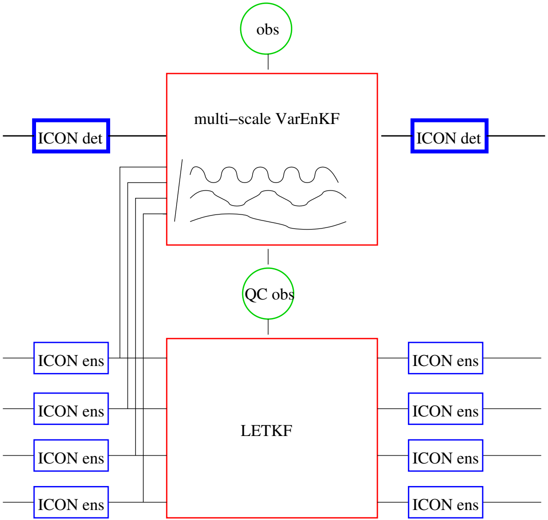
Global Data Assimilation System VarEnKF + LETKF



Global Data Assimilation System 4DVarEnKF + LETKF



Global Data Assimilation System
Multi-scale VarEnKF + LETKF



VarEnKF Basics

Idea: Use the (flow dependent) ensemble background error correlations in the deterministic variational analysis system.

$\mathbf{P}^{(b)}_{3DVar} \rightarrow \alpha\,\mathbf{P}^{(b)}_{NMC} + \beta\,\mathbf{P}^{(b)}_{EnKF}$

Goal:

Provide flow dependence to the 3D-Var $\mathbf{P}^{(b)}$

Representation:

$\mathbf{P}^{(b)}_{EnKF} = \mathbf{X}\mathbf{X}^T$ (sample covariance)

Localisation:

Required to suppress noise of the sample covariance matrix.

$\mathbf{P}^{(b)}_{EnKF} = \mathbf{C}_x \circ \mathbf{X}\mathbf{X}^T$

Schur produxt $\circ$ with localisation matrix $\mathbf{C}_x$

ensures that correlations become zero for distances $> r_{loc}$.

Multiscale Localisation

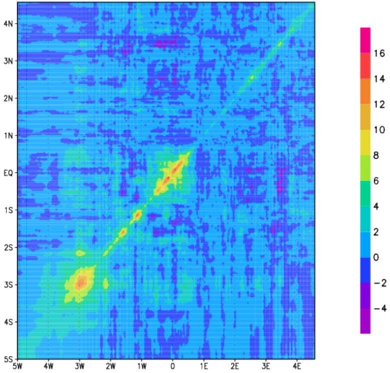
- Localisation length scales r_{loc} should be different for different situations:
  - rainy situations
  - no rain / high pressure situations

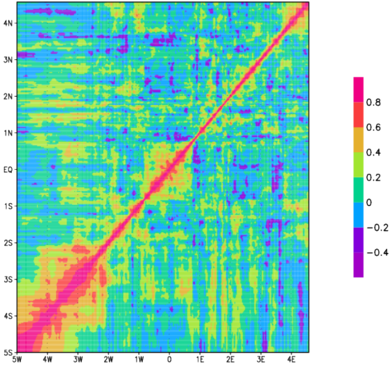
  - synoptic scale phenomena
  - smaller (convective) scale phenomena

  - in-situ observations
  - non-local observations
- May become important for grid refinement areas

Subsequent 1d-examples for raw and localised sample covariances are taken from the COSMO-DE (regular) 2.8 km grid for illustrative purposes

Raw Sample covariance and correlation matrix





Temperature covariances and correlations
along a latitude line (COSMO-DE level 29)
provided by a 40 member ensemble without localisation

Multiscale Localisation: HOWTO ?

- Localisation in physical space is an ad hoc approach:
  - $\mathbf{C}_x \circ \mathbf{X}\mathbf{X}^T$: Correlations at large distances are small.

Alternative: Localize in transformed representations:

$\mathbf{X} = Tr(\mathbf{Z}), \quad \mathbf{Z} = Tr^{-1}(\mathbf{X})$

- Localisation in spectral representation:
  - $\mathbf{C}_z \circ \mathbf{Z}\mathbf{Z}^T$: Correlations between different scales are small.
  - Corresponds to spatial smoothing of the correlation functions.
- Localisation in wavelet representation:
  - $\mathbf{C}_z \circ \mathbf{Z}\mathbf{Z}^T$: Correlations between different scales
    and at large distances (compared to the scale) are small.

