The impact of different model error covariance matrices on the performance of a particle filter applied to a coupled ocean-atmosphere climate model

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Proposal densities

$$p(x \mid y) = \frac{p(x)p(y \mid x)}{p(y)}$$

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$$p(x \mid y) = \frac{p(x)p(y \mid x)q(x,y,z)}{p(y)q(x,y,z)}$$

The freedom in the choice of q is how we design methods to avoid the curse of dimensionality.

Basic Particle Filter

Equivalent weights particle filter

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A fully nonlinear particle filter which has not been found to suffer from filter degeneracy. See van Leeuwen [2010] and Ades and van Leeuwen [2012].

Required:

- Observations y with error covariance matrix R
- (linear) observation operator H
- 2 tuning parameters
- Model *error* covariance matrix Q

HadCM3

Coupled Ocean-Atmosphere GCM [Gordon et al., 2000].
Total of 9 prognostic variables:

Atmosphere

Zonal and meridional winds, temperature, humidity and surface pressure
resolution 2.5° with 19 levels.

Ocean

Zonal and meridional flow, temperature and salinity
Resolution 1.25° with 20 levels.

State vector dimension $2,314,430$.

Twin experiment

- Assimilate daily SST data over a period of 6 months
- Full coverage of SSTs – $27,370$ variables
- 72 model timesteps between observations
- R diagonal

The big question – modelling Q.

$$\mathrm{d}x = f(x)\mathrm{d}t + \mathrm{d}\beta$$

where

$$\mathrm{d}\beta \sim \mathcal{N}(0, Q)$$

- Can always make improvements here

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- We estimate Q by looking at a long model run
- 3D, staggered grids, multiple variables, bathymytry, real data $\implies$ no elegant way of doing matrix-vector multiplication with Q
- Need for sparse BLAS routines to make efficient

Stencil

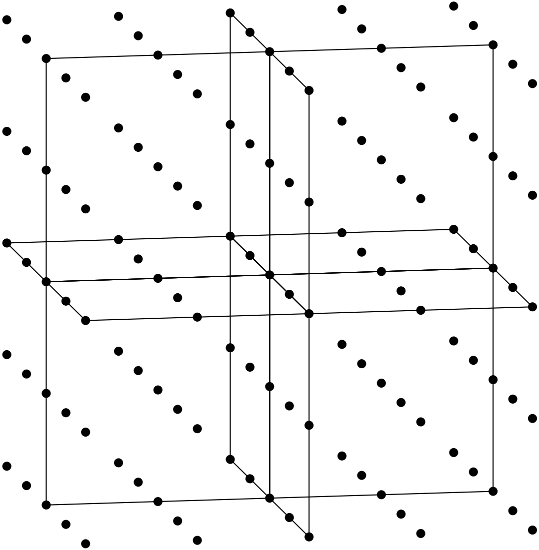

Figure : Stencil for localisation matrix

Stencil

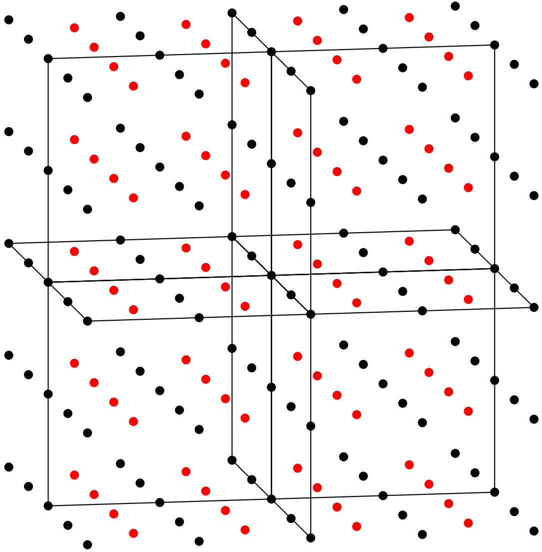

Figure : Stencil for localisation matrix

Matrix properties

- Symmetric so stored in upper triangular form
- N = 2,314,430
- NNZ = 411,266,537
- up to 1000 non-zeros per row.
- average of 354 non-zeros per row.
- 16Gb on disk.
- load Q took 228.75 seconds

Matrix-vector multiplication timings

- Serial coord_matmul_t: 1.05s
- Parallel openmp_matmul_t (24 threads):
- SPARSE BLAS matmul (12 threads):

LIBRSB (Recursive sparse blocks)Martone et al. [2010]

Matrix-vector multiplication timings

- Serial coord_matmul_t: 1.05s
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Matrix-vector multiplication timings

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LIBRSB (Recursive sparse blocks)Martone et al. [2010]

Two different Q matrices

We estimate Q from a long model run.

$$\mathsf{Cov}(X_1) = \Lambda_1^{1/2}\Sigma_1\Lambda_1^{1/2}$$
$$Q_1 \propto \Lambda_1^{1/2}\Sigma_1^2\Lambda_1^{1/2}.$$

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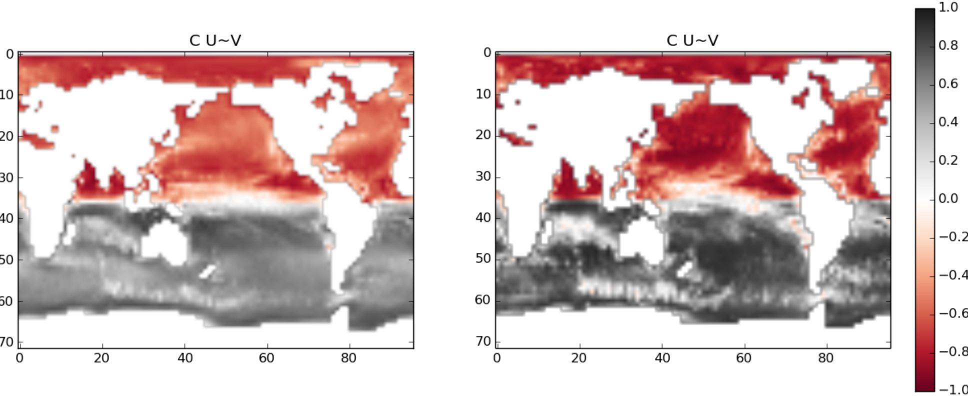
$$X_2 := X_1 - \bar{X_1}$$

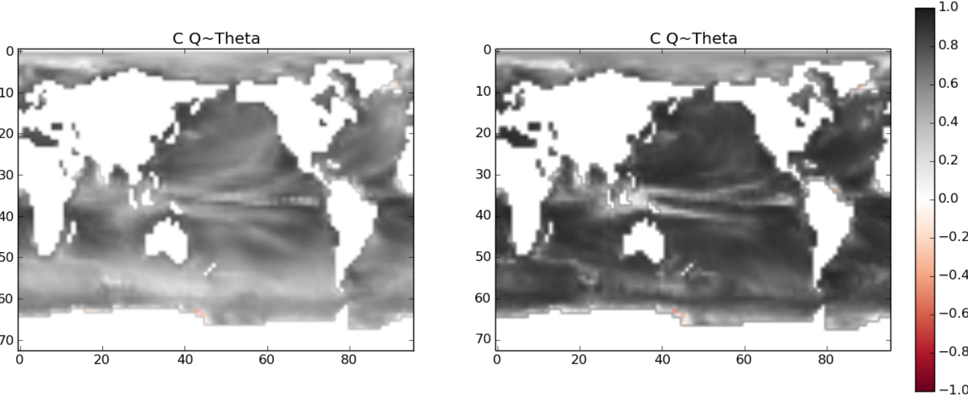
$$\mathsf{Cov}(X_2) = \Lambda_2^{1/2}\Sigma_2\Lambda_2^{1/2}$$