Discrete Wavelet transformation (DWT)

- Fast hierarchical transform with operation count $O(n)$.

coefficients (λ) of the gridded function															
$\lambda_{4,1}$	$\lambda_{4,2}$	$\lambda_{4,3}$	$\lambda_{4,4}$	$\lambda_{4,5}$	$\lambda_{4,6}$	$\lambda_{4,7}$	$\lambda_{4,8}$	$\lambda_{4,9}$	$\lambda_{4,10}$	$\lambda_{4,11}$	$\lambda_{4,12}$	$\lambda_{4,13}$	$\lambda_{4,14}$	$\lambda_{4,15}$	$\lambda_{4,16}$
			low pass $\downarrow$								high pass $\downarrow$				
$\lambda_{3,1}$	$\lambda_{3,2}$	$\lambda_{3,3}$	$\lambda_{3,4}$	$\lambda_{3,5}$	$\lambda_{3,6}$	$\lambda_{3,7}$	$\lambda_{3,8}$	$\gamma_{3,1}$	$\gamma_{3,2}$	$\gamma_{3,3}$	$\gamma_{3,4}$	$\gamma_{3,5}$	$\gamma_{3,6}$	$\gamma_{3,7}$	$\gamma_{3,8}$
		low pass $\downarrow$				high pass $\downarrow$									
$\lambda_{2,1}$	$\lambda_{2,2}$	$\lambda_{2,3}$	$\lambda_{2,4}$	$\gamma_{2,1}$	$\gamma_{2,2}$	$\gamma_{2,3}$	$\gamma_{2,4}$	$\gamma_{3,1}$	$\gamma_{3,2}$	$\gamma_{3,3}$	$\gamma_{3,4}$	$\gamma_{3,5}$	$\gamma_{3,6}$	$\gamma_{3,7}$	$\gamma_{3,8}$
	low pass $\downarrow$		high pass $\downarrow$												
$\lambda_{1,1}$	$\lambda_{1,2}$	$\gamma_{1,1}$	$\gamma_{1,2}$	$\gamma_{2,1}$	$\gamma_{2,2}$	$\gamma_{2,3}$	$\gamma_{2,4}$	$\gamma_{3,1}$	$\gamma_{3,2}$	$\gamma_{3,3}$	$\gamma_{3,4}$	$\gamma_{3,5}$	$\gamma_{3,6}$	$\gamma_{3,7}$	$\gamma_{3,8}$
low $\downarrow$	high $\downarrow$														
$\lambda_{0,1}$	$\gamma_{0,1}$	$\gamma_{1,1}$	$\gamma_{1,2}$	$\gamma_{2,1}$	$\gamma_{2,2}$	$\gamma_{2,3}$	$\gamma_{2,4}$	$\gamma_{3,1}$	$\gamma_{3,2}$	$\gamma_{3,3}$	$\gamma_{3,4}$	$\gamma_{3,5}$	$\gamma_{3,6}$	$\gamma_{3,7}$	$\gamma_{3,8}$
wavelet (γ) and scaling function (λ) coefficients															

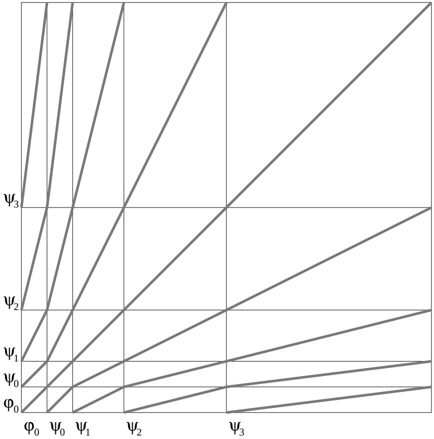
- Coefficients of the transformed vector correspond to the average value (λ) and to the deviations (γ) on different scales.
- Inverse transform is a fast hierarchical transform as well.

Wavelet transformed covariance matrices

- Wavelet transformations apply independently to each row and column of a matrix.
- Covariances of phenomena at the same scale are represented by the diagonal blocks.

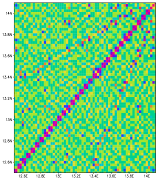
  Covariances of phenomena at different scale are represented by off-diagonal blocks.
- Covariances of phenomena at nearby locations are represented by coefficients in the vicinity of the 'branches' (diagonals of the blocks).

  Only these coefficients are considerably different from zero.