$$Q_2 \propto \Lambda_2^{1/2}\Sigma_2^2\Lambda_2^{1/2}.$$



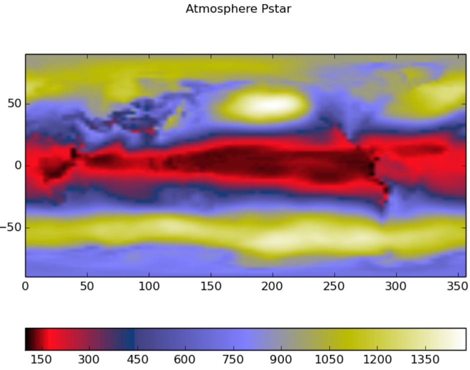
Q_1 Q_2

Correlations between atmospheric humidity and ocean θ

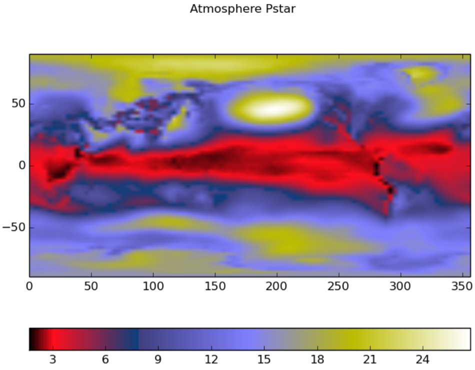


Q_1 Q_2

Atmospheric pressure variables in $\Lambda^{\frac{1}{2}}$

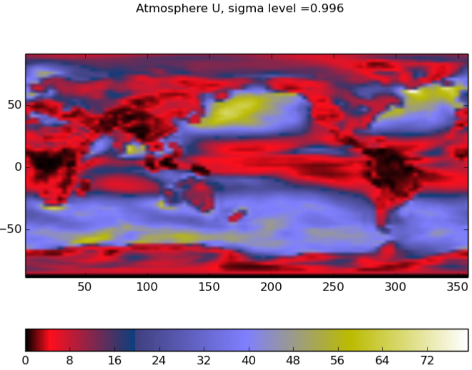


Q_1

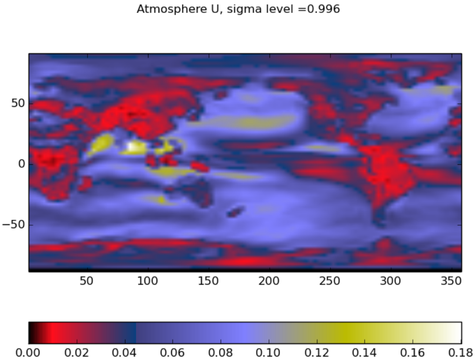


Q_2

Eastward surface winds in $\Lambda^{\frac{1}{2}}$

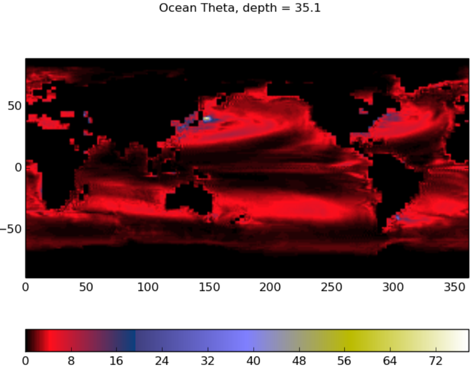


Q_1

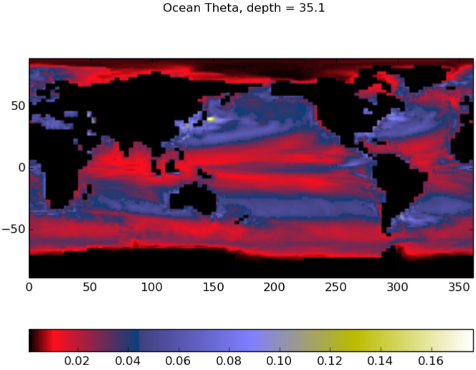


Q_2

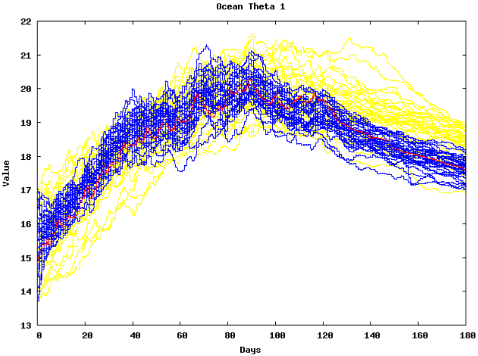
Sea water temperature at the 4^{th} depth level in $\Lambda^{\frac{1}{2}}$



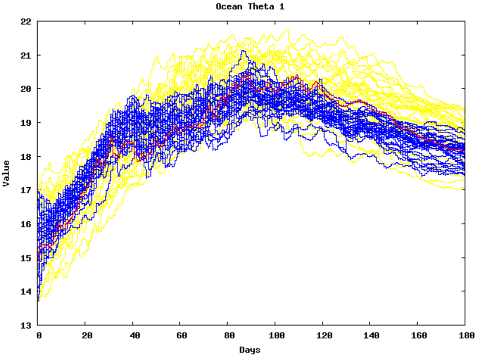
Q_1



Q_2


Ocean Theta 1
Value
Days
22
21
20
19
18
17
16
15
14
13
0
20
40
60
80
100
120
140
160
180

Q_1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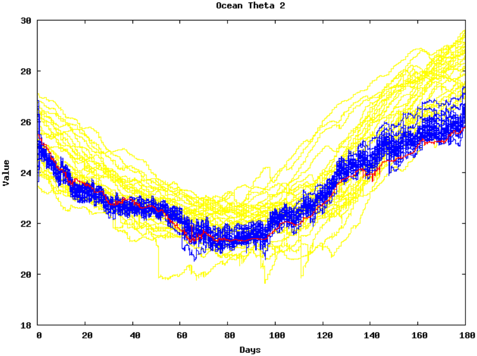
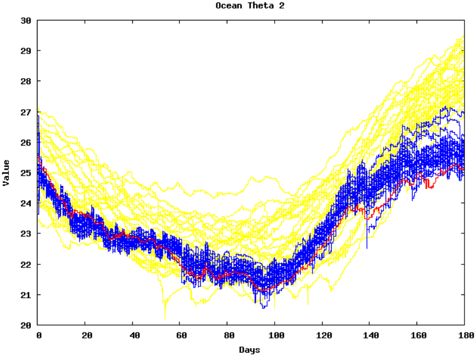
Q_2

Trajectories

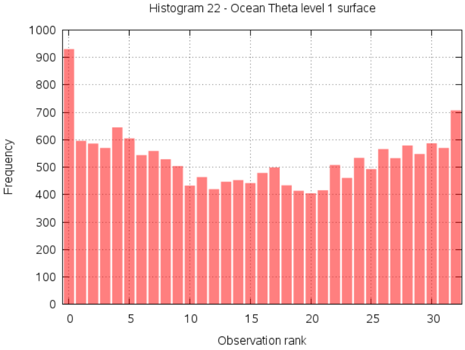


Q_1

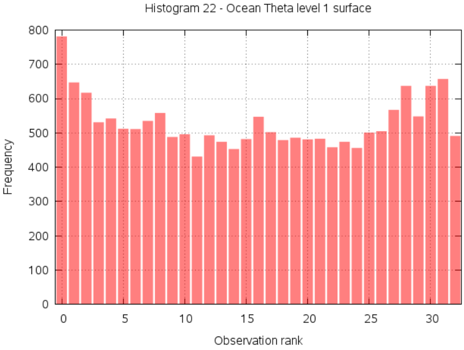


Q_2

Rank histogram of observed SSTs

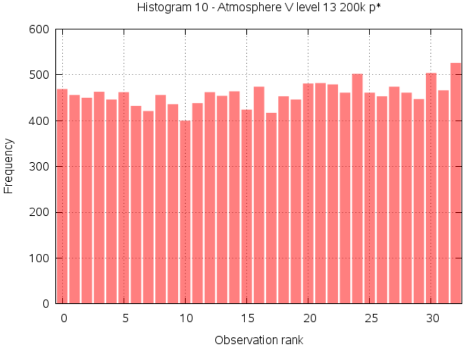


Q_1

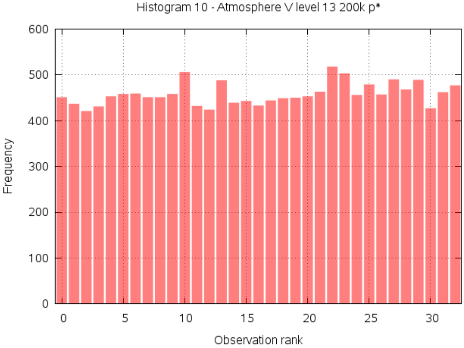


Q_2

Rank histogram of atmospheric zonal velocities

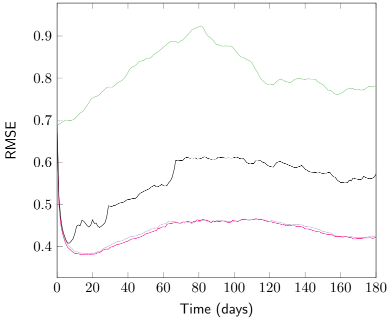


Q_1

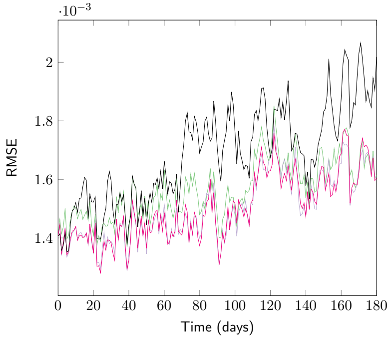


Q_2

Observed SSTs



Unobserved Surface Humidity



Conclusions and future work

Conclusions

- The equivalent weights particle filter continues to show no signs of filter degeneracy when the model has size 2.3×10^6
- Having a more physically realistic Q matrix has allowed much smaller scaling to be used and appears more robust
- We have only considered twin experiments – what will Q look like in the real system?
- New ideas and lots of research into Q is necessary for both particle filters and Weak Constrained methods.

Thank you for listening

References

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