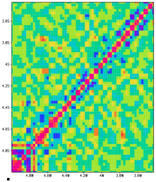


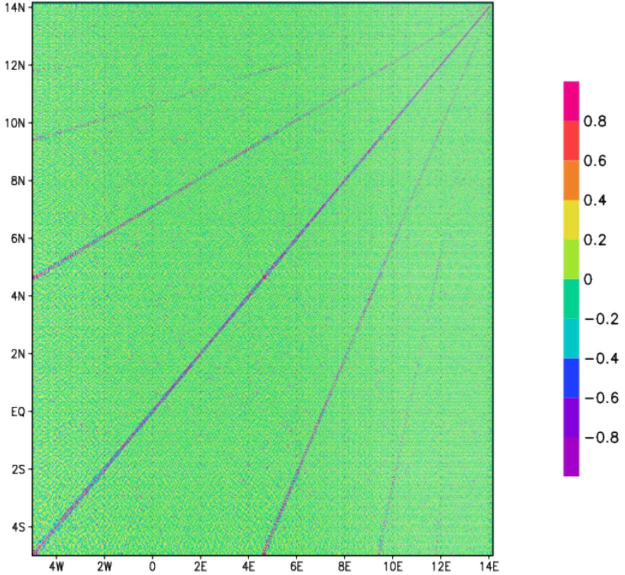
Block structure of wavelet transformed matrices

Transformed Covariance Matrices



Sample correlation matrix in wavelet representation





Characteristics of Wavelet Transformations

The wavelet transform is not unique (as FFT) but many different choices for low pass and high pass filter functions exist.

Different desiriable properties cannot be achieved at the same time:

- Compact support (for fast transform)
- Smoothness (for scale separation)
- Orthogonality or bi-orthogonality

Characteristics of Wavelet Transformations

Our Choice (so far)

- Use Frames:
  Number of variables in transformed representation is larger.
  allows smoother basis functions.
- Use the Lifting Scheme:
  allows to define wavelets with desired properties on unstructured grids,
  splits the wavelet transform into a sequence of simple operations
  which can easyly be inverted and transposed.

Lifting Scheme / Frame

This kind of wavelet transform reduces to a very simple hierarchical algorithm:

$\mathbf{W}^{-1}$ Analysis:

1 Low pass filter: Average the field (to a coarser grid)
2 High pass filter: Re-interpolate the field to the fine grid and
3 subtract the result from the original one to get the low high pass filtered component.

Iterate the algorithm with the low pass filtered field.

$\mathbf{W}$ Synthesis:

2 Interpolate the field to the fine grid

$\mathbf{W}^T$ Adjoint Synthesis:

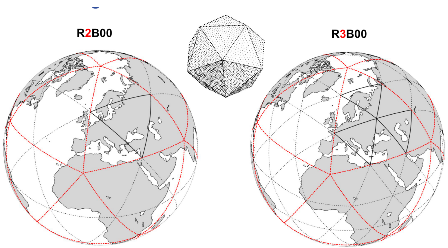
2 Adjoint of the interpolation step

This algorithm can be easyly applied to the ICON grid which is generated by a hierachy of grid refinements.

ICON Model - Icosahedral Grid

- developed by DWD, MPI for Meteorology
- main goals:
  - Unified modelling system for NWP and climate prediction
  - better conservation properties
  - flexible grid nesting in order to cover global and regional scale
  - nonhydrostatic dynamical core for capability of seamless prediction
  - scalability and efficiency on multi-core systems

Grid generation is based on the icosahedron (unstructured grid):



ICON Model - Icosahedral Grid with local Refinements

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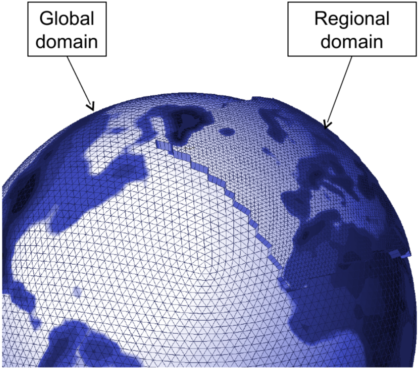
Effective grid spacing (distance between grid points):

$\Delta x \approx \frac{5050}{n\,2^k}$ km

Example:
R3B7: n=3, k=7

1st subdivision by factor of 3
7 subdivisions by factor of 2

Grid spacing 13: km



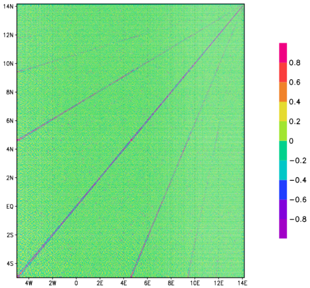
Deterministic forecast will run with 13 km resolution
Local refinement will run with 6.5 km
LETKF will run with 40 km

Localisation in Wavelet Representation

Goal: only keep the entries close to the diagonals and the 'branches' of the wavelet transformed correlation matrix:

- Close to the diagonals:
  spatial correlations between phenomena of the same scale at slightly different locations.
  - localise as usual with (now scale dependent) prescribed localisation radius.
- On the branches: correlations between different scales (at the same location)
  - perform a similar localisation in between scales

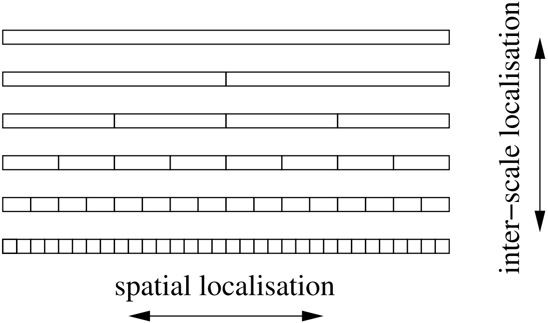
Localisation in the 2 'directions' seperate (with some constraints).



Localisation in Wavelet Representation

Different View:

Wavelet transformation can pe regarded as a process which adds a new (scale)-dimension to the existent spatial dimensions:



Localisation in the 2 'directions' seperate (with some constraints).

Localisation in Spatial Representation: Operator Approach

$\mathbf{C} \circ \mathbf{X}\mathbf{X}^T$ may be written as:

$\hat{\mathbf{X}}\,\mathbf{L}_v\,\mathbf{L}_h\,\mathbf{L}_h{}^T\,\mathbf{L}_v{}^T\,\hat{\mathbf{X}}^T$

with:

$\hat{\mathbf{X}}$	Vector of diagonal matrices, consisting of ensemble deviations
$\mathbf{L}_v$	Factorisation (root) of vertical localisation matrix $\mathbf{C}_v$ lhs: model grid rhs: coarse vertical grid
$\mathbf{L}_h$	Factorisation (root) of horizontal localisation matrix $\mathbf{C}_h$ lhs: coarse vertical grid rhs: coarse vertical & horizontal grid

Localisation in Wavelet Representation: Operator Approach

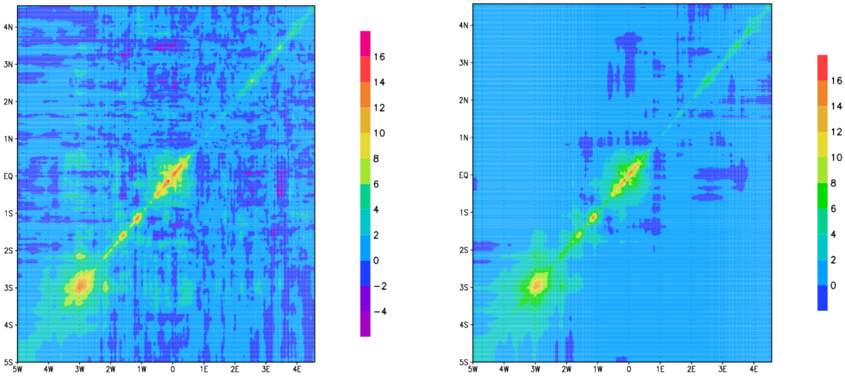
$\mathbf{C} \circ \mathbf{X}\mathbf{X}^T$ may be written as:

$$\mathbf{W}_v \, \mathbf{W}_h \, \hat{\mathbf{Z}} \, \mathbf{L}_s \, \mathbf{L}_v \, \mathbf{L}_h \, \mathbf{L}_h{}^T \, \mathbf{L}_v{}^T \, \mathbf{L}_s{}^T \, \hat{\mathbf{Z}}^T \, \mathbf{W}_h{}^T \, \mathbf{W}_v{}^T$$

with:

$\mathbf{W}_v$	Vertical wavelet transform
$\mathbf{W}_h$	Horizontal wavelet transform
$\hat{\mathbf{Z}}$	Vector of diagonal matrices, consisting of wavelet transformed ensemble deviations $\mathbf{Z} = \mathbf{W}^{-1}\mathbf{X}$
$\mathbf{L}_s$	Factorisation (root) of inter-scale localisation matrix $\mathbf{C}_s$
$\mathbf{L}_v$	Factorisation (root) of vertical localisation matrix $\mathbf{C}_v$
$\mathbf{L}_h$	Factorisation (root) of horizontal localisation matrix $\mathbf{C}_h$

Sample Covariance Localised in Wavelet Representation



Temperature covariances and correlations

left: Raw sample

right: reconstructed after localisation in wavelet representation

Outlook

- The method is half way implemented

Thank you for your attention